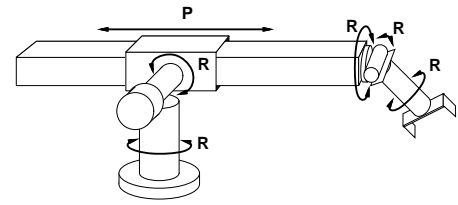


Manipulators



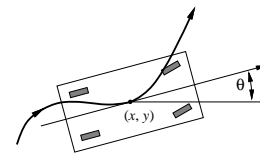
Configuration of robot specified by 6 numbers  
 ⇒ 6 degrees of freedom (DOF)

6 is the minimum number required to position end-effector arbitrarily.  
 For dynamical systems, add velocity for each DOF.

Outline

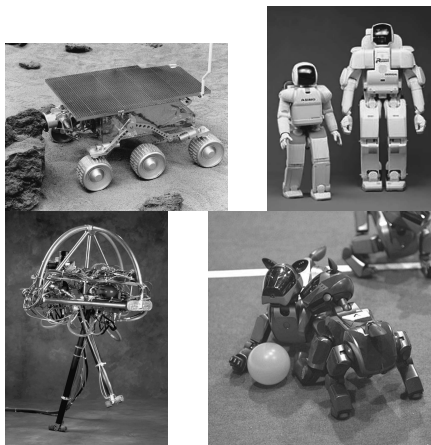
- Robots, Effectors, and Sensors
- Localization and Mapping
- Motion Planning
- Motor Control

Non-holonomic robots



A car has more DOF (3) than controls (2), so is **non-holonomic**;  
 cannot generally transition between two infinitesimally close configurations

Mobile Robots



Sensors

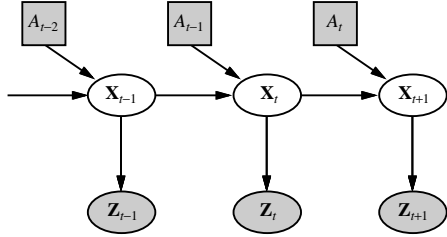
**Range finders:** sonar (land, underwater), laser range finder, radar (aircraft), tactile sensors, GPS



**Imaging sensors:** cameras (visual, infrared)  
**Proprioceptive sensors:** shaft decoders (joints, wheels), inertial sensors, force sensors, torque sensors

## Localization—Where Am I?

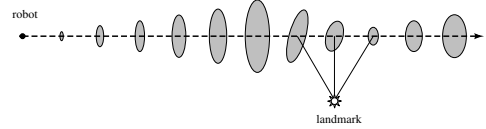
Compute current location and orientation (pose) given observations:



Chapter 25 7

## Localization contd.

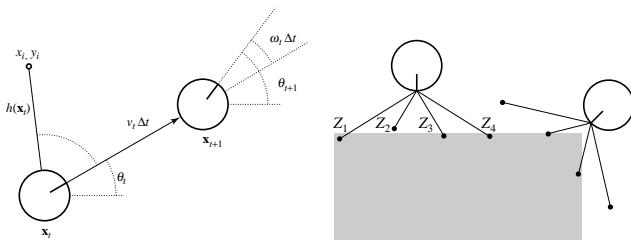
Can also use **extended Kalman filter** for simple cases:



Assumes that landmarks are *identifiable*—otherwise, posterior is multimodal

Chapter 25 10

## Localization contd.



Assume Gaussian noise in motion prediction, sensor range measurements

Chapter 25 8

## Mapping

Localization: given map and observed landmarks, update pose distribution

Mapping: given pose and observed landmarks, update map distribution

SLAM: given observed landmarks, update pose and map distribution

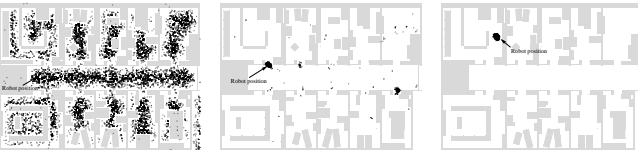
Probabilistic formulation of SLAM:

add landmark locations  $L_1, \dots, L_k$  to the state vector,  
proceed as for localization

Chapter 25 11

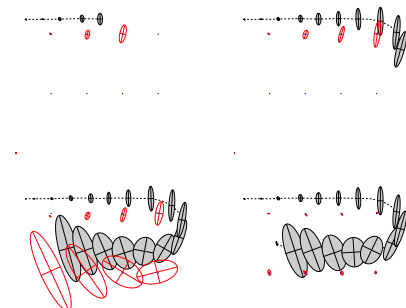
## Localization contd.

