Some Fundamental Limitations for Cognitive Radio

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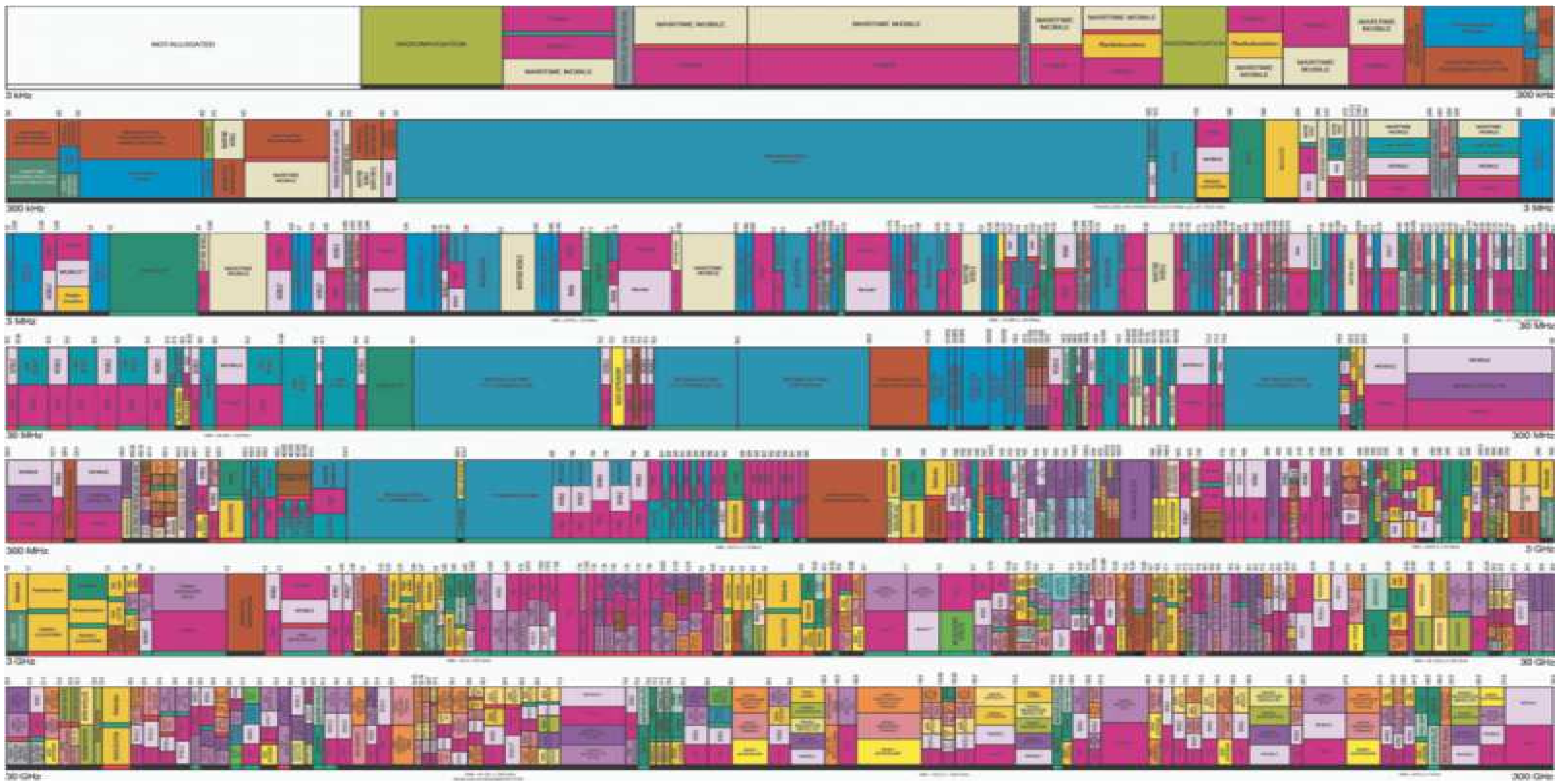
Outline

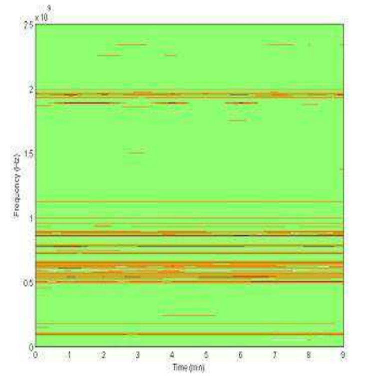
1. Why cognitive radios?
2. Fundamental need to detect very weak primary signals
3. Knowledge of modulation does not help but knowledge of pilot signals does
4. Receiver uncertainty and quantization's impact on detection
5. Conclusions

Apparent spectrum allocations



- Traditional spectrum allocation picture
- Apparent spectrum scarcity

Apparent spectrum usage



Actual measurements show that $< 20\%$ of spectrum is used, but:

- Some users listen for very weak signals
  - GPS
  - Weather radar and remote sensing
  - Radio astronomy
  - Satellite communications
- Spectrum use can vary with space and time on all scales.

Cognitive radio justification

Wireless interference is primarily a local phenomenon.

"If a radio system transmits in a band and nobody else is listening, does it cause interference?"

Ambitious Goal

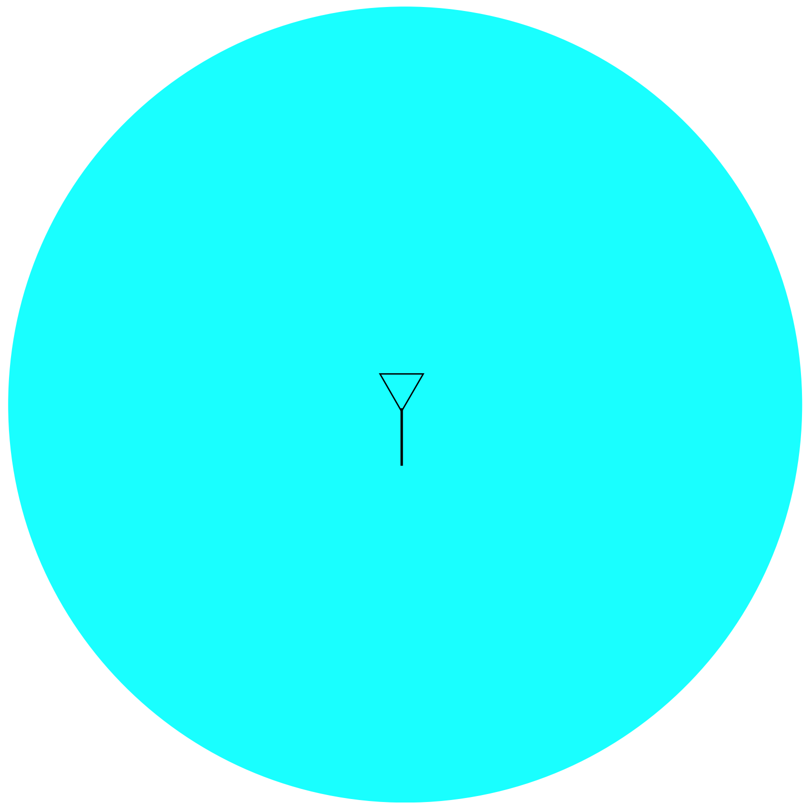
Would like to take advantage of plentiful spectrum without requiring a lot of regulation or assumed coordination among users.

Objectives

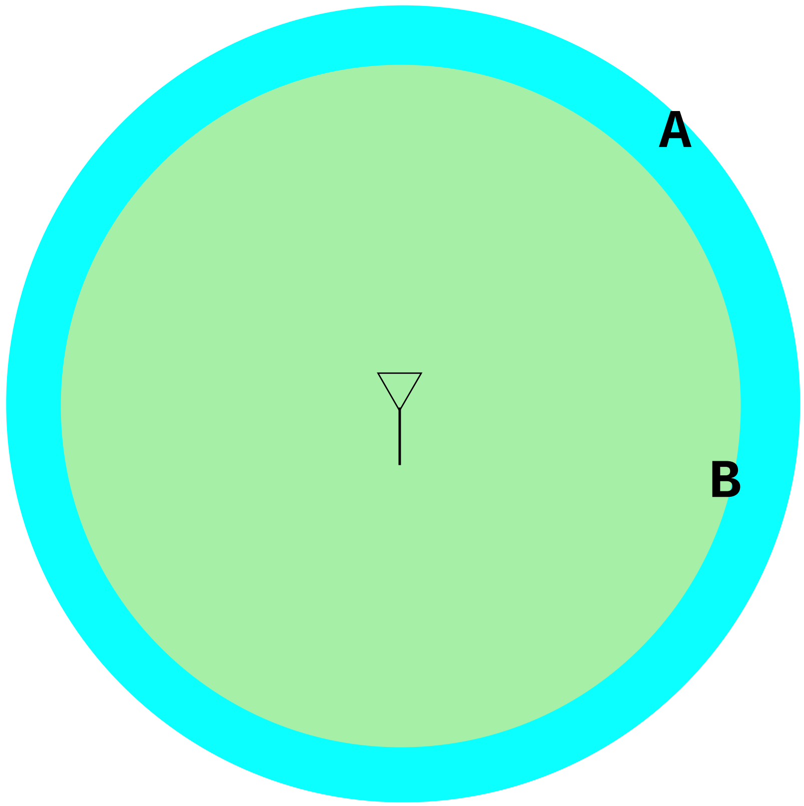
- **Protect primary users of the spectrum**
  - Socially important services may deserve priority on band
  - Legacy systems may not be able to change
- Allow for secondary users to use otherwise unused bands
  - Not the UWB approach: "speak softly but use a wide band"
  - May have to coordinate/coexist with other secondary users

See what happens for the case of a single secondary user first.

