IMPLICATION
PREMISE
CONCLUSION

$\Rightarrow$ (implies). A sentence such as $(P \wedge Q) \Rightarrow R$ is called an **implication** (or conditional). Its **premise** or **antecedent** is $P \wedge Q$, and its **conclusion** or **consequent** is R. Implications are also known as **rules** or **if–then** statements. The implication symbol is sometimes written in other books as $\supset$ or $\rightarrow$.

EQUIVALENCE

$\Leftrightarrow$ (equivalent). The sentence $(P \wedge Q) \Leftrightarrow (Q \wedge P)$ is an **equivalence** (also called a **biconditional**).

NEGATION

$\neg$ (not). A sentence such as $\neg P$ is called the **negation** of P. All the other connectives combine two sentences into one; $\neg$ is the only connective that operates on a single sentence.

ATOMIC SENTENCES
COMPLEX SENTENCES
LITERAL

Figure 6.8 gives a formal grammar of propositional logic; see page 854 if you are not familiar with the BNF notation. The grammar introduces **atomic sentences**, which in propositional logic consist of a single symbol (e.g., P), and **complex sentences**, which contain connectives or parentheses (e.g., $P \wedge Q$). The term **literal** is also used, meaning either an atomic sentence or a negated atomic sentence.

$$
\begin{array}{rcl}
\textit{Sentence} & \rightarrow & \textit{AtomicSentence} \mid \textit{ComplexSentence} \\
\\
\textit{AtomicSentence} & \rightarrow & \textbf{\textit{True}} \mid \textbf{\textit{False}} \\
 & \mid & P \mid Q \mid R \mid \ldots \\
\textit{ComplexSentence} & \rightarrow & (\ \textit{Sentence}\ ) \\
 & \mid & \textit{Sentence Connective Sentence} \\
 & \mid & \neg \textit{Sentence} \\
\\
\textit{Connective} & \rightarrow & \wedge \mid \vee \mid \Leftrightarrow \mid \Rightarrow
\end{array}
$$

Figure 6.8 A BNF (Backus–Naur Form) grammar of sentences in propositional logic.

Strictly speaking, the grammar is ambiguous—a sentence such as $P \wedge Q \vee R$ could be parsed as either $(P \wedge Q) \vee R$ or as $P \wedge (Q \vee R)$. This is similar to the ambiguity of arithmetic expressions such as $P + Q \times R$, and the way to resolve the ambiguity is also similar: we pick an order of precedence for the operators, but use parentheses whenever there might be confusion. The order of precedence in propositional logic is (from highest to lowest): $\neg$, $\wedge$, $\vee$, $\Rightarrow$, and $\Leftrightarrow$. Hence, the sentence

$$\neg P \vee Q \wedge R \Rightarrow S$$

is equivalent to the sentence

$$((\neg P) \vee (Q \wedge R)) \Rightarrow S.$$