Can use **particle filtering** to produce approximate position estimate



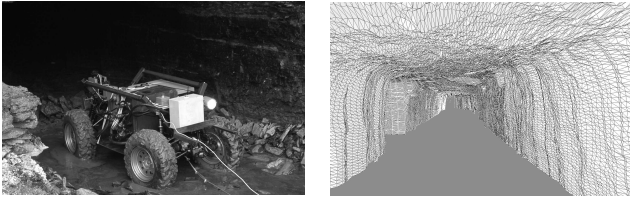
Chapter 25 9

## Mapping contd.



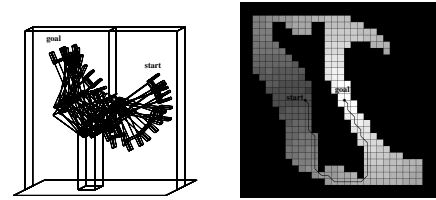
Chapter 25 12

## 3D Mapping example



Chapter 25 13

## Cell decomposition example

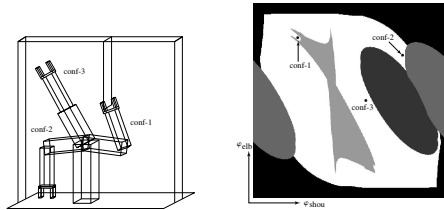


Problem: may be no path in pure freespace cells  
Solution: recursive decomposition of mixed (free+obstacle) cells

Chapter 25 16

## Motion Planning

Idea: plan in **configuration space** defined by the robot's DOFs

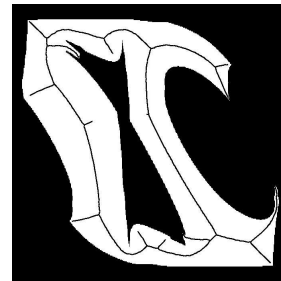


Solution is a point trajectory in free C-space

Chapter 25 14

## Skeletonization: Voronoi diagram

Voronoi diagram: locus of points equidistant from obstacles



Problem: doesn't scale well to higher dimensions

Chapter 25 17

## Configuration space planning

Basic problem:  $\infty^d$  states! Convert to **finite** state space.

**Cell decomposition:**

divide up space into simple **cells**,  
each of which can be traversed "easily" (e.g., convex)

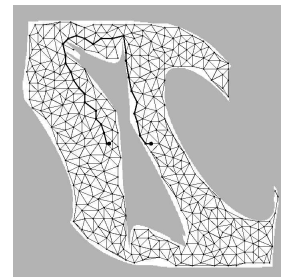
**Skeletonization:**

identify finite number of easily connected points/lines  
that form a graph such that any two points are connected  
by a path on the graph

Chapter 25 15

## Skeletonization: Probabilistic Roadmap

A probabilistic roadmap is generated by generating random points in C-space and keeping those in freespace; create graph by joining pairs by straight lines



Problem: need to generate enough points to ensure that every start/goal pair is connected through the graph

Chapter 25 18

## Motor control

Can view the motor control problem as a search problem in the **dynamic** rather than **kinematic** state space:

- state space defined by  $x_1, x_2, \dots, \dot{x}_1, \dot{x}_2, \dots$
- continuous, high-dimensional (Sarcos humanoid: 162 dimensions)

Deterministic control: many problems are exactly solvable esp. if linear, low-dimensional, exactly known, observable

Simple **regulatory control** laws are effective for specified motions

Stochastic **optimal control**: very few problems exactly solvable  
 ⇒ approximate/adaptive methods

## Simple learning algorithm: Stochastic gradient

Minimize  $E_\theta[y^2]$  by gradient descent:

$$\begin{aligned} \nabla_{\theta_0} E_\theta[y^2] &= \nabla_{\theta_0} \int P_{\theta_0}(\theta) F(\theta)^2 d\theta \\ &= \int \frac{\nabla_{\theta_0} P_{\theta_0}(\theta)}{P_{\theta_0}(\theta)} F(\theta)^2 P_{\theta_0}(\theta) d\theta \\ &= E_\theta \left[ \frac{\nabla_{\theta_0} P_{\theta_0}(\theta)}{P_{\theta_0}(\theta)} y^2 \right] \end{aligned}$$

