CHAPTER 7

FIRST-ORDER LOGIC
Outline

- Wumpus world in FOL
- Fun with sentences
- Syntax and semantics of FOL
\[ \exists A \equiv \text{Equality} \]
\[ \equiv \ \left\langle \land \land x, y, a, b \right\rangle \]
\[ \left\langle \land \land \text{Variables} \right\rangle \]
\[ \left\langle \land \land \text{Functions} \right\rangle \]
\[ \left\langle \land \land \text{Predicates} \right\rangle \]
\[ \left\langle \land \land \text{Constants} \right\rangle \]

**Syntax of FOL: Basic Elements**
\[
\begin{align*}
(\text{King of France}) & < (\text{Father of Richard}) \\
E.G., \text{ Brother (King John, Richard, The Lionheart)} & \\
\text{or constant or variable} & \\
\text{Term} = \text{function} (\text{term}_1, \ldots, \text{term}_n) \\
\text{or term}_1 = \text{term}_2 \\
\text{Atomic sentence} = \text{predicate} (\text{term}_1, \ldots, \text{term}_n) \\
\end{align*}
\]
Complex sentences

Complex sentences are made from atomic sentences using connectives

\[ \neg S, \ S_1 \land S_2, \ S_1 \lor S_2, \ S_1 \Rightarrow S_2, \ S_1 \Leftrightarrow S_2 \]

E.g. \( \text{Sibling}(\text{King John}, \text{Richard}) \Rightarrow \text{Sibling}(\text{Richard}, \text{King John}) \)
\[ > (1, 2) \lor \leq (1, 2) \]
\[ > (1, 2) \land \neg > (1, 2) \]
An atomic sentence \( \text{predicate}(\text{term}_1, \ldots, \text{term}_n) \) is true if the objects referred to by \( \text{term}_1, \ldots, \text{term}_n \) are in the relation referred to by \( \text{predicate} \).

Interpretation specifies referents for constants, symbols, objects, relations, and function symbols.

A model contains objects and relations among them.

Sentences are true with respect to a model and an interpretation.

Truth in first-order logic
relations: sets of tuples of objects

\{ \ldots \} \\

functional relations: all tuples of objects + "value" object

\{ \ldots \} \\

relations: sets of tuples of objects

\{ \ldots \}
Everyone is at Berkeley and everyone is smart:

\[
A \forall x (A(t,x, \text{Berkeley}) \land \text{smart}(x))
\]

Typically, is the main connective with \(\forall\).

\[
\ldots \forall
\]

\[
A(t, \text{Berkeley}, \text{Berkeley}) \land \text{smart}(\text{Richard}) \land \text{smart}(\text{Richard}) \land \text{smart}(\text{Richard}) \land \text{smart}(\text{Richard})
\]

is equivalent to the conjunction of instantiations of \(P\):

\[
A \forall x (A(t, x, \text{Berkeley}) \land \text{smart}(x))
\]

Everyone at Berkeley is smart:

\[
A(t, \text{sentence}) \land \text{variables}
\]

universal quantification
Is true if there is anyone who is not at Stanford:

\[
\exists x \in \text{At}(x, \text{Stanford}) \quad \iff \quad \text{Smart}(x)
\]

Common mistake: using as the main connective with \(\exists\).

Typically, \(\forall\) is the main connective with \(\exists\).

\[
\ldots \land \text{At}(\text{Richard}, \text{Stanford}) \land \text{Smart}(\text{Richard})
\]

\[
\text{At}(\text{John}, \text{Stanford}) \land \text{Smart}(\text{John})
\]

is equivalent to the disjunction of instantiations of \(\exists x \in \text{At}(x, \text{Stanford}) \land \text{Smart}(x)\).