Primary decodability region

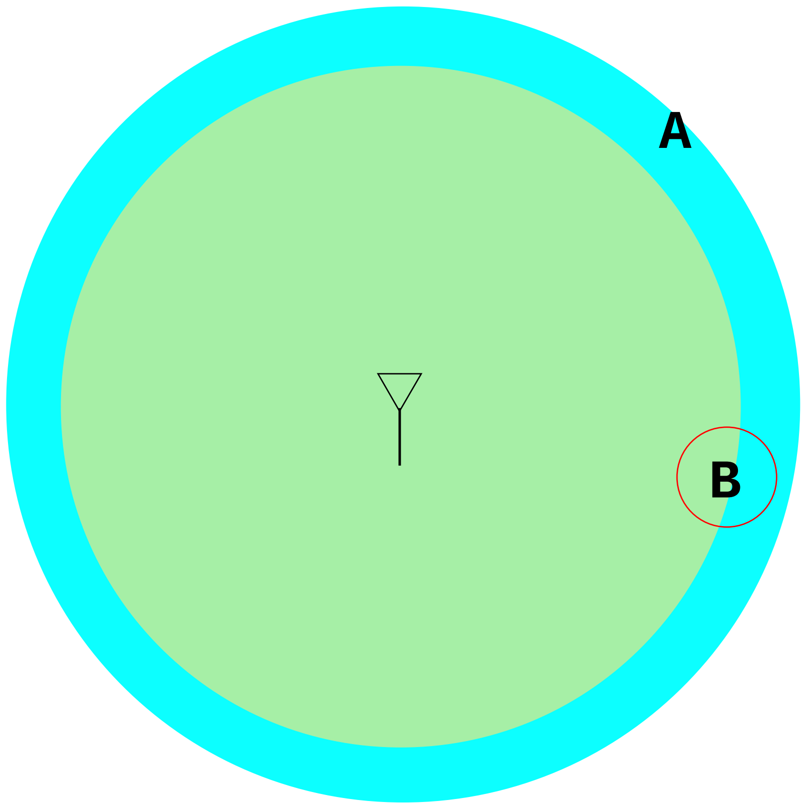


Primary protected region



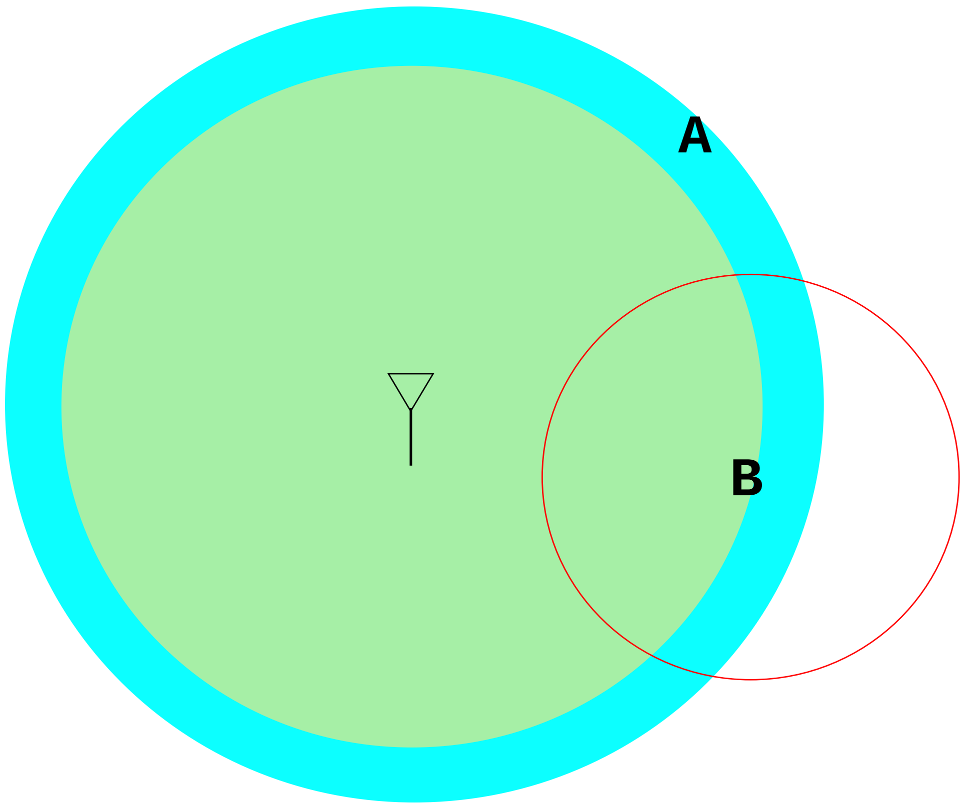
Users at the very edge of the currently decodable region will do worse under any changes to the exclusive-use model.

“No talk” zones



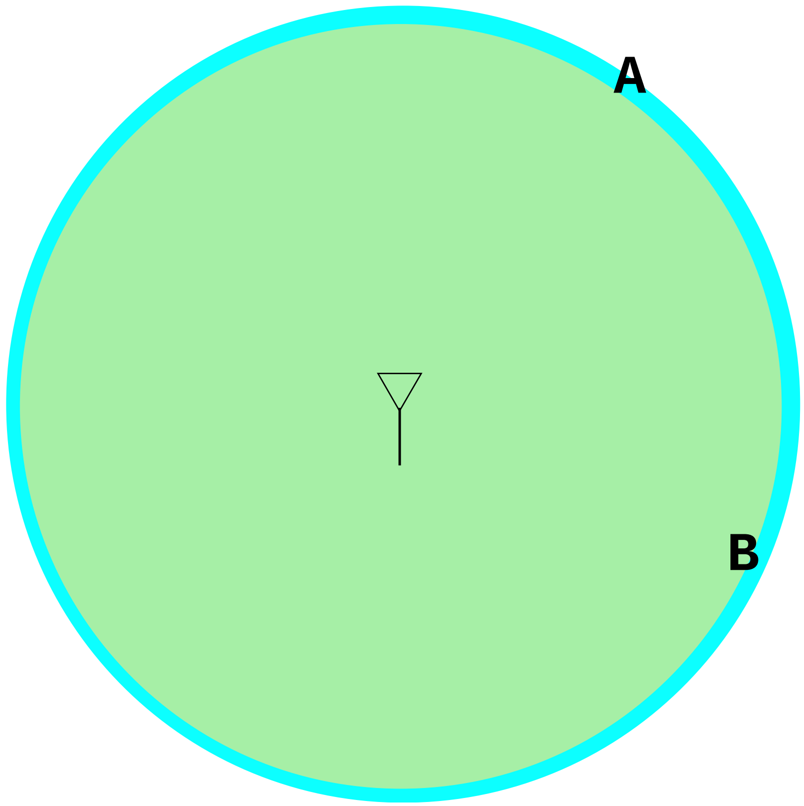
Mice can get close...

“No talk” zones

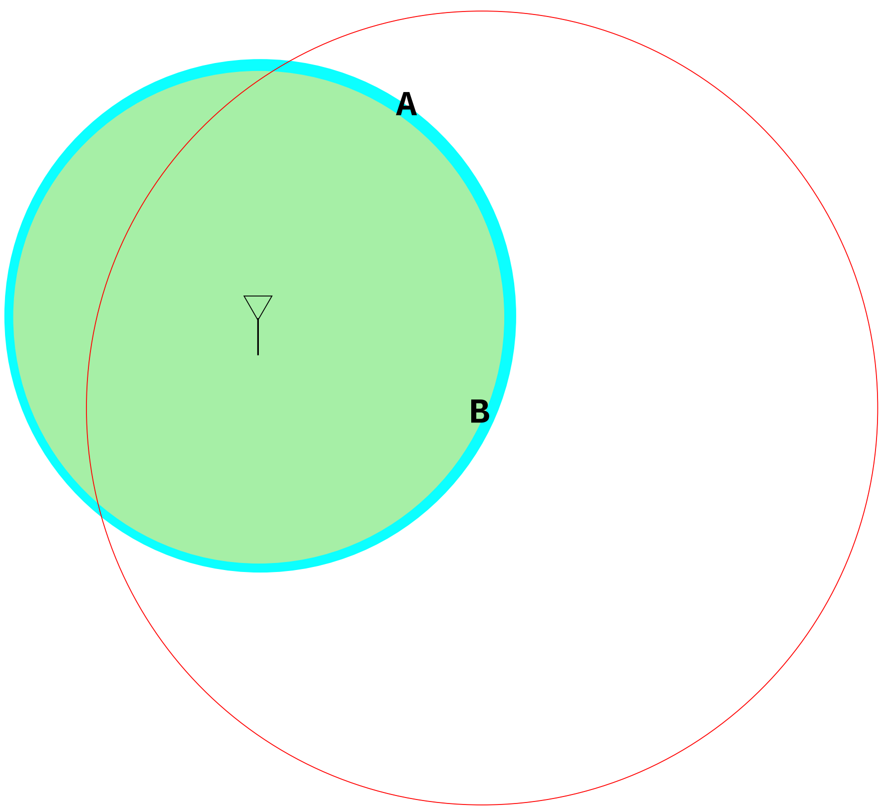


But keep the lions far away!

