CS 289 Knowledge representation and reasoning
Fall 1995 Stuart Russell **Answers, Weeks 1–3**

**8.2** One tricky part is that English is ambiguous. One sense of the word "water" includes ice ("that's frozen water"), while another sense excludes it: ("that's not water—it's ice"). We will stick with the first sense. The other tricky part is that we are dealing with objects that change (freeze and melt) over time. Thus, it won't do to say $w \in Liquid$, because $w$ (a mass of water) might be a liquid at one time and a solid at another. For simplicity, we will use a situation calculus representation, with sentences such as $T(w \in Liquid, s)$.

**a.** "Water is a liquid between 0 and 100 degrees." We will translate this as "For any water and any situation, the water is liquid iff and only if the water's temperature in the situation is between 0 and 100 centigrade."

$$\begin{aligned}&\forall\, w, s \;\; w \in Water \Rightarrow \\ &\quad (Centigrade(0) < Temperature(w, s) < Centigrade(100)) \Leftrightarrow T(w \in Liquid, s)\end{aligned}$$

**b.** "Water boils at 100 degrees." It is a good idea here to do some tool-building. On page 243 we used $MeltingPoint$ as a predicate applying to individual instances of a substance. Here, we will define $SBoilingPoint$ to denote the boiling point of all instances of a substance. The basic meaning of boiling is that instances of the substance becomes gaseous above the boiling point:

$$\begin{aligned}&SBoilingPoint(c, bp) \Leftrightarrow \\ &\quad \forall\, x, s \;\; x \in c \Rightarrow (\forall\, t \;\; T(Temperature(x, t), s) \land t > bp \Rightarrow T(x \in Gas, s))\end{aligned}$$

Then we need only say $SBoilingPoint(Water, Centigrade(100))$.

**c.** "The water in John's water bottle is frozen." We will use the constant $Now$ to represent the situation in which this sentence holds. Note that it is easy to make mistakes in which one asserts that only some of the water in the bottle is frozen.

$$\begin{aligned}&\exists\, b \;\; \forall\, w \;\; w \in Water \land b \in WaterBottles \land Has(John, b, Now) \land Inside(w, b, Now) \\ &\quad \Rightarrow T(w \in Solid, Now)\end{aligned}$$

**d**. "Perrier is a kind of water."

$$Perrier \subset Water$$

**e**. "John has Perrier in his water bottle."

$$\exists\, b\ \ \forall\, w\ \ w \in Water \wedge b \in WaterBottles \wedge Has(John, b, Now) \wedge Inside(w, b, Now)$$
$$\Rightarrow w \in Perrier$$

**f**. "All liquids have a freezing point." Presumably what this means is that all substances that are liquid at room temperature have a freezing point. If we use $RTLiquidSubstance$ to denote this class of substances, then we have

$$\forall\, c\ \ RTLiquidSubstance(c) \Rightarrow \exists\, t\ \ SFreezingPoint(c, t)$$

where $SFreezingPoint$ is defined similarly to $SBoilingPoint$. Note that this statement is false for categoris such as "blue liquid."

**8.6** This is mainly a matter of keeping the quantification straight and having a clear mental picture as in Figure 8.4.

**a**. $T(c, i)$ means that some event of type $c$ occurs throughout the interval $i$:

$$\forall\, c, i\ \ T(c, i) \Leftrightarrow (\exists\, e\ \ e \in c \wedge During(i, e))$$

**b**. A $p \wedge q$ event (also known as an $And(p, q)$ event) is one in which both $p$ and $q$ occur throughout the duration of the event. There is only one way this can happen: both $p$ and $q$ have to persist for the whole interval. Thus $p \wedge q$ and $p \underline{\wedge} q$ coincide.

**c**. $T(p \vee q, i)$ means that a $p$ event or a $q$ event is going on at any point in the interval $i$:

$$\forall\, p, q, i\ \ T(p \vee q, i) \Leftrightarrow (\forall\, j\ \ During(j, i) \Rightarrow (E(p, j) \vee E(q, j)))$$

Remember (page 238) that $T(p \underline{\vee} q, i)$ means that there is one big event that persists through all of interval $i$ that is either a $p$ or a $q$:

$$\forall\, p, q, i\ \ T(p \underline{\vee} q, i) \Leftrightarrow T(p, i) \vee T(q, i)$$

**d**. $T(\underline{\neg} p, i)$ should mean that there is no event of type $p$ going on in any subinterval of $i$, while $T(\neg p, i)$ should mean that there is no single event of type $p$ that spans all of $i$, even though there may be one or more events of type $p$ for subintervals of $i$:

$$\forall\, p, i\ \ T(\underline{\neg} p, i) \Leftrightarrow \neg \exists\, j\ \ SubEvent(j, i) \wedge T(p, j)$$
$$\forall\, p, i\ \ T(\neg p, i) \Leftrightarrow \neg T(p, i)$$

One way to decide if we made the right choice for what $\neg$ and $\underline{\neg}$ should mean is to insist that they obey De Morgan's laws, with consistent absence or presence of the underline:

$$T(p \vee q, i) \Leftrightarrow T(\neg(\neg p \wedge \neg q), i)$$
$$T(p \underline{\vee} q, i) \Leftrightarrow T(\underline{\neg}(\underline{\neg} p \wedge \underline{\neg} q), i)$$