Some Fundamental Limits on Cognitive Radio

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Abstract

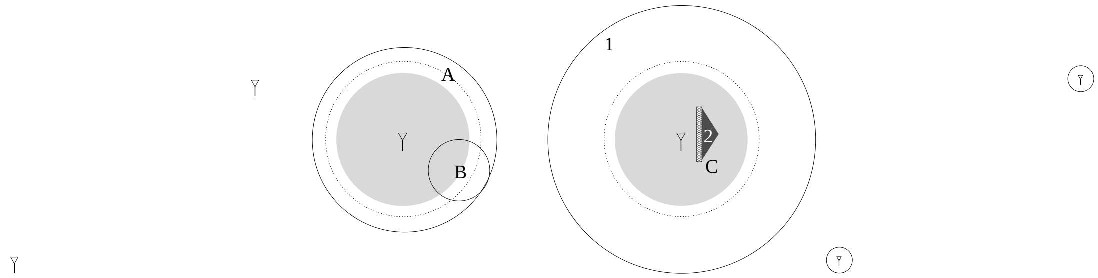
Cognitive radio refers to wireless architectures in which a communication system does not operate in a fixed assigned band, but rather searches and finds an appropriate band in which to operate. In this paper we explore, from first principles, the fundamental requirements for such system that tries to avoid interference to potential primary users of a band. We first show that in order to deliver real gains, cognitive radios must be able to detect undecodable signals. This is done by showing how to evaluate the tradeoff between secondary user power, available space for secondary operation, and interference protection for the primary receivers. We prove that in general, the performance of the optimal detector for detecting a weak unknown signal from a known zero-mean constellation is like that of the energy detector (radiometer). However we show that the presence of a known pilot signal can help greatly. We further motivate the need for pilot signals by showing that the radiometer is rendered useless by just moderate noise uncertainty. Finally, we show that quantization combined with noise uncertainty can make the detection of signals by any detector *absolutely impossible* below a certain SNR threshold.

1 Description of problem and some fundamental geographic tradeoffs

The enormous success of the ISM bands has strengthened criticism of the FCC's traditional process which allocates bands to a single use, issues exclusive licenses to a single entity within a geographical area, and prohibits other devices from transmitting significant power within these bands. As a result, the FCC is considering revising its spectrum allocation policies [1], and is moving ahead with the process. Cognitive radios are one proposed idea to take advantage of a more open spectrum policy.[2, 3]. A cognitive radio would be designed to dynamically adapt its transmissions to find and utilize frequencies while minimizing interference. This is inspired by actual measurements showing that most of the allocated spectrum is vastly underutilized [4]. Cognitive radios promise great societal benefits by allowing new applications that can flexibly use what spectrum happens to be available. However, fundamental theoretical questions remain as to the exact requirements for engineering a practical cognitive radio system.

For illustrative purposes, consider the world depicted in (1a). Each of the two antennas represents a bustling metropolis. Under current regulations, users in the middle of nowhere are prohibited from using these spectrum bands, even if there are no receivers in the vicinity that might be interfered with. However, cognitive radios may someday be allowed to share the spectrum, on the condition that they not interfere with primary users. These primary users may be providing more socially important services, or they might simply be legacy systems that are unable to change.

Figure (1b) depicts the primary transmitter in one metropolis. The dotted circle represents the boundary of decodability for a single-user system. That is, in the absence of all interference,



(a) **(b)** **(c)** **(d)**

Figure 1: The case for cognitive radios

a user within the dotted line would be able to decode a signal from the transmitter, while a user outside the circle would not. However, if we want to actually create a system in which we guarantee performance to primary users within the decodability circle, we run into a difficulty, even when we consider the simplified case of only a single secondary user.

Let a primary receiver's "no-talk zone" refer to the area of the world where secondary users must be silent to allow the primary receiver to continue decoding its received signals. Now consider receiver A, located on the border of the decodability region. Its no-talk zone includes a very large region indeed! Any amount of interference, no matter how infinitesimal, will cause A to lose its ability to decode. Therefore, it is clear that we must build some sort of buffer into our protected radius.