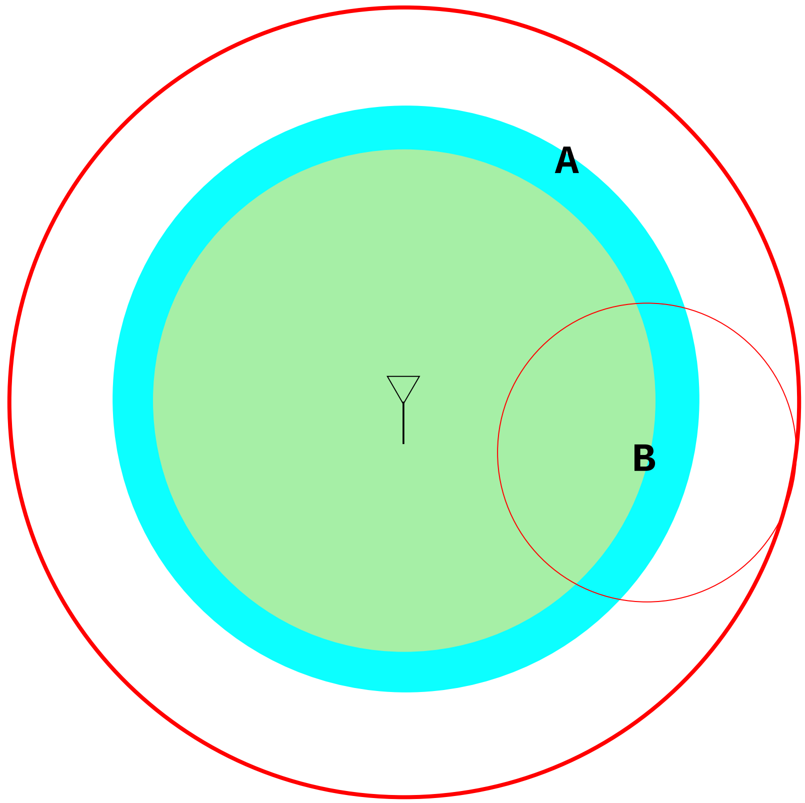
If the protected region nears the decodability bound...



The "no talk" zones grow dramatically



Union of “no talk” zones



Need to be able to detect an undecodable transmission if either the protected radius or the allowed secondary power is large.

Fundamental Tradeoffs

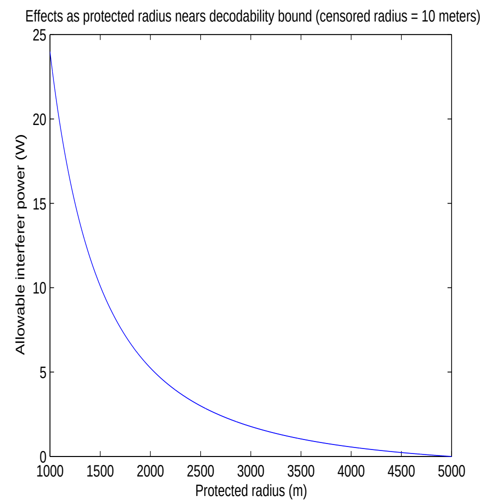
Assume $\frac{1}{d^2}$ attenuation of signals:

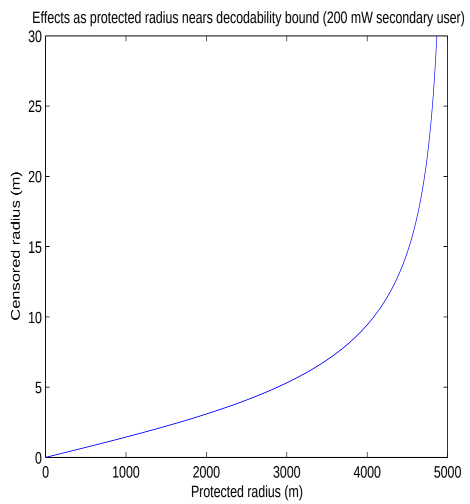
$$r_{censored} = \sqrt{\frac{P_s}{\frac{P_p}{(2^{2R}-1)r_{protected}^2} - \sigma^2}}$$

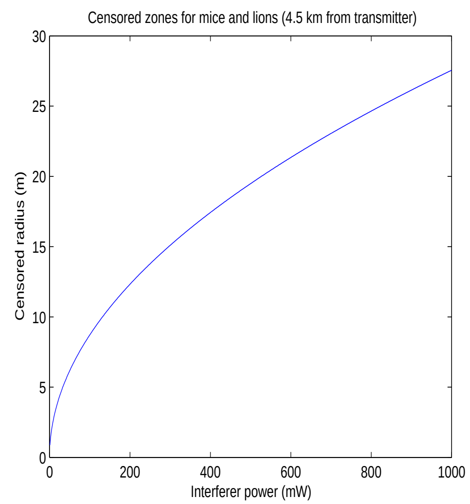
$$P_s = r_{censored}^2 \left[\frac{P_p}{(2^{2R}-1)r_{protected}^2} - \sigma^2\right]$$

- To allow secondary power to increase, we must be able to detect weak primary signals in order to protect primary receivers from interference.
- To protect primary receivers with already marginal reception, the censored radius must grow and so even weaker primary signals must be able to be detected.

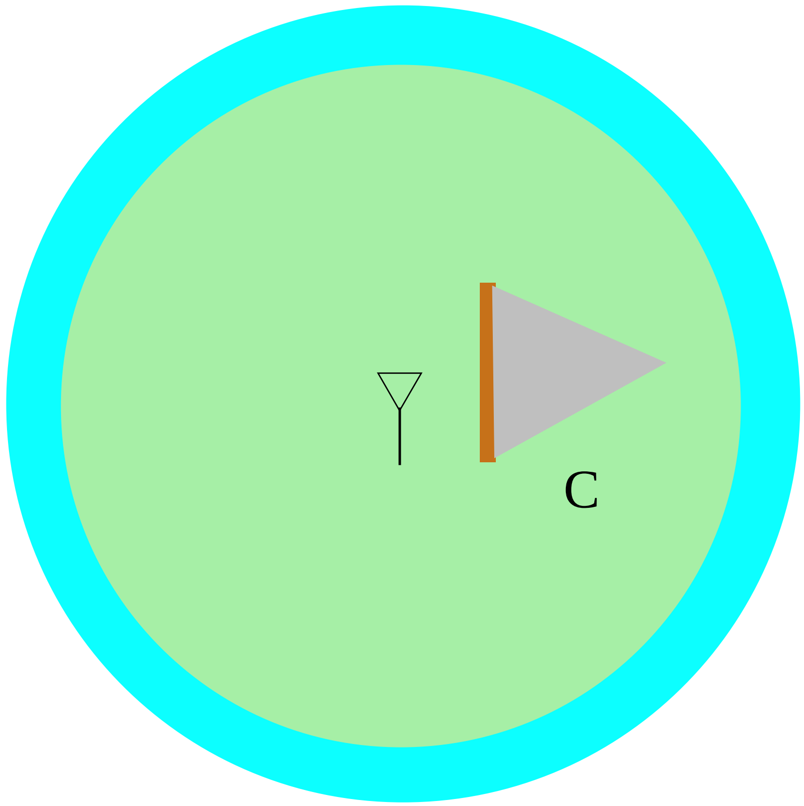
Censored radius vs. interferer power and protected radius