Someone at Stanford is smart:

\[
\exists \text{sentence} \langle \text{variables} \rangle
\]

Existential quantification
Quantifier duality: each can be expressed using the other

Everyone in the world is loved by at least one person

$(\forall x \in \text{People}(x), \exists y \in \text{People}(y))$

There is a person who loves everyone in the world

$(\exists y \in \text{People}(y), \forall x \in \text{People}(x))$

$x \in \text{People}(x)$ is not the same as $x \in \text{People}(x)$

$(\exists y \in \text{People}(y), x \in \text{People}(x)$ is the same as $y \in \text{People}(y)$

$(\exists y \in \text{People}(y), x \in \text{People}(x)$ is the same as $x \in \text{People}(x)$

Properties of quantifiers
A first cousin is a child of a parent’s sibling.

One’s mother is one’s female parent.

“Sibling” is reflexive.

Brothers are siblings.

Fun with sentences.
\[ \forall (d', p', \text{parent}(d') \land \text{sibling}(d')) \land \exists p', p \in [p'] \Rightarrow (\forall x, y. \text{Question}(x, y) \land \text{parent}(x) \land \text{parent}(y) \Rightarrow \text{made}(x, y) \land \text{parent}(x, y)) \land \forall x, y. \text{parent}(x, y) \Rightarrow (\text{mother}(x, y) \land \text{parent}(x, y)) \land (\text{sibling}(x, y) \land \text{parent}(x, y)) \land (\text{brother}(x, y) \land \text{parent}(x, y)) \]
\[ ((\text{parent}^f, f) \land (\text{parent}^{m, x} \land (x, f) \land \text{parent}^m) \land (f = m) \land f, m \in x \lor (x = x) ) \] \[ \iff (x, y \in \text{siblings}, x, y) \land \text{A x, y} \text{ siblings in terms of parent:} \]

*E.g.* definition of (full) siblings in terms of parent:

\[ \exists 2 = 2 \text{ is valid} \]

*E.g.* 1 = 2 and \( x = (x) \times (x) \times \) are satisfiable

*E.g.* 1 = 2 and \( x \quad \) and \( x \quad \) refer to the same object

it and only if \( \text{term} \quad \) and \( \text{term} \quad \) refer to the same object

\[ \text{term} \quad \text{term} \quad \text{true under a given interpretation} \]

Equality
ASK(KB, S) returns some/all o such that KB |= S

\[ S \circ = \text{Simulate}(H \\
\{x/x, y/y \} = o \\
S = \text{Simulate}(x, y) \]

\^ denotes the result of plugging o into S.

Given a sentence S and a substitution o,

\text{substitution}(binding \ list) \rightarrow \{a/\text{Shoot} \}

Answer: yes.

L.e., does the KB entail any particular actions at t = 5?

\( \text{ASK}(KB, \text{Action}(a, 5)) \)

\( \text{TTL}(KB, \text{Precept}(\text{Small, breeze, None}, 5)) \)

Suppose a wumpus-world agent is using an FOL KB and perceives a small and a breeze (but no glitter) at t = 5:

Interacting with FOL KBS
Keeping track of changes is essential.

\[ \text{Holding}(\text{Gold}, t) \iff \text{Action}(\text{Grab}, t) \]

At \( \text{ActGold}(t) \land \neg \text{Holding}(\text{Gold}, t) \)

Reflex with internal state: do we have the gold already?

\[ \text{Reflex: ActGold}(t) \iff \text{Action}(\text{Grab}, t) \]

At \( q, l \rightarrow \text{Perception}(\text{Scent}, q, l) \)

At \( q, g \rightarrow \text{Perception}(\text{Smell}, q, g) \)

"Perception" knowledge base for the Wumpus world.
\[
[(y \land \text{Breezy}(y)) \lor (x \land \text{Pit}(x, y) \land \text{Adjacent}(x, y))] \iff (\forall y \text{ Breezy}(y)),
\]

Definition for the Breezy predicate:

Neither of these is complete—e.g., the causal rule doesn't say whether squares far away from pits can be breezy.

\[
(\forall x, y \text{ Breezy}(y) \land \text{Adjacent}(x, y)) \iff (\forall x \text{ Pit}(x, y) \lor \text{Adjacent}(x, y)),
\]

Causal rule—infer effect from cause.

\[
(\forall y \text{ Breezy}(y) \land \text{Adjacent}(x, y) \land \text{Pit}(x, y)) \iff (\forall y \text{ Breezy}(y)),
\]

Diagnostic rule—infer cause from effect.