Let the shaded circle represent the "protected region" where we guarantee decodability to primary receivers. The more we shrink the bound of the protected region inside the decodability region, the smaller the necessary no-talk zones become. Similarly, the power of the secondary user's transmissions is important. If they are "mice," who squeak softly, then the no-talk zones around each receiver can be much smaller. If they are "lions," roaring with high power transmissions, the radius of the no-talk zones will become much larger. For the simplified case of $\frac{1}{d^2}$ propagation loss[1], we have:

$$P_s = r_{\text{censored}}^2 \left[\frac{P_p}{(2^{2R}-1) r_{\text{protected}}^2} - \sigma^2 \right]$$

$$r_{\text{censored}} = \sqrt{\frac{P_s}{\frac{P_p}{(2^{2R}-1) r_{\text{protected}}^2} - \sigma^2}}$$

where r_{censored} is the radius of the "no-talk zone" around the receiver, $r_{\text{protected}}$ is the protected radius of the primary transmitter in which we want to preserve decodability for the primary receivers, P_s is the power of the potential secondary transmissions, and (P_p, R) are the power and rate of the primary user.

Even if the individual no-talk zones are small, uncertainty in the primary receivers' locations can result in a large global no-talk zone. Secondary users must stay out of the space that is the union of all possible no-talk zones (1b). We note that in the hypothetical example, the prohibited region for secondary users has already extended beyond the decodability region.

If we take shadowing into account (1c), the prohibited region continues to grow. Local shadowing or fading can cause the secondary user to see a very low power primary transmission[2]

[1] In reality, the situation depends a little on the application. If secondary users are likely to be indoors or located at low elevations among buildings, their interfering power will die off much faster than $\frac{1}{d^2}$.

[2] How low is low? This has to be an engineering and policy decision that weighs the practical range of fades

even though he actually may be within the no-talk zone. As a result, a secondary has no way to tell if it is well outside the protected region (user 1), or in the global quiet zone but behind a building (user 2). To avoid secondary users in a local shadow interfering with unshadowed primary users (user C), the no-talk zone must be pushed out even further. The outermost circle represents the quiet zone such that the maximum SNR on its border equals the minimum SNR within the shaded circle.

Since this global quiet zone is well outside the region of decodability for all but the weakest secondary users, we see that the "don't transmit if you can decode a signal" rule is inadequate in that it does not allow us to exploit the possibility for higher power secondary spectrum usage in the vast majority of geographic contexts where even high power transmissions would result in no interference. These users are still well outside the quiet zones (1d).

In the following sections, we consider the case where the transmission is an undecodable BPSK signal, a signal composed of symbols from a zero-mean constellation, or a sinusoid of unknown parameters. In all cases we assume the signal power is much less than the noise power. We then consider the effects of noise uncertainty and quantization on detector performance.

2 Background

We consider the problem of detecting of a signal in additive white Gaussian noise (AWGN). Our goal is to distinguish between the hypotheses:

$$\begin{aligned} &\mathcal{H}_0: \quad && Y[n] = W[n] \quad && n = 1, \ldots, N \\ &\mathcal{H}_s: \quad && Y[n] = X[n] + W[n] \quad && n = 1, \ldots, N \end{aligned}$$

If $\overrightarrow{x}$ is known at the receiver, the optimal detector is just a matched filter [5]. Earlier work has shown that a matched filter requires $O(1/SNR)$ samples to meet a predetermined probability of error constraint.

On the other hand, if the transmitted signal lacks any features exploitable by a matched filter, the detector performs much more poorly. For example, if the transmitter only transmits random Gaussian noise of known power, the optimal detector is just an energy detector (radiometer) [6, 7]. In this case, $O(1/SNR^2)$ samples are required to meet a probability of error constraint.

The number of samples is significant because it imposes a limit on the frequency agility of a cognitive radio. If it requires too many samples to determine if a band is being used, then we can not take advantage of short duration gaps in band usage. On a practical level, it also imposes a limit on our ability to quickly set up connections or networks.

3 Detection of undecodable BPSK signals

In this section we try to detect the presence of a BPSK modulated signal in AWGN. We assume the receiver has no information about the sequence of bits transmitted. As such, we assume that the $X[n]$ are i.i.d. $\sim$ Bernoulli(1/2) and independent of the noise.

Finding the maximum likelihood decision[3] rule is straightforward and yields:

and shadowing with the desire to limit no-talk zones. However, we expect that we will want to add at least $10-20$ dB of extra protection for primary users.