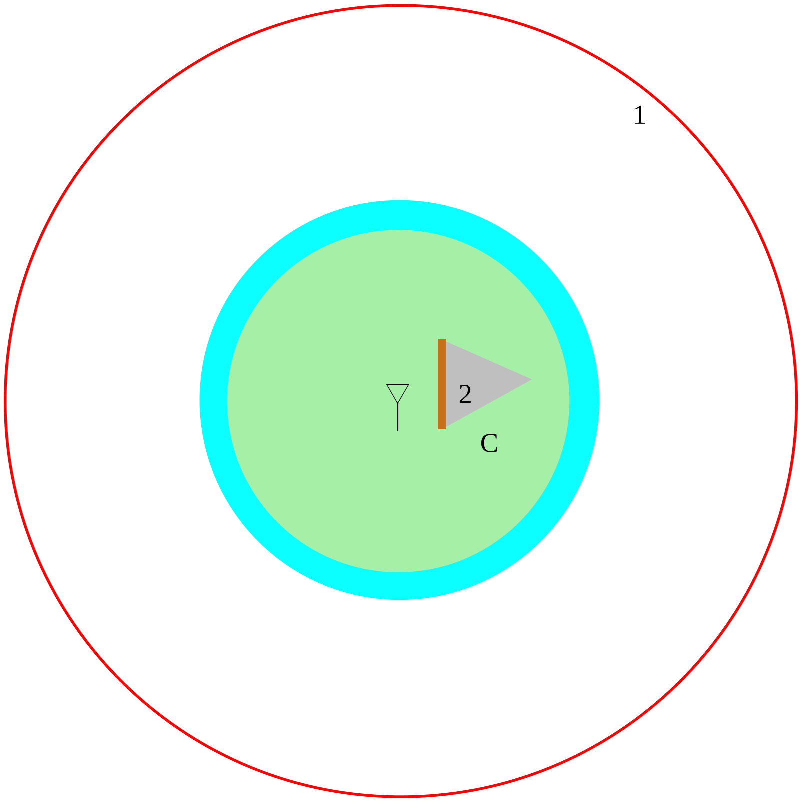


Shadowing



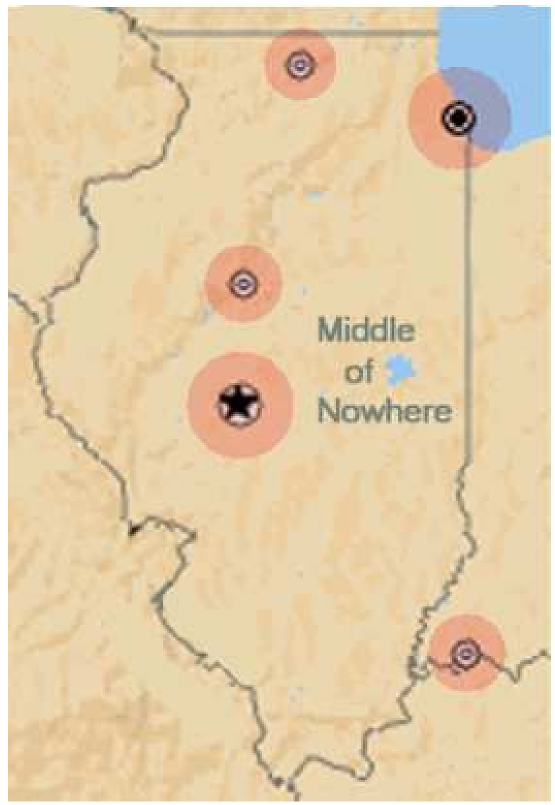
A secondary user might be in a local shadow while his transmissions could still reach an unshadowed primary receiver.

Shadowing



Secondary user can not distinguish between positions (1) and (2) — must be quiet in both: **must detect even weaker primary signals.**

Cognitive radio is still potentially useful



Even while protecting primary users, a large geographic area may still be available for secondary users in any given band.

Lessons so far

- “Don’t transmit if you can decode” is a poor rule
- Could do much better if we could detect undecodable signals
  - Better protect primary users
  - Allow longer range and higher rate secondary uses
- How hard is this?

Model

- Hypothesis testing problem: is the primary signal out there?

$$\mathcal{H}_0 : Y[n] = W[n]$$
$$\mathcal{H}_s : Y[n] = W[n] + x[n]$$

- Moderate P_{fa}, P_{md} targets.
- Potentially very low SNR at the detector: will need many samples to distinguish hypotheses.
- Proxy for difficulty: How long must we listen?

Assume perfect knowledge

- $x[n]$ known exactly at receiver
- Optimal detector is a matched filter

$$\sum_{k=1}^{N} y[n] \frac{x[n]}{\|x\|} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_s}{\gtrless}} \frac{\|x\|}{2}$$

The power of processing gain: we only require $O(1/SNR)$ samples

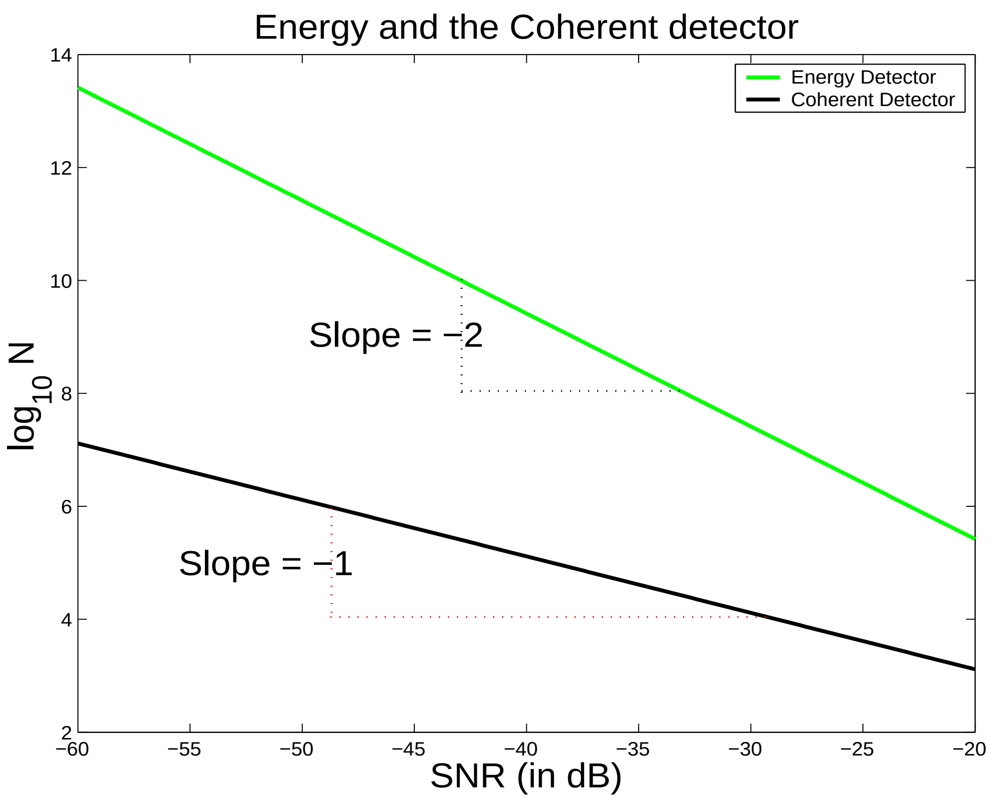
Assume minimal knowledge

- Only know power and signal is like white Gaussian noise
- No processing gain available
- Optimal detector is an energy detector (radiometer)

$$\sum_{k=1}^{N} y[n]^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_s}{\gtrless}} N \left(\sigma^2 + \frac{P}{2} \right)$$

- We require $O(1/SNR^2)$ samples

Energy detector vs. Coherent detector



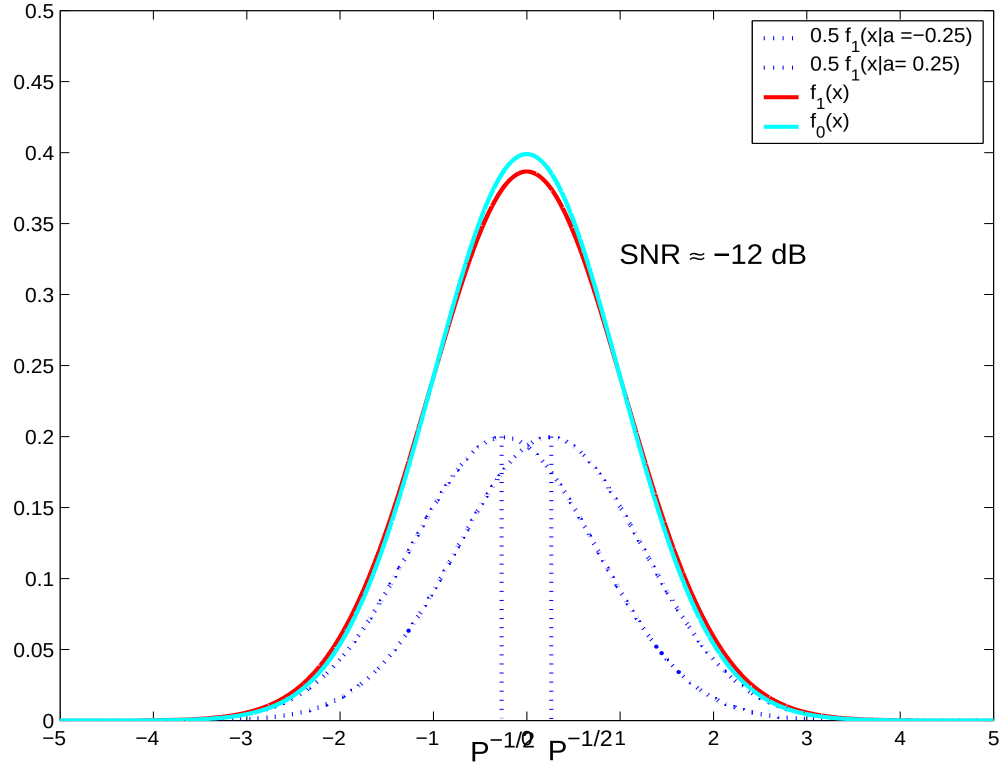
Undecodable BPSK

What if we had a little more information?