Given samples  $(\theta_j, y_j)$ ,  $j = 1, \dots, N$ , we have

$$\nabla_{\theta_0} \hat{E}_\theta[y^2] = \frac{1}{N} \sum_{j=1}^N \frac{\nabla_{\theta_0} P_{\theta_0}(\theta_j)}{P_{\theta_0}(\theta_j)} y_j^2$$

For Gaussian noise with covariance  $\Sigma$ , i.e.,  $P_{\theta_0}(\theta) = N(\theta_0, \Sigma)$ , we obtain

$$\nabla_{\theta_0} \hat{E}_\theta[y^2] = \frac{1}{N} \sum_{j=1}^N \Sigma^{-1} (\theta_j - \theta_0) y_j^2$$

## Biological motor control

Motor control systems are characterized by massive redundancy

Infinitely many trajectories achieve any given task

E.g., 3-link arm moving in plane throwing at a target  
 simple 12-parameter controller, one degree of freedom at target  
 11-dimensional continuous space of optimal controllers

Idea: if the arm is noisy, only "one" optimal policy minimizes error at target

I.e., noise-tolerance might explain actual motor behaviour

Harris & Wolpert (*Nature*, 1998): signal-dependent noise explains eye saccade velocity profile perfectly

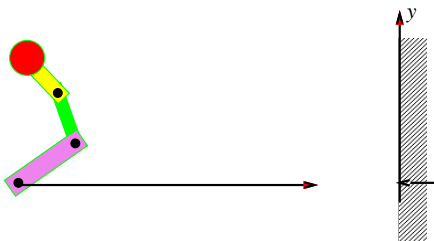
## What the algorithm is doing



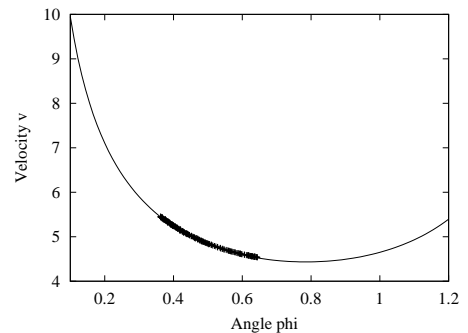
## Setup

Suppose a controller has "intended" control parameters  $\theta_0$  which are corrupted by noise, giving  $\theta$  drawn from  $P_{\theta_0}$

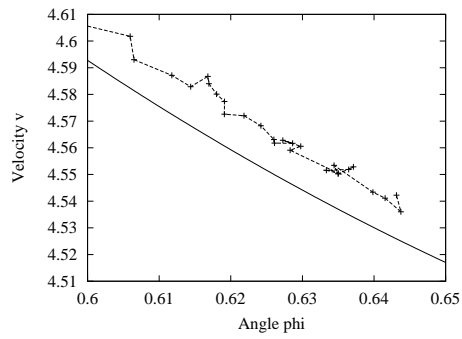
Output (e.g., distance from target)  $y = F(\theta)$ ;



## Results for 2-D controller

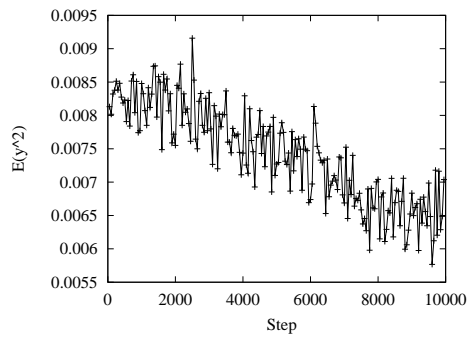


## Results for 2-D controller



Chapter 25 25

## Results for 2-D controller



Chapter 25 26

## Summary

The rubber hits the road

Mobile robots and manipulators

Degrees of freedom to define robot configuration

Localization and mapping as probabilistic inference problems  
(require good sensor and motion models)

Motion planning in configuration space  
requires some method for finitization

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