[3]In reality, we would want to tilt the probability of false alarm and missed detection to favor false alarms over missed detections. After all, missed detection would cause us to cause interference while false alarms would just cause us to move to another band. In a world with many secondary users, keeping the total number of missed detections will be important. However, even in the asymmetric case, the required probabilities are likely to be "moderate" enough that the central limit theorem continues to apply. The situation is not like the probability of decoding error in a code where extremely tiny probabilities are targeted.

$$\sum_{n=1}^{N} \ln\left[\cosh\left(\frac{\sqrt{P}}{\sigma^2} Y[n]\right)\right] \underset{H_0}{\overset{H_S}{\gtrless}} \frac{NP}{2\sigma^2}$$

Since the decision statistic is the sum of many (we assume N large) independent random variables and the probability for false alarm is moderate, we can use the central limit theorem to approximate the probability of error. To do so, we must first find the mean and variance of the likelihood ratio under $\mathcal{H}_0$ and $\mathcal{H}_s$. This can be done using Taylor series approximations, and we find that in the limit of low SNR, the decision rule performs almost exactly like an energy detector. See Fig. 2a for the performance curves of this detector.

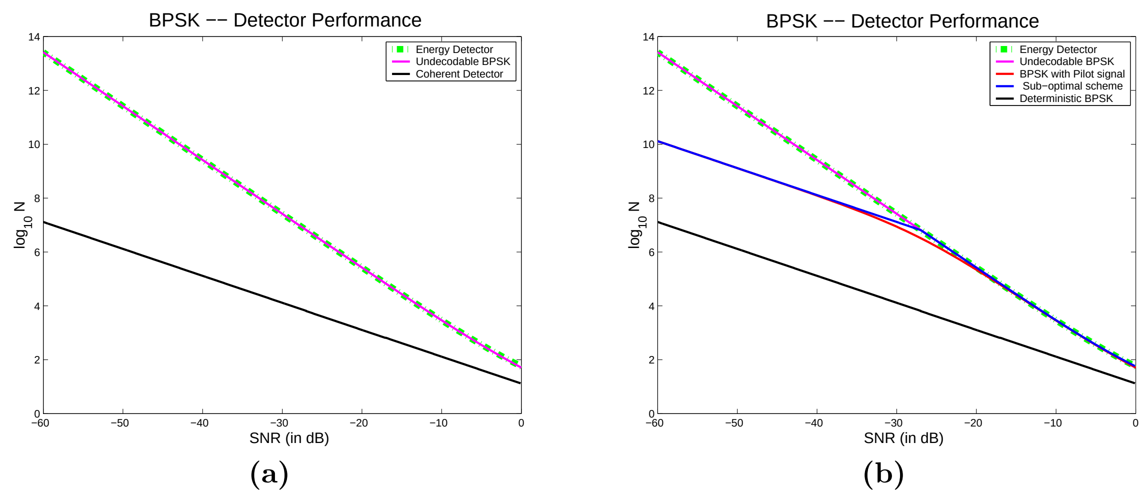


Figure 2: The plot on the left compares the performance of an undecodable BPSK signal to the energy detector. The plot of the right includes the performance curve for an undecodable BPSK signal with a known pilot signal.

4 Detection of zero-mean constellations

4.1 Model

We examine the maximum likelihood decision rule for a constellation known to the receiver, subject to the constraint that the symbols have low energy. We show that the optimal detector for a zero-mean constellation behaves like a energy detector in the limit of low SNR.

We assume a constellation of 2^{LR} symbols. Each symbol has dimension L and R is greater than the channel's capacity. The transmitter picks a sequence of N (N large) i.i.d. symbols and transmits it over the channel.

$$\begin{aligned} \mathcal{H}_0 : \overrightarrow{Y}[n] &= \overrightarrow{W}[n] \\ \mathcal{H}_s : \overrightarrow{Y}[n] &= \overrightarrow{X}[n] + \overrightarrow{W}[n] \end{aligned}$$

where $\overrightarrow{x}[n] = \overrightarrow{c}_i$, $i \in (1, 2^{LR})$ and $\overrightarrow{W}[n] \sim \mathcal{N}(0, \sigma^2 \mathbf{I}_L)$.The probability that the ith symbol in the constellation, denoted $\overrightarrow{c}_i$, is transmitted is $Pr(\overrightarrow{c}_i)$. We denote the energy in the ith symbol by $\Gamma(i)$, and the average energy of the constellation by $\overline{\Gamma}$.