- Power is low.
- Modulation scheme (BPSK) is known
- Assume perfect synchronization to both the carrier frequency and symbol timing.

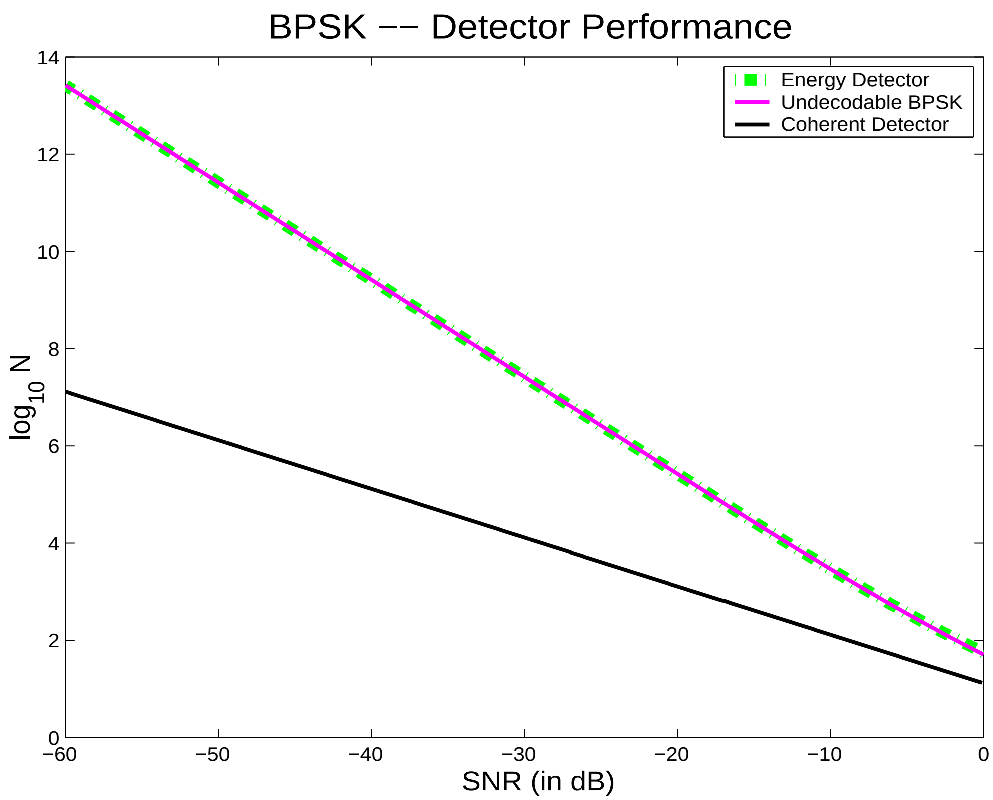
$$\mathcal{H}_0 : Y[n] = W[n]$$
$$\mathcal{H}_s : Y[n] = W[n] + X[n]\sqrt{P}$$
$$X[n] \sim \text{iid. Bernoulli}(1/2)$$

Undecodable BPSK cont.



- Optimal detector turns out to be like the energy detector at low SNR
- We require $O(1/SNR^2)$ samples

Numerical plots for number of required samples..



General symbol constellations

- Is the story bad only for BPSK?

$$\mathcal{H}_0 : \overrightarrow{Y}[n] = \overrightarrow{W}[n]$$

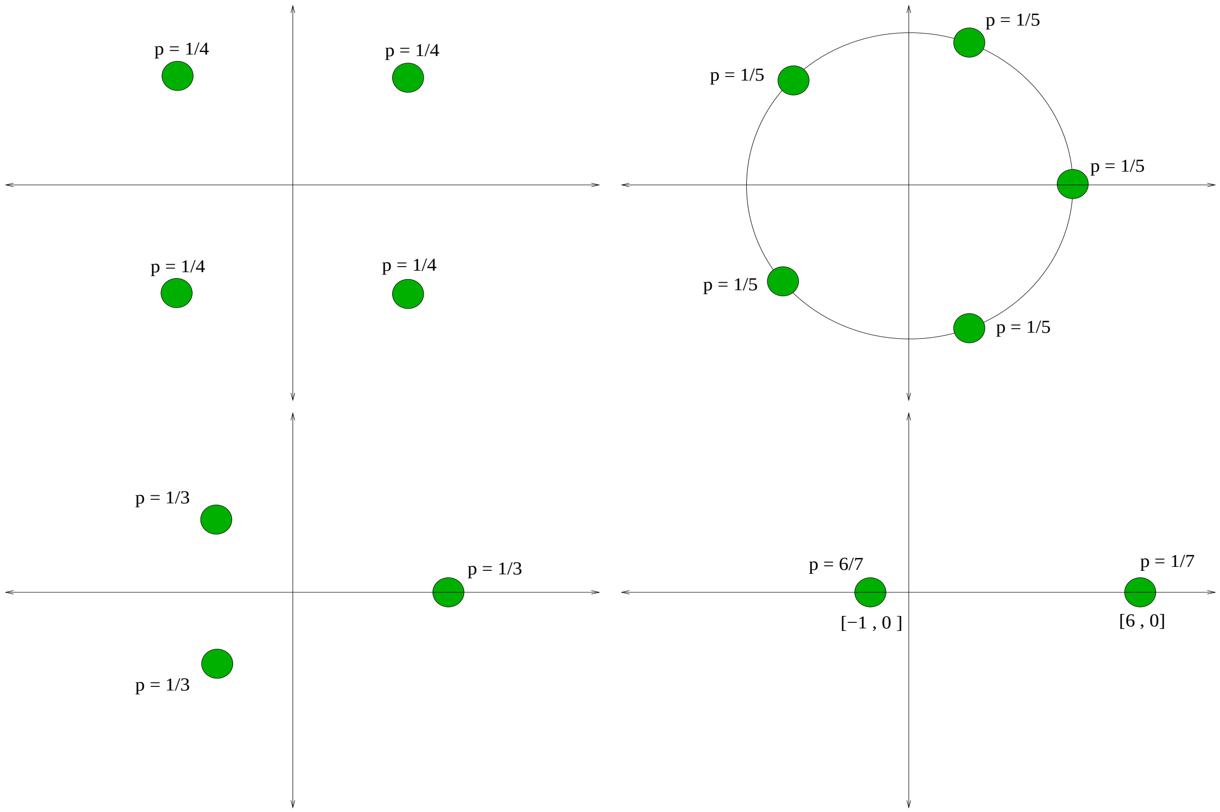
$$\mathcal{H}_s : \overrightarrow{Y}[n] = \overrightarrow{W}[n] + \overrightarrow{X}[n]$$

$$\overrightarrow{x}[n] = \overrightarrow{c}_i \ , \qquad i \in \{1, 2, \ldots, 2^{LR}\}$$

$$\overrightarrow{w}[n] \sim \mathcal{N}(0, \sigma^2 \mathbf{I}_L)$$

- Assumptions
  - Short symbols $\vec{c}_i$ of length L.
  - 2^{LR} symbols known to the receiver
  - Symbol constellation is zero-mean
  - Symbols independent
  - Very little energy in any individual symbol

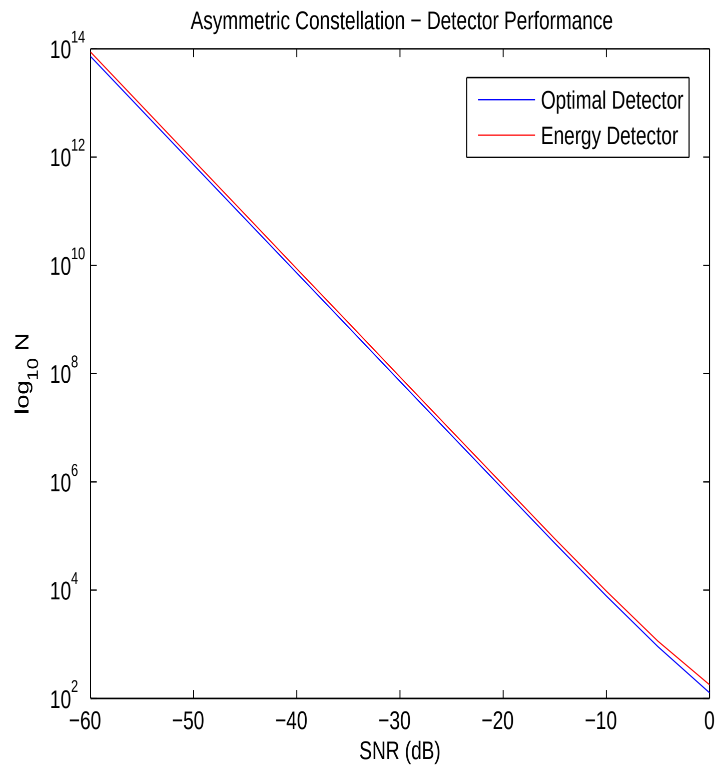
Examples of zero-mean symbol constellations



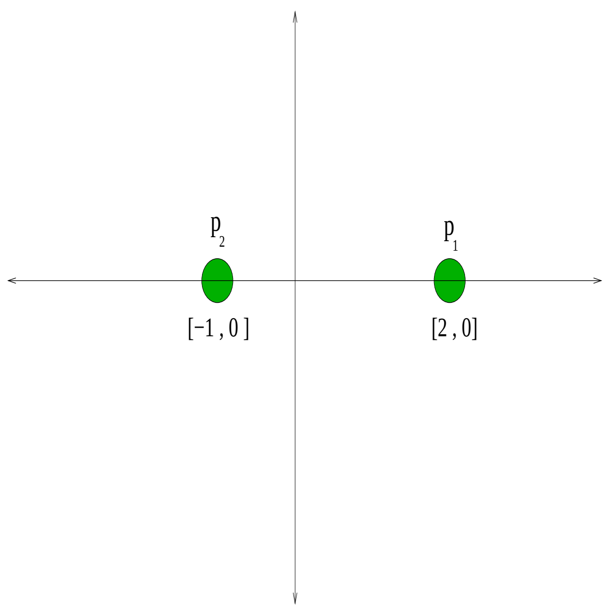
Also includes symbols multiplied by short PRN sequences, short OFDM packets, etc.

