Noise calibration, delay coherence and SNR walls for signal detection

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Problem setup

\[ H_0 : W[n] \]

\[ H_1 : H(X[n]) + W[n] \]

\( \{Y[n]\}_1^N \) → Detector → \( \hat{H} \)

Faded Signal

Noise
Spectrum sensing: traditional picture

Energy Detector
SNR = 1 dB
N = 50

Detection Threshold

Test-statistic Support
Spectrum sensing: traditional picture

Energy Detector
SNR = 1 dB
N = 100

Detection Threshold

Test-statistic Support
Spectrum sensing: traditional picture

Energy Detector
SNR = 1 dB
N = 500

Detection Threshold

Test-statistic Support
Spectrum sensing: noise uncertainty

Energy Detector
SNR = 1 dB
SNR Wall = -4 dB
N = 50

Detector Threshold
Worst-case distributions

Test-statistic Support

m₀

m₁
Spectrum sensing: noise uncertainty

Energy Detector
SNR = 1 dB
SNR Wall = -4 dB
N = 500

Detector Threshold
Worst-case distributions

Test-statistic Support

m_0

m_1
Spectrum sensing: noise uncertainty

Energy Detector
SNR = 1 dB
SNR Wall = -4 dB
N = 5000

Detector Threshold
Worst-case distributions

Test-statistic Support

m₀
Spectrum sensing: SNR walls

Energy Detector
SNR = -6 dB
SNR Wall = -4 dB

Detector Threshold

Worst-case distributions

$P_{MD}$

$P_{FA}$

$m_0$  $m_1$

Test-statistic Support
Impact of SNR walls — sensing overhead

Time Overhead

Model:
- Energy Detector
- Coherent Detector

Parameters:
- $P_{MD} = P_{FA} = 0.01$
- Pilot Power = 10%
- Coherence Time = 100
- SNR walls with noise uncertainty = 1 dB
- SNR walls with noise uncertainty = 0.001 dB

Graph shows the relationship between SNR [dB] and time overhead on a logarithmic scale.
Narrowband pilot signals

Sinusoidal pilot plus noise

Magnitude response of sinusoidal pilot plus noise
Narrowband pilot signals

- Single-tap Gauss-Markov fading process

- Magnitude response of a single-tap Gauss-Markov process
Narrowband pilot signals

Single-tap Gauss-Markov faded sinusoidal pilot

Magnitude response of faded pilot plus noise
Cyclostationary signals

Raised-Cosine modulated BPSK signal

Time (n)
Cyclostationary feature detection

\[
\{Y[n]\}_{n=1}^{N} \xrightarrow{\text{Cyclostationary Feature Transform}} \tilde{S}_{Y}[f,\alpha] \xrightarrow{\text{Correlation with } S_{Y}[f,\alpha]} T(Y) \geq \lambda
\]

\[N = 3200\]
Cyclostationary feature detection

\[
\{Y[n]\}_{1}^{N} \xrightarrow{\text{Cyclostationary Feature Transform}} \tilde{S}_{Y}[f,\alpha] \xrightarrow{\text{Correlation with}} S_{Y}[f,\alpha] \xrightarrow{} T(Y) \approx \lambda
\]

\[
N = 32000
\]
Cyclostationary feature detection

\[
\{Y[n]\}^N_1 \xrightarrow{\text{Cyclostationary Feature Transform}} \tilde{S}_Y[f,\alpha] \xrightarrow{\text{Correlation with } S_Y[f,\alpha]} T(Y) \gtrsim \lambda
\]
Feature detection: single-tap Gauss-Markov fading

Single-tap Gauss-Markov fading process

$S_{Y}(f, \alpha)$

$N = 6400$
Feature detection: single-tap Gauss-Markov fading

Single-tap Gauss-Markov fading process

$N = 64000$
Feature detection: single-tap Gauss-Markov fading

Single-tap Gauss-Markov fading process

N = 640000
Feature detection: multi-tap Gauss-Markov fading

N = 6400
Feature detection: multi-tap Gauss-Markov fading

$N = 64000$
Feature detection: multi-tap Gauss-Markov fading

\[ N = 640000 \]
Feature detection: finite-delay fading process

Time-varying single-tap finite delay fading

Single-tap time-varying finite delay fading, N = 3200
Feature detection: finite-delay fading process

Time-varying single-tap finite delay fading

Single-tap time-varying finite delay fading, $N=320000$
Phase coherence vs Delay coherence

Delay spread = 2 micro secs
Carrier frequency = 1 GHz

Single mobile receiving antenna
16 randomly located scatterers
Phase coherence vs Delay coherence

Bandwidth = 400 KHz

- First lap, $h_1[n]$
- Second lap, $h_2[n]$
- Third lap, $h_3[n]$
- Fourth lap, $h_4[n]$
Phase coherence vs Delay coherence

Bandwidth = 400 KHz

First tap, $h_1[m]$

Phase (in radians)

Time (in secs)

0 0.02 0.04 0.06 0.08 0.1

0 0.02 0.04 0.06 0.08 0.1

First tap, $h_2[m]$

Magnitude

0 1 2 3

t=0 sec

0 1 2 3

t=20 ms
Phase coherence vs Delay coherence

Bandwidth = 1 KHz

First tap, $h_0[m]$

First tap, $h_2[m]$

$t=0$

$t=20$ msec

$t=several$ secs
Noise calibration: pilot case

Pilot

Band pass filters

$\textbf{f}_p$ $\textbf{f}$

$\textbf{f}_m$
Noise calibration: cyclostationary feature detection

\[ \{Y[n]\}_1^N \rightarrow \{\hat{Y}[n]\}_1^N \rightarrow \hat{S}_\hat{Y}[f,\alpha] \rightarrow T(\hat{Y}) \geq \lambda \]

AGC calibration:

\[ \hat{Y}[n] = \frac{1}{\sqrt{\frac{1}{N} \sum_{n=1}^{N} |Y[n]|^2}} Y[n] \]

As \( N \to \infty \)

\[ \frac{1}{\sqrt{\frac{1}{N} \sum_{n=1}^{N} |Y[n]|^2}} \rightarrow \begin{cases} \frac{1}{\sigma_a} & \text{Under } \mathcal{H}_0 \\ \frac{1}{\sqrt{P+\sigma_a^2}} & \text{Under } \mathcal{H}_1 \end{cases} \]
Robustness gains from noise calibration
Robustness gains from noise calibration

Means after AGC calibration

Delay Coherence Time

Means of test-statistic $\times 10^3$
Robustness gains from noise calibration

Colored noise model:

\[ W[n] = M[n] - \beta M[n - 1], \]

\( M[n] \) is iid white Gaussian noise, \( \beta \in [0, \beta_{max}] \).