The signal $\overrightarrow{Y}$ of length N is received, and a maximum likelihood rule is used to determine whether the signal is purely noise, or whether a symbol was transmitted.

4.2 Results

The approximate maximum likelihood decision rule is shown below. (See next section for derivation.)

$$\sum_{n=1}^{N}\left(\frac{1}{\sigma^2}\left[\sum_{i=1}^{2^{LR}} Pr(\overrightarrow{c}_i)\overrightarrow{c}_i^T\right]\overrightarrow{Y}[n]-\frac{\overline{\Gamma}}{2\sigma^2}+\frac{1}{8\sigma^4}\sum_{i=1}^{2^{LR}} Pr(\overrightarrow{c}_i)(2\overrightarrow{c}_i^T\overrightarrow{Y}[n]-\Gamma_i)^2\right)\underset{\mathcal{H}_0}{\overset{\mathcal{H}_s}{\gtrless}} 0 \qquad (1)$$

Assuming low SNR, under what further conditions does our optimal detector reduce to a energy detector? The bracketed term, like a matched filter, is a linear function of $\overrightarrow{Y}$. If this term equals zero, the $\overrightarrow{Y}^T\overrightarrow{Y}$ terms, smaller by a factor of SNR, become the dominant terms.

$\sum_{i=1}^{2^{LR}} Pr(\overrightarrow{c}_i)\overrightarrow{c}_i^T$ is just the average of the symbol constellation. Therefore, if we have a zero-mean signal constellation, the optimal detector at low SNR behaves qualitatively like an energy detector in its dependence on SNR (Fig. 3).

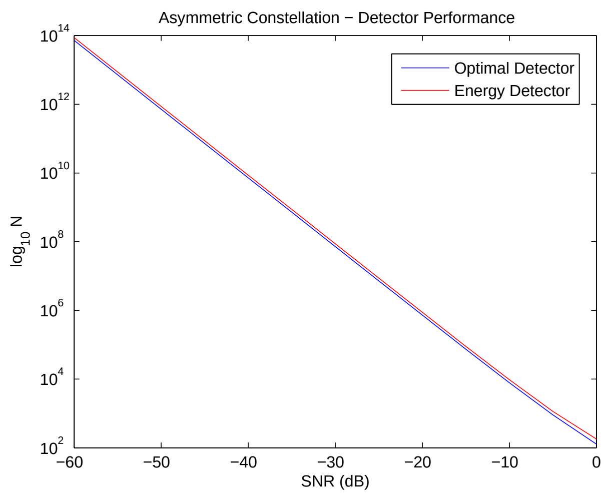


Figure 3: Zero-mean constellation detector and energy detector

4.3 Analysis

We derive expressions for the probability of receiving $\overrightarrow{Y}$ under the $\mathcal{H}_0$ and $\mathcal{H}_s$ hypotheses:

$$\begin{aligned} Pr(\overrightarrow{Y} \mid \mathcal{H}_0) &= \frac{1}{(2\pi\sigma^2)^{L/2}}\exp\left(\frac{-1}{2\sigma^2}\overrightarrow{Y}^T\overrightarrow{Y}\right) \\ Pr(\overrightarrow{Y} \mid \mathcal{H}_s) &= \sum_{i=1}^{2^{LR}} Pr(\overrightarrow{c}_i)\frac{1}{(2\pi\sigma^2)^{L/2}}\exp\left(\frac{-1}{2\sigma^2}(\overrightarrow{Y}-\overrightarrow{c}_i)^T(\overrightarrow{Y}-\overrightarrow{c}_i)\right) \end{aligned}$$

The independence of the N transmitted symbols results in the maximum likelihood rule:

$$\prod_{n=1}^{N}\left[\sum_{i=1}^{2^{LR}} Pr(\overrightarrow{c}_i)\exp\left(\frac{1}{2\sigma^2}(2\overrightarrow{c}_i^T\overrightarrow{Y}[n]-\overrightarrow{c}_i^T\overrightarrow{c}_i)\right)\right]\underset{\mathcal{H}_0}{\overset{\mathcal{H}_s}{\gtrless}} 1$$

We assume $\frac{1}{2\sigma^2}(2\overrightarrow{c}_i^T\overrightarrow{Y}[n]-\overrightarrow{c}_i^T\overrightarrow{c}_i) \ll 1$. Then applying the approximation $e^x \approx 1+x+\frac{x^2}{2}$, taking logarithms, and using $\ln(1+x)\approx x$ yields the decision rule in (1).

