The uniqueness quantifier $\exists!$

We have seen how to use $\exists$ to say that *some* objects exist, but there is no concise way to say that a *unique* object satisfying some predicate exists. Some authors use the notation

$$\exists!\, x \;\; King(x)$$

to mean "there exists a unique object x satisfying $King(x)$" or more informally, "there's exactly one King." You should think of this not as adding a new quantifier, $\exists!$, but rather as being a convenient abbreviation for the longer sentence

$$\exists\, x \;\; King(x) \land \forall\, y \;\; King(y) \;\Rightarrow\; x = y$$

Of course, if we knew from the start there was only one King, we probably would have used the constant *King* rather than the predicate $King(x)$. A more complex example is "Every country has exactly one ruler":

$$\forall\, c \;\; Country(c) \;\Rightarrow\; \exists!\, r \;\; Ruler(r, c)$$

The uniqueness operator ι

It is convenient to use $\exists!$ to state uniqueness, but sometimes it is even more convenient to have a term representing the unique object directly. The notation $\iota\, x\, P(x)$ is commonly used for this. (The symbol ι is the Greek letter iota.) To say that "the unique ruler of Freedonia is dead" or equivalently "the r that is the ruler of Freedonia is dead," we would write:

$$Dead(\iota\, r\, Ruler(r, Freedonia))$$

This is just an abbreviation for the following sentence:

$$\exists!\, r \;\; Ruler(r, Freedonia) \land \forall\, s \;\; Ruler(s, Freedonia) \;\Rightarrow\; Dead(s)$$

Notational variations

The first-order logic notation used in this book is the *de facto* standard for artificial intelligence; one can safely use the notation in a journal article without defining it, because it is recognizable to most readers. Several other notations have been developed, both within AI and especially in other fields that use logic, including mathematics, computer science, and philosophy. Here are some of the variations:

Syntax item	**This book**	**Others**
Negation (not)	$\neg P$	$\sim P \quad \overline{P}$
Conjunction (and)	$P \land Q$	$P \& Q \quad P \cdot Q \quad PQ \quad P, Q$
Disjunction (or)	$P \lor Q$	$P \mid Q \quad P; Q \quad P + Q$
Implication (if)	$P \;\Rightarrow\; Q$	$P \rightarrow Q \quad P \supset Q$
Equivalence (iff)	$P \;\Leftrightarrow\; Q$	$P \equiv Q \quad P \leftrightarrow Q$
Universal (all)	$\forall\, x \;\; P(x)$	$(\forall x)P(x) \quad \bigwedge x\, P(x) \quad P(x)$
Existential (exists)	$\exists\, x \;\; P(x)$	$(\exists x)P(x) \quad \bigvee x\, P(x) \quad P(Skolem_i)$
Relation	$R(x, y)$	$(R\; x\; y) \quad Rxy \quad xRy$