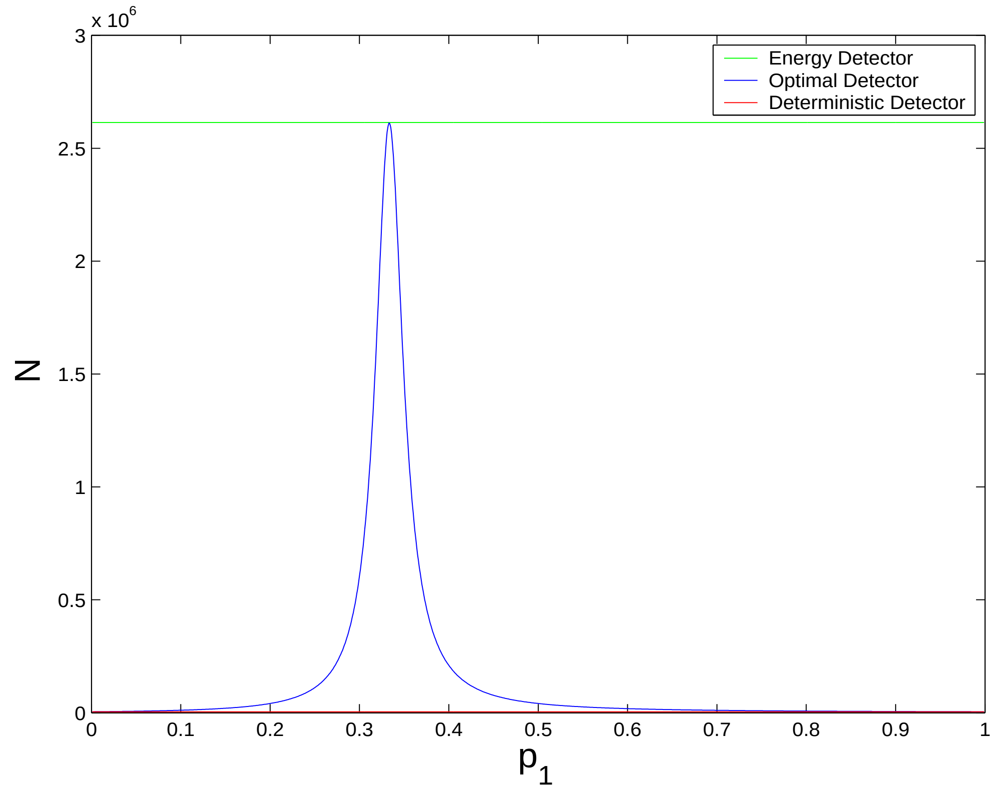
Result

- Optimal detector is like an energy detector at low SNR
- We require $O(1/SNR^2)$ samples



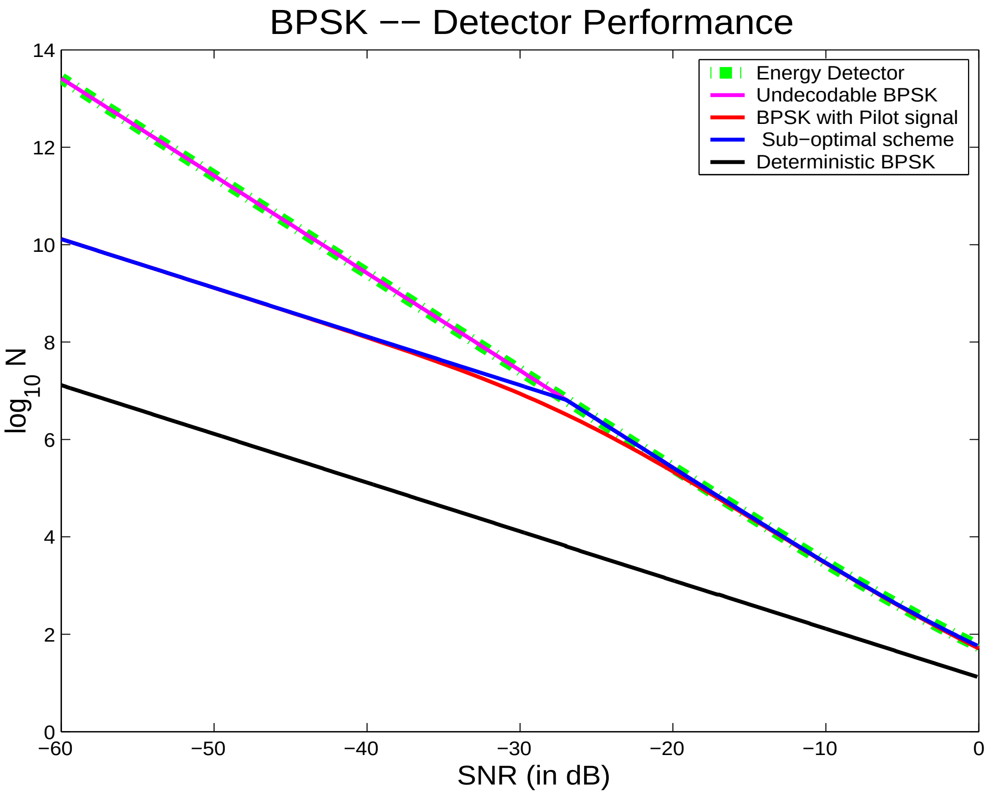
The importance of the zero-mean assumption





The peak occurs when the constellation has zero mean.

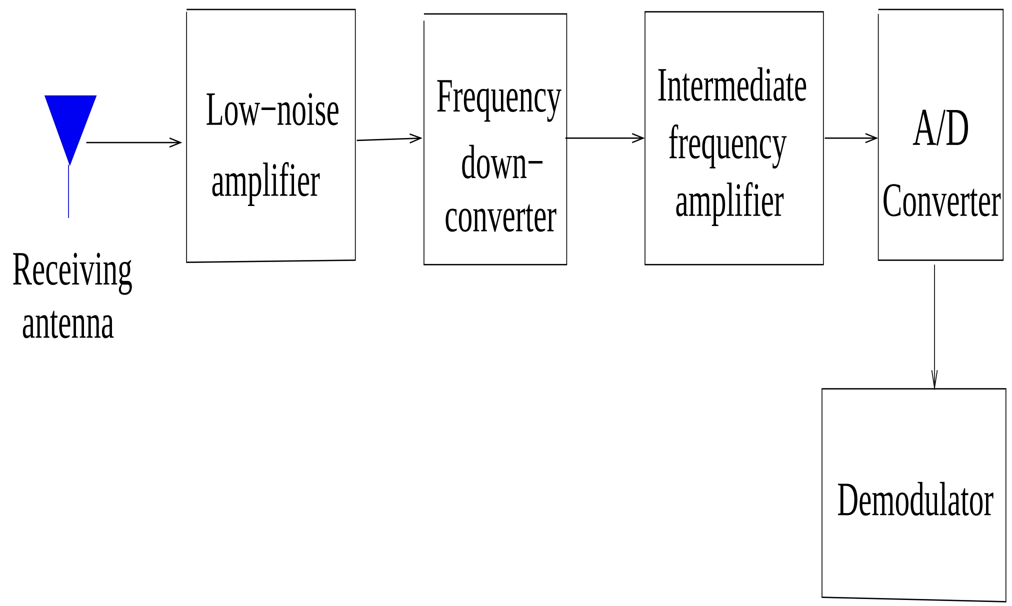
Assume a weak pilot signal alongside the BPSK



The story so far

- Without help from the primary user, the secondary users require a long time to detect free bands.
  - Less agility
  - Overhead in searching for unused bands
- It gets a whole lot worse.
  - Noise uncertainty
  - Quantization

Receiver chain structure



Noise uncertainty

- Let residual noise uncertainty be x dB within the band.
- Receiver faces an SNR within