5 Pilot signals

We have shown that zero-mean signals are difficult to detect at low SNR and knowing the modulation scheme does not help. We can significantly improve the performance of our detector by transmitting an additional pilot signal. At high SNR, an energy detector is nearly optimal [8], but it performs far worse than a coherent detector at low SNR.

If the transmitter sends a pilot signal simultaneously with its transmissions, we can design a suboptimal detector that just detects the pilot at low SNR. Because of the power of processing gain, this scheme provides a substantial reduction in the number of samples required to detect, even for extremely weak pilot signals. Simulations further show that this scheme is nearly optimal at low SNR, i.e. a secondary receiver can just look for a pilot signal and ignore the remainder of the transmission. Refer to Fig. 2b for the performance curves in the presence of a pilot signal.

5.1 The impact of imperfect synchronization

Until now, we have seen examples of signals which require $O(1/SNR)$ or $O(1/SNR^2)$ samples for detection. Are these the only possible behaviors in the low SNR regime? Or are there other examples of signals which require $O(1/SNR) < N < O(1/SNR^2)$ samples for detection?

Consider the detection of sinusoid signals with unknown frequency, which could be thought of as a pilot signal with some uncertainty about the clock syntony between the secondary user and the primary transmitter. The detector performance for a sinusoidal signal with unknown frequency is given by $P_D = Q_{\mathcal{X}'^2_2(\lambda)}\left(2\ln\frac{\frac{N}{2}-1}{P_{FA}}\right)$, where $\lambda = N \cdot SNR$ and $Q_{\mathcal{X}'^2_2(\lambda)}$ is the tail probability of the standard chi-squared random variable with 2 degrees of freedom and noncentrality parameter λ.

It is easy to see that while this imperfect synchronization does cause us to need more samples, the gap is small compared to the gap between the energy detector and the coherent detector. It is a general feature of coherent detection that processing gain gives us robustness to a wide range of uncertainties in the system.

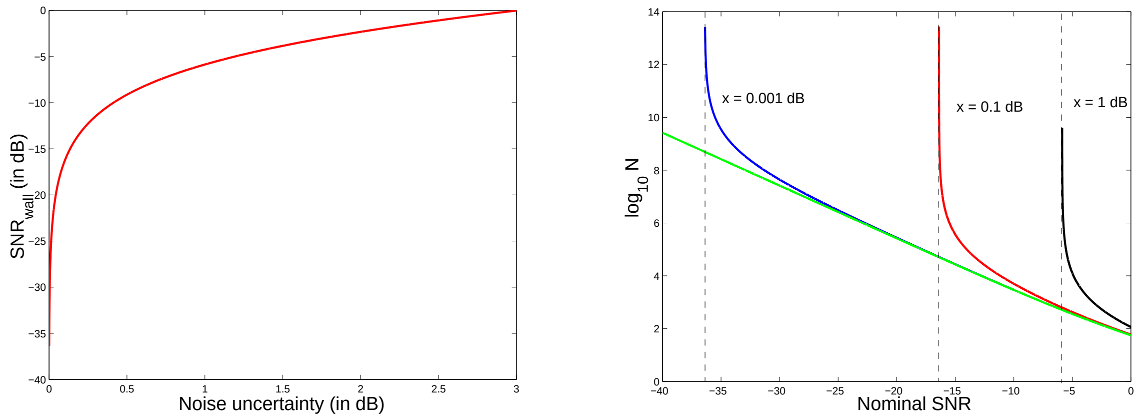
6 Noise Uncertainty and Quantization

6.1 Noise Uncertainty

We have seen that using a radiometer requires $N = O(1/SNR^2)$ samples to detect the presence of the primary transmission, and the previous sections have shown that knowledge of the modulation can not help us to do better. However, it turns out that the energy detector suffers from a problem more severe than just the increased number of samples.

