CHAPTER 6
Logical agents
Outline

- Inference rules
- Normal forms
- Propositional (Boolean) logic
- Logic in general
- Wumpus world
- Knowledge bases
Or at the implementation level:

<table>
<thead>
<tr>
<th>Knowledge base = set of sentences in a formal language</th>
</tr>
</thead>
<tbody>
<tr>
<td>domain-independent algorithms</td>
</tr>
<tr>
<td>domain-specific content</td>
</tr>
</tbody>
</table>
Deuce appropriate actions
Deuce hidden properties of the world
Update internal representations of the world
Incorporate new percepts
Represent states, actions, etc.

The agent must be able to:

```
function KB-Agent(percept) returns an action

static KB, a knowledge base

tell KB, AMAKE-AGENT-SENTENCE(action, t)

tell KB, MAKE-AGENT-SENTENCE(QUERY, t)

ask KB, MAKE-AGENT-QUERY(QUERY, t)

tell KB, MAKE-PERCEPT-SENTENCE(percept, t)

if a counter, initially 0, indicating time
```

A simple knowledge-based agent
Releasing drops the gold in the same square
Grabbing picks up the gold if in the same square
Shooting uses up the only arrow
Shooting kills the wumpus if you are facing it
Glitter if and only if gold is in the same square
Squares adjacent to pit are breezy
Squares adjacent to wumpus are smelly

Environment
Without entering pit or wumpus square
Goals: Get gold back to start

Percepts Breeze, Glitter, Smell
Actions Left turn, Right turn,
Forward, Grab, Release, Shoot

Wumpus World Page description
Is the world discrete?

Is the world static?

Is the world fully accessible?

Is the world deterministic?
Is the world discrete? Yes

Is the world static? Yes

Is the world fully accessible? No—only local perception

Is the world deterministic? Yes—outcomes exactly specified

Wumpus World characterization
<table>
<thead>
<tr>
<th></th>
<th>A</th>
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<tbody>
<tr>
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<td>OK</td>
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</tbody>
</table>

**Exploring a Wumpus World**
wumpus wasn’t there safe
wumpus was there dead safe
shoot straight ahead
Can use a strategy of action:
cannot move
Smell in (1, 1)

(2, 2) is most likely to have a pit
Assuming pits uniformly distributed,
no safe actions

Breeze in (1, 2) and (2, 1)
Logic in General
<table>
<thead>
<tr>
<th>Propositional Logic</th>
<th>First-order Logic</th>
<th>Temporal Logic</th>
<th>Probability Theory</th>
<th>Fuzzy Logic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Facts</td>
<td>Facts, objects, relations, times</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>False, unknown</td>
<td>False, objects, relations, times</td>
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</tbody>
</table>

Epistemological commitment: what states of knowledge?


Logics are characterized by what they commit to as „primitives“

**Types of Logic**
Either the Giants won or the Reds won

E.g., the KB containing "the Giants won" and "the Reds won"

is true in all worlds where KB is true if and only if

Knowledge base KB entails sentence \( \mathcal{L} \)

\[ KB \models \mathcal{L} \]
\[ \models \emptyset \]

\[ \models \emptyset \]

We say \( m \) is a model of a sentence \( \alpha \) if \( \alpha \) is true in \( m \)

structured worlds with respect to which truth can be evaluated

Logicians typically think in terms of models, which are formally

Models
from what is known by the $\gamma B$.

That is, the procedure will answer any question whose answer follows
sound and complete inference procedure.

Preview: we will define a logic (first-order logic) which is expressive

\[ \text{Completeness: } \forall \alpha, \text{ if true that } \gamma B \vdash \alpha \]

\[ \text{Soundness: } \forall \alpha, \text{ if true that } \gamma B \models \alpha \]

Sentence $\alpha$ can be derived from $\gamma B$ by procedure $\gamma$.
If $S_1$ and $S_2$ is a sentence, $S_3 \iff S_4$ is a sentence.

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The proposition symbols $P_1$, $P_2$, etc. are sentences.