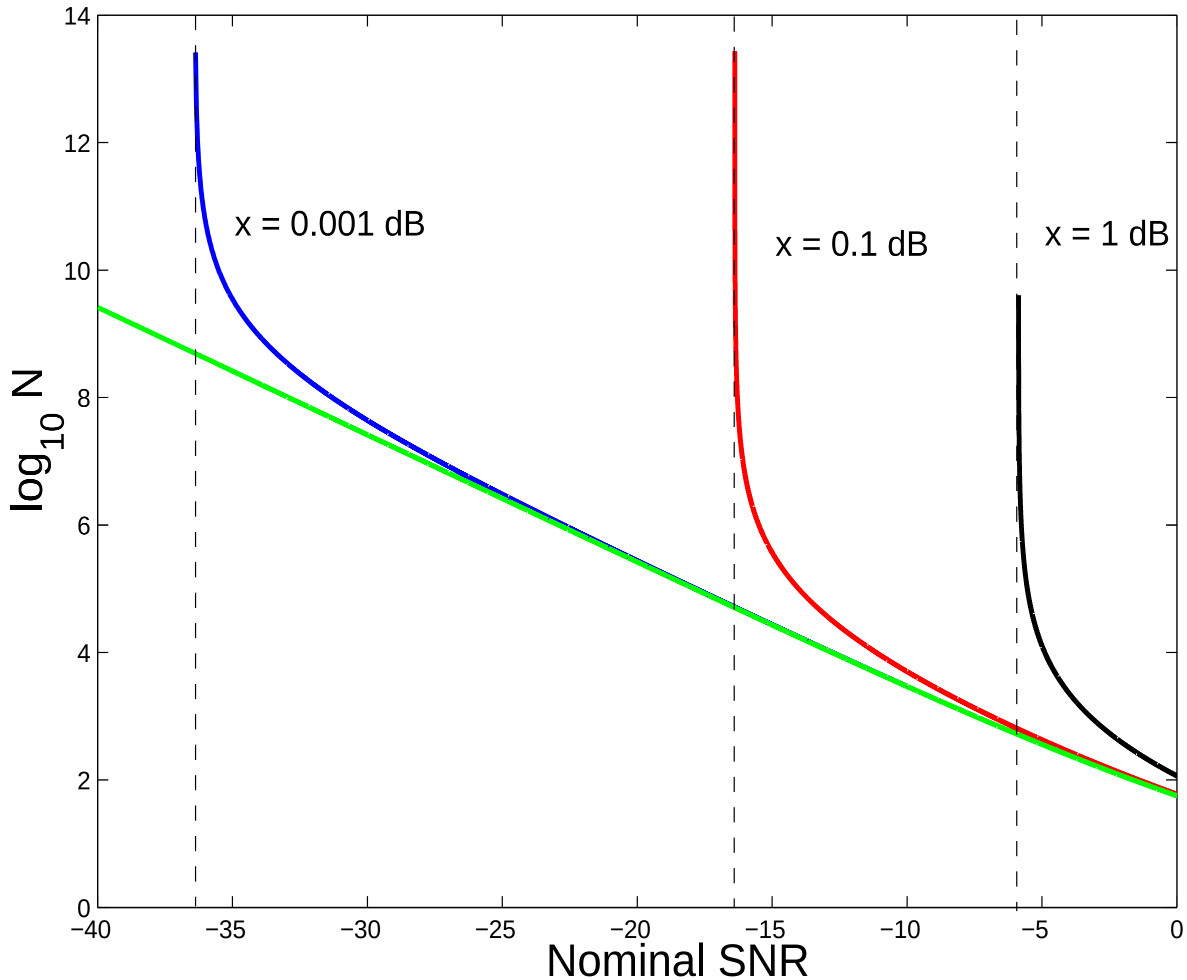
$$[SNR_{nominal},\ SNR_{nominal} + x]$$

- $P_{noise} \in [P_{nominal},\ \alpha \cdot P_{nominal}]\ , \qquad \alpha = 10^{(x/10)}$
- Therefore, the energy detector fails if:

$$P_{noise} \geq P_{nominal} + P_{signal}$$

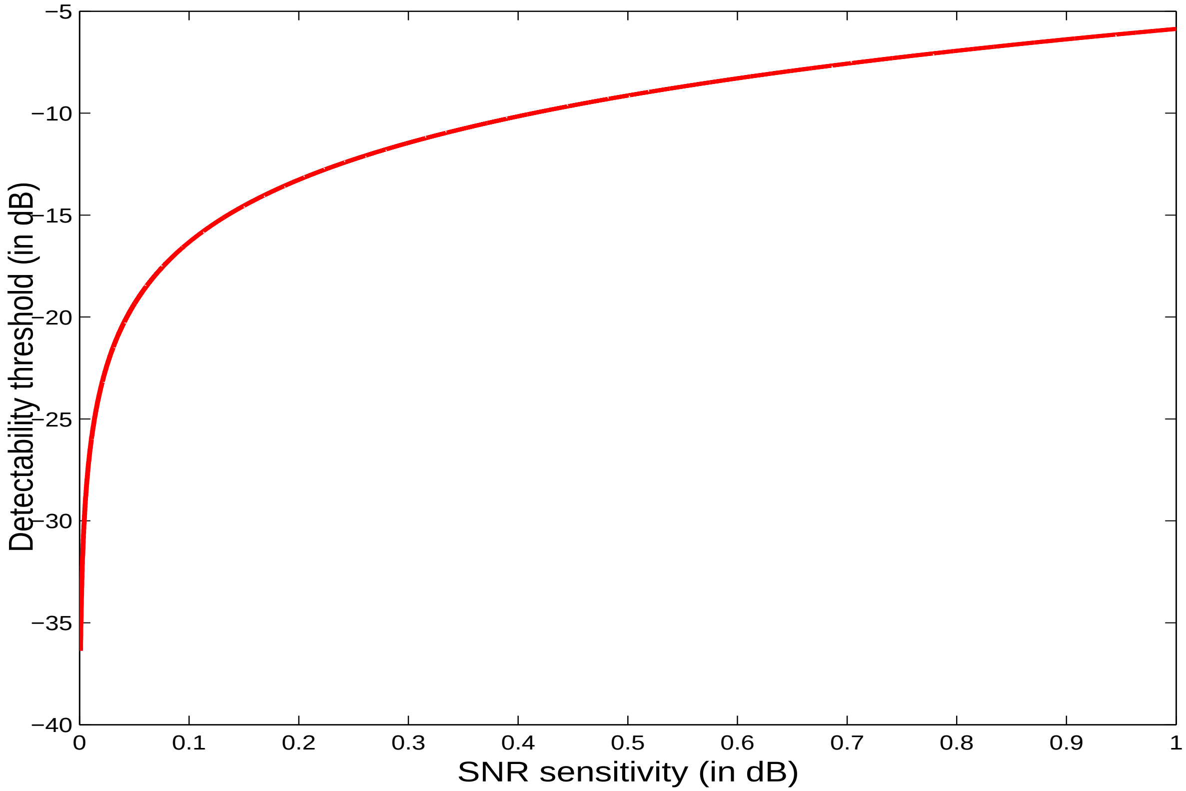
$$\Rightarrow \qquad SNR_{nominal} \leq 10 \log_{10} [10^{(x/10)} - 1]$$