In the analysis done so far we assumed that the receiver noise is Gaussian and its variance is known exactly. In practice the noise is only approximately Gaussian and its variance is uncertain within some band. Suppose, that the noise is Gaussian but its variance is unknown within a certain range, i.e., let the noise uncertainty be x dB. Then the noise variance lies in $[\sigma^2_{nominal}, \alpha \cdot \sigma^2_{nominal}]$, where $\alpha = 10^{x/10}$. In this case it is easy to see that if $\sigma_0^2 = \sigma_1^2 + P_{signal}$, the radiometer could have the same sample mean under both hypothesis and hence it cannot differentiate between the two hypotheses on the basis of the mean alone. Thus, if the SNR is sufficiently low the radiometer can be rendered useless. Specifically, for a given uncertainty of x dB, there exists a threshold (Fig. 4) below which the radiometer cannot detect the signal, $SNR_{wall} = 10\log_{10}[10^{(x/10)} - 1]$.

Hence there is more than just a cost of a factor of $O(1/SNR)$ samples associated with the radiometer, which is *non-detectability.* A pilot signal, on the other hand, can be detected even



(a) (b)

Figure 4: The location of the SNR_{wall} as a function of the noise uncertainty x is plotted in the left figure. The performance of the radiometer for different values of the noise uncertainty is show in the right figure.

under noise uncertainty because it is highly unlikely for the noise to pick an exact pilot signal to confuse the matched filter.

6.2 Quantization

Another aspect of practical systems that has not been considered yet is quantization. Almost all the receivers have an internal A/D converter following the RF chain. Therefore, all the demodulation/detection that the receiver does is based on quantized versions of the received signal, not the received signal itself [9]. In the previous sections we ignored this aspect and assumed that our cognitive radio had infinite precision. It is intuitively clear that quantization should make detection harder. The question we try to answer in this section is: "How is the effect of quantization on detection different from its effect on demodulation?"

Specifically, we first discuss the effect of a quantizer on the detection of a coherent signal, then we discuss the case when the signal is unknown at the receiver (the more relevant case for cognitive radios) and evaluate its performance.

Classically, a quantizer is often viewed as an additional source of noise. Thus in the coherent case, quantizing the signal effectively reduces the SNR at the receiver, i.e, quantization induces an "SNR loss" in the performance of the detector. We now ask if this is a good model for cognitive radios (non-coherent detection).

We assume that the receiver uses a standard A/D converter designed for coherent detection of data symbols. This makes sense because the RF chain and A/D converter would likely be shared between detection and demodulation functionalities within a cognitive radio. Fig. 5 shows our abstract model of the receiver in a cognitive radio.

Assume that we use a $2M$ bin symmetric quantizer with the bins located at $\pm d_i$, $i = 0, 1, \ldots, M-1$, $d_0 = 0$. Then the quantized signal can be viewed as a discrete random variable Z taking values in the set $\{0, 1, 2, \ldots, M-1\}$ with $P(Z = i|\mathcal{H}_j) = p_i^{(j)}$, for $i = 0, 1, 2, \ldots, M-1$ and $j = 0, 1$. Here the event $\{Z = i\}$ refers to the fact that the received signal falls in bin $B_i \triangleq [d_{i-1}, d_i) \bigcup (-d_i, -d_{i-1}]$, $i = 1, 2, \ldots, M$ with the notation that $d_M = \infty$, $-d_M = -\infty$. Hence $\sum_{i=0}^{M-1} p_i^{(j)} = 1$ for $j = 0, 1$. Now, we can write the likelihood ratio as a function of the received quantized signal $\bar{z}$:

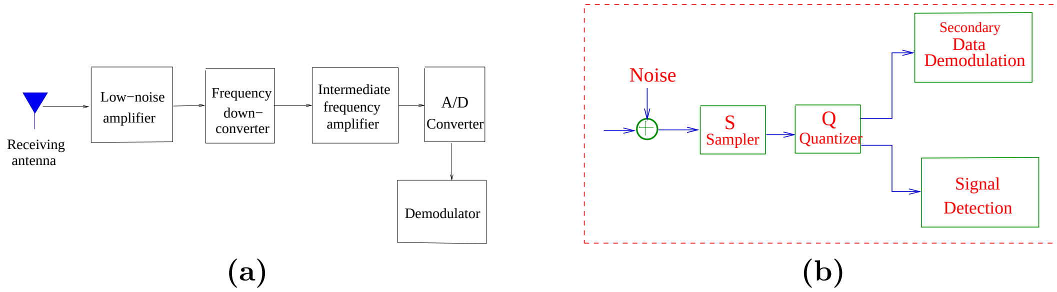


Figure 5: The receiver structure is shown in the left plot and our abstracted model of the receiver is shown in the right most plot.

