function grows no faster than the logarithm of the actual path cost. In mathematical notation, the condition for subexponential growth is that

$$|h(n) - h^*(n)| \leq O(\log h^*(n)),$$

where $h^*(n)$ is the *true* cost of getting from n to the goal. For almost all heuristics in practical use, the error is at least proportional to the path cost, and the resulting exponential growth eventually overtakes any computer. Of course, the use of a good heuristic still provides enormous savings compared to an uninformed search. In the next section, we will look at the question of designing good heuristics.

Computation time is not, however, A*'s main drawback. Because it keeps all generated nodes in memory, A* usually runs out of space long before it runs out of time. Recently developed algorithms have overcome the space problem without sacrificing optimality or completeness. These are discussed in Section 4.3.

4.2 HEURISTIC FUNCTIONS

So far we have seen just one example of a heuristic: straight-line distance for route-finding problems. In this section, we will look at heuristics for the 8-puzzle. This will shed light on the nature of heuristics in general.

The 8-puzzle was one of the earliest heuristic search problems. As mentioned in Section 3.3, the object of the puzzle is to slide the tiles horizontally or vertically into the empty space until the configuration matches the goal configuration (Figure 4.7).

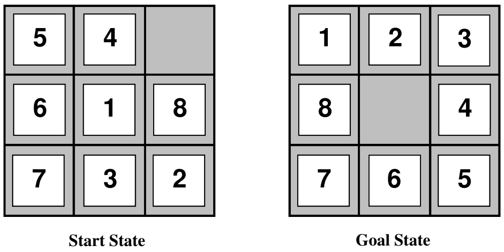

Figure 4.7 A typical instance of the 8-puzzle.

The 8-puzzle is just the right level of difficulty to be interesting. A typical solution is about 20 steps, although this of course varies depending on the initial state. The branching factor is about 3 (when the empty tile is in the middle, there are four possible moves; when it is in a corner there are two; and when it is along an edge there are three). This means that an exhaustive search to depth 20 would look at about $3^{20} = 3.5 \times 10^9$ states. By keeping track of repeated states, we could cut this down drastically, because there are only $9! = 362,880$ different arrangements of 9 squares. This is still a large number of states, so the next order of business is to find a good