Propositional Logic is the simplest logic—illustrates basic ideas.
Rules for evaluating truth with respect to a model $M$:

Each model specifies true/false for each proposition symbol.

Propositional Logic: Semantics
<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$KB$</th>
<th>$B \land \neg C$</th>
<th>$B \land C$</th>
<th>$A \land C$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{True}$</td>
<td>$\text{True}$</td>
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Check all possible models—$\alpha$ must be true whenever $KB$ is true.

Is it the case that $KB \models \alpha$?

Let $\alpha = B \land A$ and $KB = B \land C = A \land C$.

**Propositional Inference: Enumeration Method**
### Propositional Inference: Solution

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>KB</th>
<th>A \land C</th>
<th>B \land \neg C</th>
<th>C</th>
<th>B</th>
<th>A</th>
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This table illustrates the truth values for the given propositions under various conditions. The columns represent different logical expressions and their outcomes based on the truth values of A, B, and C. The rows display the corresponding truth assignments for each expression.
\[ B \iff (D \lor C) \text{ and } A \iff B \]

Often written as set of implications:

\[ (D \leftarrow \lor C \lor \leftarrow \lor B) \lor (B \leftarrow \lor C \lor \leftarrow \lor A) \lor (B \lor \leftarrow \lor A) \]

\textit{E.g.,} "\textbf{Horn Form} (restricted)"

\[ (D \leftarrow \lor B \leftarrow ) \land (C \leftarrow \lor B \leftarrow ) \land (D \leftarrow \lor A) \land (C \leftarrow \lor A) \land (B \lor \leftarrow \lor A) \]

\textit{E.g.,} "\textbf{Disjunctive Normal Form} (DNF—universal)"

\[ (D \leftarrow \lor C \leftarrow \lor B \leftarrow ) \lor (A \leftarrow \lor B \leftarrow ) \lor (A \leftarrow \lor C \leftarrow \lor B) \land (B \lor \leftarrow \lor C) \land (C \lor \leftarrow \lor B) \land (A \leftarrow \lor C) \land (A \leftarrow \lor B) \land (B \leftarrow \lor A) \land (C \leftarrow \lor A) \land (D \lor \leftarrow \lor A) \]

\textit{E.g.,} "\textbf{Conjunctive Normal Form} (CNF—universal)"

Often expressed in \textit{standardized forms},

\textit{Other approaches to inference use syntactic operations on sentences,}

\textbf{Normal Forms}
Validity and Satisfiability

\[ \forall x \exists y \quad (x \neq y) \]

A sentence is unsatisfiable if it is true in no models.

\[ \forall A \neq \forall A \]

A sentence is satisfiable if it is true in some model.

\[ \forall B \subseteq \forall \forall A \]

A sentence is valid if it is true in all models.
Can use inference rules as operators in a standard search alg.

Proof = a sequence of inference rule applications

Legitimate (sound) generation of new sentences from old

Application of inference rules

E.g., the GSAT algorithm (Ex. 6.15)

Heuristic search in model space (sound but incomplete)

Truth table enumeration (sound and complete for propositional)

Model checking

Proof methods divide into (roughly) two kinds:

Proof methods
Can be used with forward chaining or backward chaining

\[ \models \phi \]

\[ \models \psi \iff \forall \phi \vee \cdots \vee \psi \]

Modus Ponens (for Horn form): Complete for Horn KBS

\[ \models \phi \lor \phi \]

Resolution (for CNF): Complete for propositional logic

Inference rules for propositional logic
Truth table method is sound and complete for propositional logic

Propositional logic suffices for some of these tasks.

Propositional logic, reasoning by cases, etc.

Wumpus world requires the ability to represent partial and negated sentences

- completeness: derivations only entail sentences
- soundness: derivations produce only entail sentences
- inference: deriving sentences from other sentences
- entailment: necessary truth of one sentence given another
- semantics: truth of sentences wrt models
- syntax: formal structure of sentences

Basic concepts of logic:

- to derive new information and make decisions
- logical agents apply inference to a knowledge base

Summary