Quantization bins	SNR loss (coherent detector)	SNR loss (non-coherent detector)
2 bin	2 dB	∞
4 bin	0.5 dB	1.4 dB
6bin	0.3 dB	0.7 dB

Table 1: Table showing the SNR loss due to quantization

$$\Lambda(\bar{Z}) = \frac{P(\bar{Z}=\bar{z}|H=1)}{P(\bar{Z}=\bar{z}|H=1)} = \prod_{n=1}^{N} \frac{P(Z_n = z_n|H=1)}{P(Z_n=z_n|H=0)} = \prod_{i=0}^{M-1}\left[\frac{p_i^{(1)}}{p_i^{(0)}}\right]^{n_i}$$

where $n_i \in \{0,1,2,\ldots,N\}$ and $\sum_{i=0}^{M-1} n_i = N$. Taking the log-likelihood ratio we get the test statistic $T(\bar{Z}) = \sum_{i=1}^{M} C_i n_i$, where $C_i = \log{(p_i^{(1)}/p_i^{(0)})}$. The decision rule compares the test statistic to a threshold γ chosen to meet probability of false alarm and probability of missed detection requirements.

We can write the test statistic as $\sum_{i=1}^{N} Y_i$, where Y_i is a discrete random variable taking values in the set $\{C_0, C_1, \ldots, C_{M-1}\}$ with corresponding probabilities $p_i^{(j)}$, $i = 0,1,\ldots,M-1$ under hypothesis $\mathcal{H}_j$. Thus, the test statistic is a sum of N i.i.d. discrete random variables Y_i. We can again use the central limit theorem and approximate the test statistic as a Gaussian random variable with appropriate mean and variance under both hypotheses:

$$T(\bar{Z}) \sim \begin{cases} \mathcal{N}(Nm_0, N\sigma_0^2) & \text{under } \mathcal{H}_0 \\ \mathcal{N}(Nm_1, N\sigma_1^2) & \text{under } \mathcal{H}_1 \end{cases}$$

Where $m_j = \mathbb{E}(Y_i|\mathcal{H}_j)$, and $\sigma_j^2 = \mathbb{V}ar(Y_i|\mathcal{H}_j)$ for $j = 0,1$. Using this approximation and working out the details, the number of samples required is given by $N = \left[\frac{\sigma_0 Q^{-1}(P_{FA}) - \sigma_1 Q^{-1}(P_D)}{m_1 - m_0}\right]^2$. Note that N depends on the SNR through m_j and σ_j^2, which in turn depend on the number of quantization bins as well as the position of the quantization bins. We compute the above expression for various values of the number of quantization bins M and the performance curves are plotted in Fig. 6. Note that all the curves in the figure are parallel shifts of each other. This shows that the number of samples required in the quantized case is also $O(\frac{1}{SNR^2})$. Thus, we observe an SNR loss for the non-coherent detector.

One important thing to notice is that the SNR loss in the coherent and non-coherent cases are different quantitatively. Table 6.2 illustrates this effect.

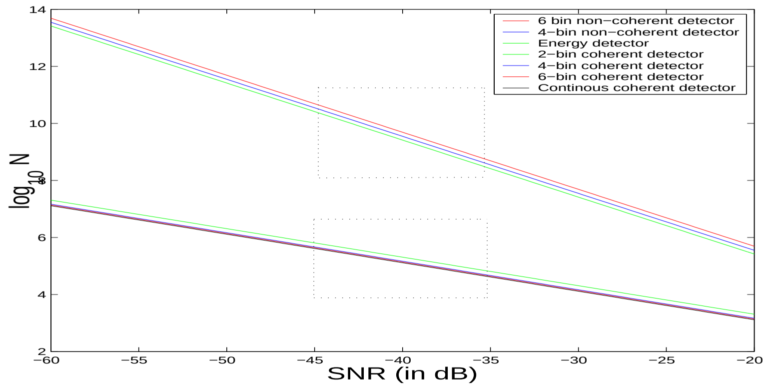


Figure 6: The performance of quantized detectors for both coherent and non-coherent receivers. Here we have plotted the coherent detectors for $M = 2, 4, 6$ and the non-coherent detectors for $M = 4, 6$.

7 Quantization with noise uncertainty

In section 6.1 we have seen that noise uncertainty renders the radiometer useless if the SNR falls below a particular threshold. Does this same behavior hold if the signal is quantized?