Squares are breezy near a pit:

\[
(\forall x \text{ Pit}(x) \land \text{Agent}(x)) \iff (\forall t \text{ Breezy}(t)),
\]

\[
(\forall t \text{ Agent}(t), \text{Pit}(t) \land \text{Smelly}(t)) \iff (\forall t \text{ Smelly}(t)),
\]

Properties of locations:

\[
\text{Breezing hidden properties}
\]
Situations are connected by the \textit{Result} function. Result \((a, s)\) is the situation that results from doing \(a\) in \(s\).

Add a situation argument to each non-external predicate.

\textit{E.g.}, \(\text{Hold}(\text{gold}, \text{now})\) rather than just \(\text{Hold}(\text{gold})\). Facts hold in situations, rather than externally.

\textbf{Keeping track of change}
... what about the dust on the gold, wear and tear on gloves,  

Ramiﬁcation problem: real actions have many secondary consequences—

... caveats—what if gold is slippery or nailed down or ...

Qualiﬁcation problem: true descriptions of real actions require endless

\[ (b) \text{ inference—avoid repeated "copy-overs" to keep track of state} \]

\[ (a) \text{ representation—avoid frame axioms} \]

Frame problem: find an elegant way to handle non-change

\[ (\text{HaveAction RESULT (Crab, s)}) \iff (\text{HaveAction \{ s \}) \text{ result\{Crab, s\}}) \]

"Frame" axiom—describe non-changes due to action

\[ (\text{HaveAction RESULT (Crab, s)}) \iff (\text{ActGold \{ s \}) \text{ result\{Crab, s\}}) \]

"Effect" axiom—describe changes due to action

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Describing actions
\[[(\text{Holding}(\text{Gold}, s) \lor \text{Release}) \wedge \\
(\text{Grad} \lor \text{AIGold}(s)) \Rightarrow ((\text{Grad}, \text{Result}(a,s)) \wedge \text{A's Holding}(\text{Gold}, \text{Result}(a,s)))]\\
\Rightarrow (\text{For holding the Gold:})\\
\]

\[p \text{ true already and no action made} \quad \wedge \quad \\
\left[p \text{ true afterwards, an action made} \right] \quad \Rightarrow \quad \text{p true afterwards}
\]

Each axiom is "about a predicate (not an action per se):"

Successor-state axioms solve the representational frame problem

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**Describing actions II**
that \( S_0 \) is the only situation described in the KB. This assumes that the agent is interested in plans starting at \( S_0 \) and

i.e. 'Go forward and then grab the gold'

\[
\{ \text{Result(Grab, Result(Forward, S_0))} \}
\]

Answer: \( s' \in \text{Result(Grab, Result(Forward, S_0))} \)

i.e. 'In what situation will I be holding the gold?'

\[
\text{Query: ASK} \{ \text{KB} \models \exists s' \in \text{Result(Grab, Result(Forward, S_0))} \}
\]

Initial condition in KB: \( \forall t' [t', T \models \text{Gold}(T, t')] \) \( \forall t' [t', T \models \text{Agent}(T, t')] \)
Planning systems are special-purpose reasoners designed to do this type of inference more efficiently than a general-purpose reasoner.

Given a plan \( P \) and a state \( s \), the \( PlanResult(p, s) \) is the result of executing \( P \) in \( s \). The \( PlanResult(p, s) \) is the result of executing \( P \) in \( s \).

Represent plans as action sequences [\( a_1, a_2, \ldots, a_n \)]

Making plans: a better way
can formulate planning as inference on a situation calculus KB

- conventions for describing actions and change in FOL

Situational calculus:

- increased expressive power: sufficient to define wumpus world

- syntax: constants, functions, predicates, equality, quantifiers

- objects and relations are semantic primitives

First-order logic:

Summary