Energy detector with noise uncertainty

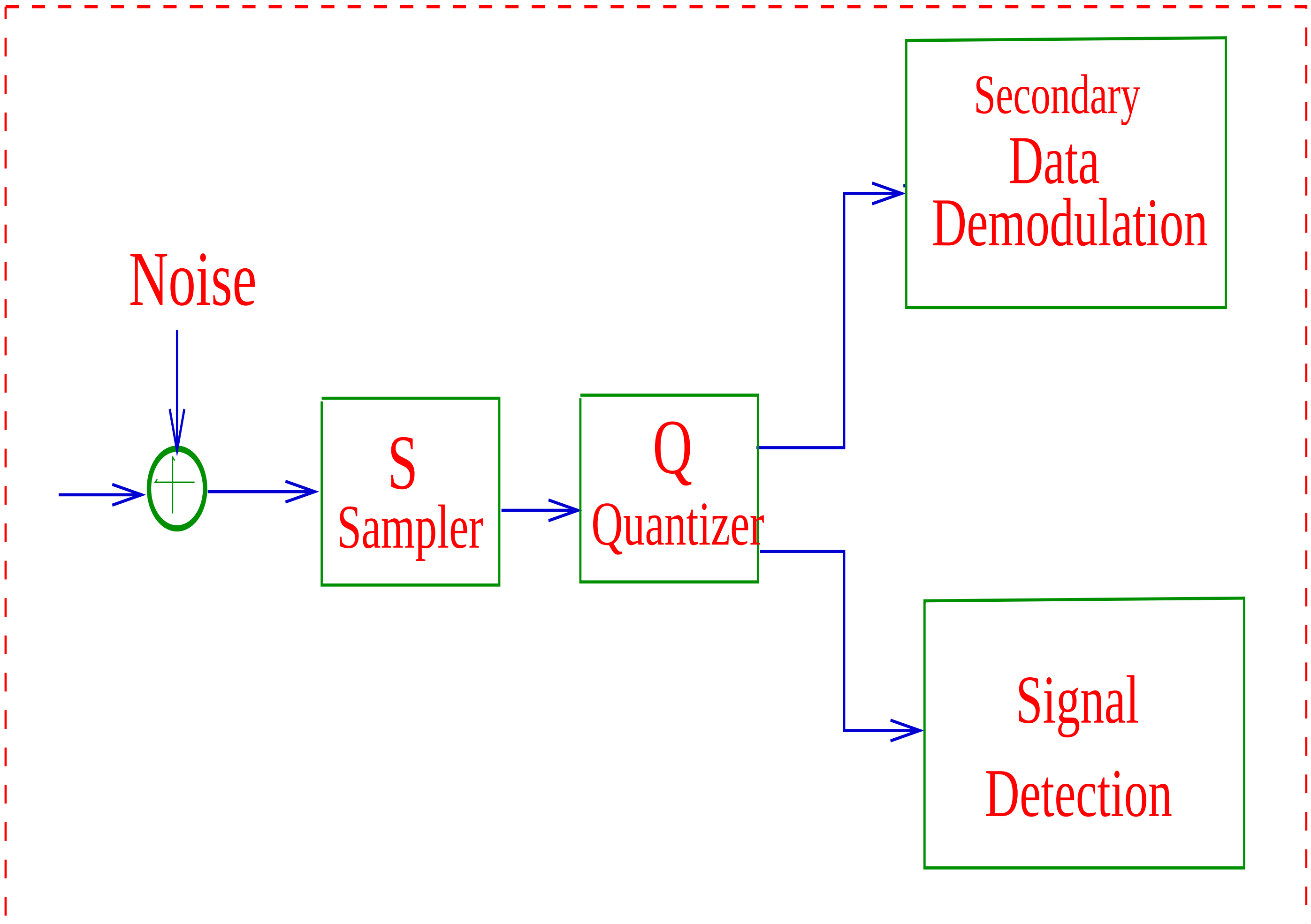


“Wall” position as a function of uncertainty

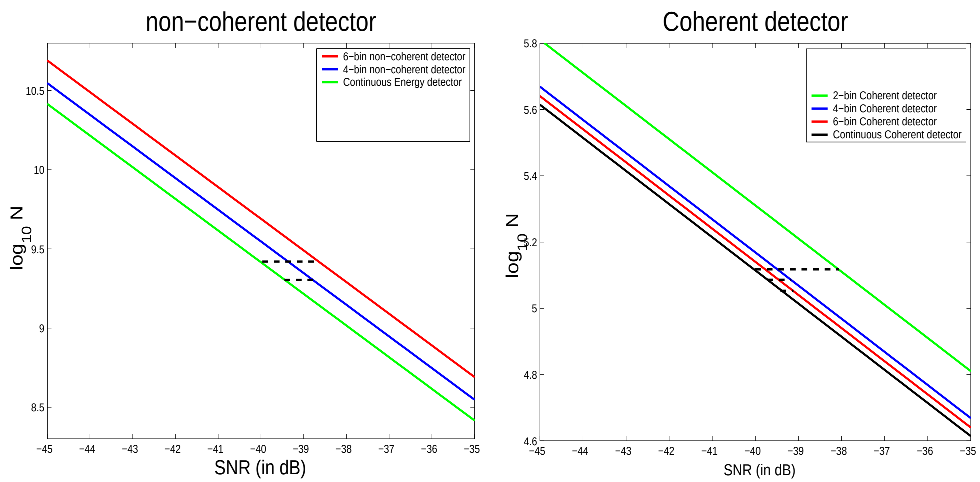
$$SNR_{wall} = 10\log_{10}\left[10^{(x/10)} - 1\right]$$



Quantization: Our abstraction



SNR loss from quantization



Quantization bins	SNR loss (coherent detector)	SNR loss (non-coherent detector)
2 bins	2 dB	∞
4 bins	0.5 dB	1.4 dB
6 bins	0.3 dB	0.7 dB

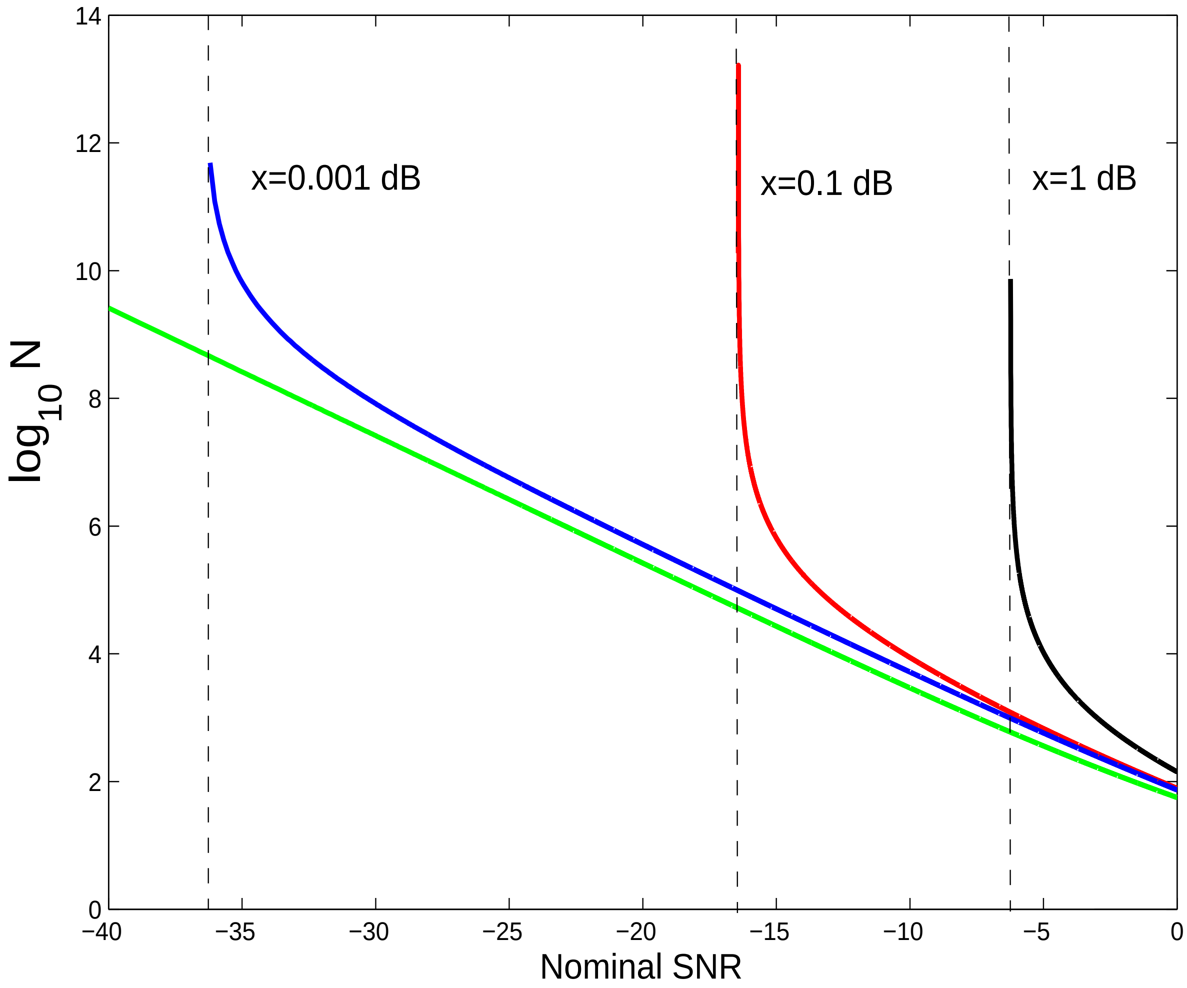
Noise uncertainty under quantization

- Detection can be **absolutely impossible** for 2-bit quantizer under noise variance uncertainty alone.
  - Can make the distributions identical under both hypotheses if

$$Q\left(\frac{d_1}{\sigma_0}\right) = \frac{1}{2}\left[Q\left(\frac{d_1+\sqrt{P}}{\sigma_1}\right) + Q\left(\frac{d_1-\sqrt{P}}{\sigma_1}\right)\right]$$

  - σ_i^2 is noise variance under hypothesis i
  - d_1 is the quantization bin boundary
- Wall *always exists* for any detector.

Quantized detector with noise uncertainty: 2-bit quantizer



Conclusions

- Cognitive radios must be able to detect the presence of undecodable signals
  - Just knowing the modulation scheme and codebooks is nearly useless: stuck with energy detector performance.
  - Even small noise uncertainty causes serious limits in detectability.
  - Quantization makes matters even worse.
- Primary users should transmit pilot signals or sirens.
- If not, some serious "infrastructure" will be needed to support cognitive radio deployment.

Key future questions

- Multiuser situations
  - Control channel use and coordination
  - Distributed reliable environmental proofs
  - Efficiency and robustness
- How to ensure forward compatibility
  - Can future computational capabilities help systems engineered today?
  - Poorly engineered systems today will hinder future systems.
- Congestion is still potentially possible in the future.