We analyze a 2-bit quantizer with noise uncertainty. Assume that the noise variance is σ_i^2 under hypothesis $\mathcal{H}_i$. In this case, it is easy to verify that the quantizer output is a binary random variable with the following pmf's

$$p_1^{(1)} = 1 - p_0^{(1)} = \left[Q\left(\frac{d_1 - \sqrt{P}}{\sigma_1}\right) + Q\left(\frac{d_1 + \sqrt{P}}{\sigma_1}\right)\right]; \qquad p_1^{(0)} = 1 - p_0^{(0)} = 2Q\left(\frac{d_1}{\sigma_0}\right) \tag{2}$$

Clearly, if $p_1^{(0)} = p_1^{(1)}$, then the two hypotheses become identical and hence the signal is *undetectable by any detector*. Fig. 7 shows the performance of a 2-bit quantizer under noise uncertainty and the variation of the location of the wall with the noise uncertainty x. Comparing Fig. 7 with Fig. 4 we see that the location of the SNR walls are almost the same.

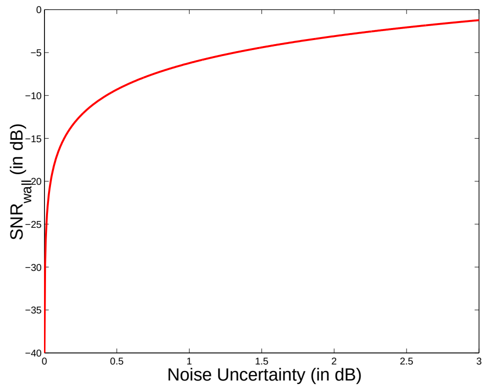
It is important to note that the walls depicted in Fig. 4 were specifically for radiometers,[4] and hence other feature detectors could possibly overcome them. However, the walls for the quantized detector are *absolute*; the signal is completely undetectable with enough noise uncertainty. We believe that an adequate model of receiver noise uncertainty will induce absolute walls for any depth quantizer, though the walls might shift slightly with the quantization depth.

8 Conclusion

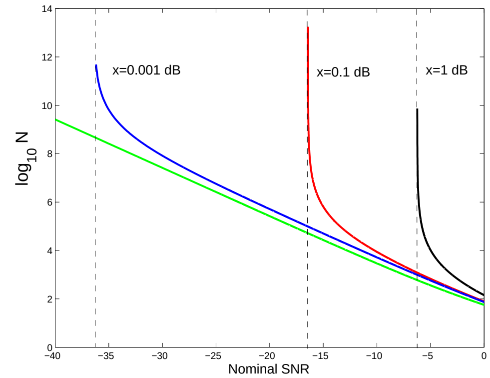
Cognitive radio has the potential to enable the use of vast amounts of underutilized spectrum. However, care must be taken to protect legacy users. The commonsense rule of "Don't transmit if you can decode" is inadequate, both from the standpoint of protecting legacy users, and maximizing the potential for spectrum reuse.

On a practical level, cognitive radios must be able to detect the presence of undecodable signals. Just knowing the modulation scheme and codebook is nearly useless. In the best of

[4]In general, we have made the "crowded spectrum" model and so have not considered the possibility of using out of band measurements to help us detect usage within the band. However, if a known unused band was available *that was known to share identical receiver noise characteristics with the target band,* then we could compare those two bands to detect signals even when we could not decode them. However, we believe that this is unlikely to be helpful in practice for any sort of high-rate signal. That is because much of the uncertainty in receiver noise is not band-specific and we expect that variations of receiver noise characteristics (especially unintentional and intentional transmitters in the vicinity) will cause the walls to exist in general.



(a)



(b)

Figure 7: The left hand plot shows the variation of the SNR_{wall} with noise uncertainty x, and the right hand plot are the performance of quantized detectors for a 2-bit quantization.

cases we achieve only the performance of an energy detector. Even small noise uncertainty causes serious limits in detectability. Quantization makes matters even worse and translates noise uncertainty into absolute barriers to detection. Fortunately, these barriers can all be overcome if the primary user transmits a pilot signal. Ideally, the pilots would be transmitted by the primary receivers themselves but this seems unlikely for practical reasons.

This suggests that cognitive radio architectures need serious thought if we are to get any significant benefits.

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