Propositional inference, propositional agents

Chapter 7.5–7.7

Outline

- ♢ Inference rules and theorem proving
  - – forward chaining
  - – backward chaining
  - – resolution
- ♢ Efficient model checking algorithms
- ♢ Boolean circuit agents

Proof methods

Proof methods divide into (roughly) two kinds:

Application of inference rules

- Legitimate (sound) generation of new sentences from old
- Proof = a sequence of inference rule applications
  Can use inference rules as "actions" in a standard search alg.
- Typically require translation of sentences into a normal form

Model checking

truth table enumeration (always exponential in n)
improved backtracking, e.g., Davis–Putnam–Logemann–Loveland
heuristic search in model space (sound but incomplete)
  e.g., min-conflicts-like hill-climbing algorithms

Forward and backward chaining

Horn Form (restricted)

KB = **conjunction** of **Horn clauses**

Horn clause =

- ♢ proposition symbol; or
- ♢ (conjunction of symbols) $\Rightarrow$ symbol

E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

Modus Ponens (for Horn Form): complete for Horn KBs

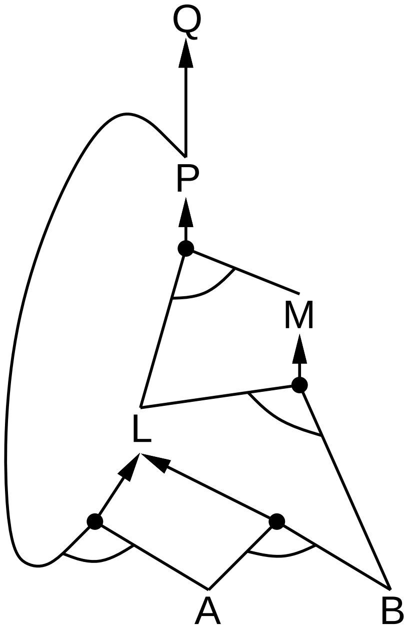
$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \cdots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

Can be used with forward chaining or backward chaining.
These algorithms are very natural and run in **linear** time

Forward chaining

Idea: fire any rule whose premises are satisfied in the KB,
add its conclusion to the KB, until query is found

$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B

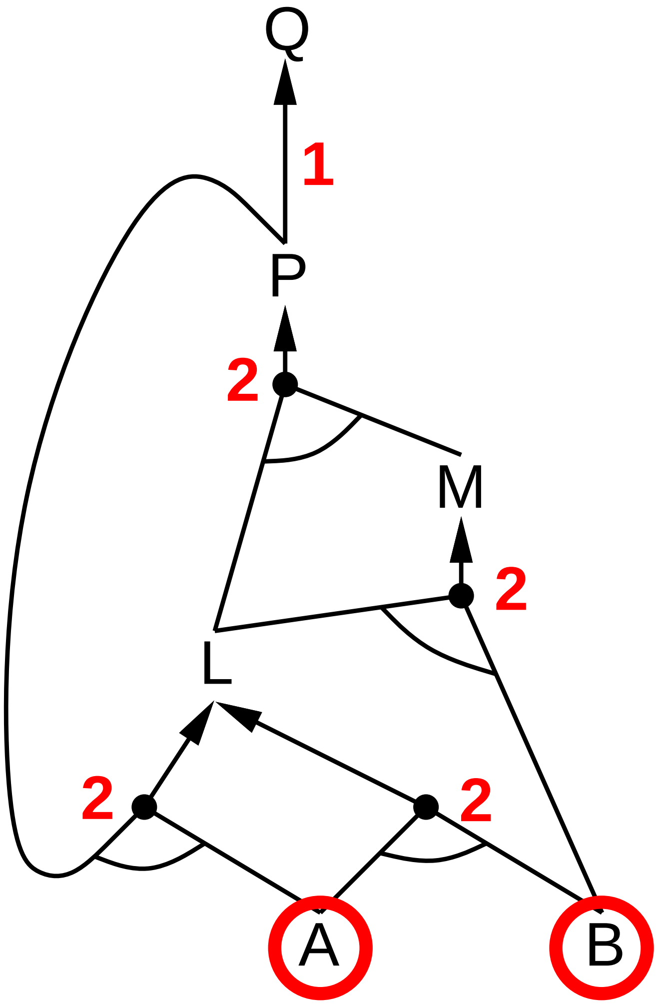


Forward chaining algorithm

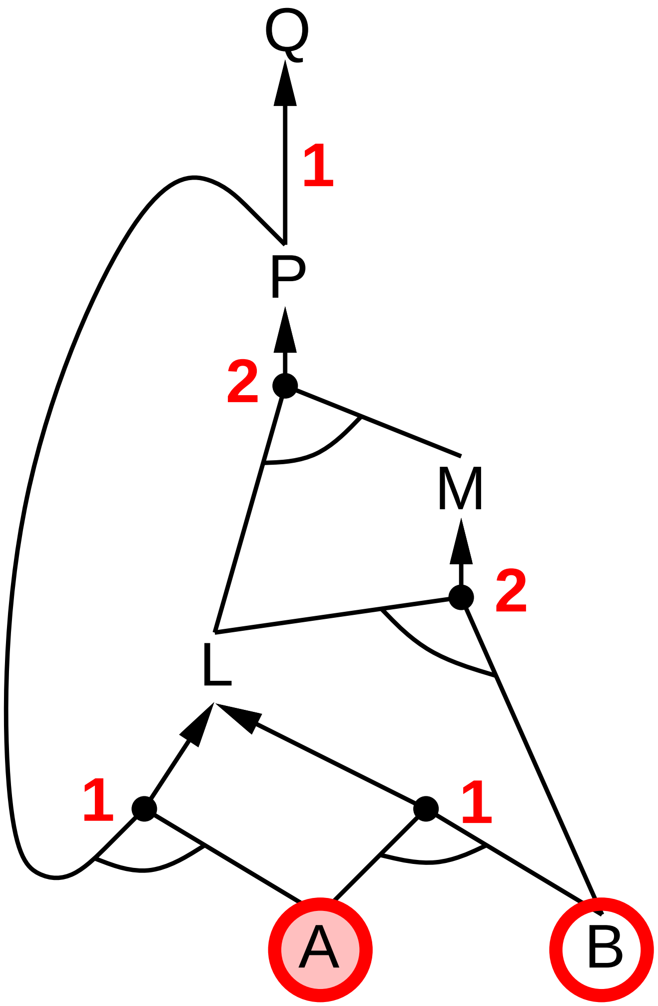
```
function PL-FC-ENTAILS?($KB$, $q$) returns $true$ or $false$
    inputs: $KB$, the knowledge base, a set of propositional Horn clauses
            $q$, the query, a proposition symbol
   local variables: $count$, a table, indexed by clause, initially the number of premises
                    $inferred$, a table, indexed by symbol, each entry initially $false$
                    $agenda$, a list of symbols, initially the symbols known in $KB$

    while $agenda$ is not empty do
        $p \leftarrow$ POP($agenda$)
        unless $inferred[p]$ do
            $inferred[p] \leftarrow true$
            for each Horn clause $c$ in whose premise $p$ appears do
                decrement $count[c]$
                if $count[c] = 0$ then do
                    if HEAD[$c$] $= q$ then return $true$
                    PUSH(HEAD[$c$], $agenda$)
    return $false$
```

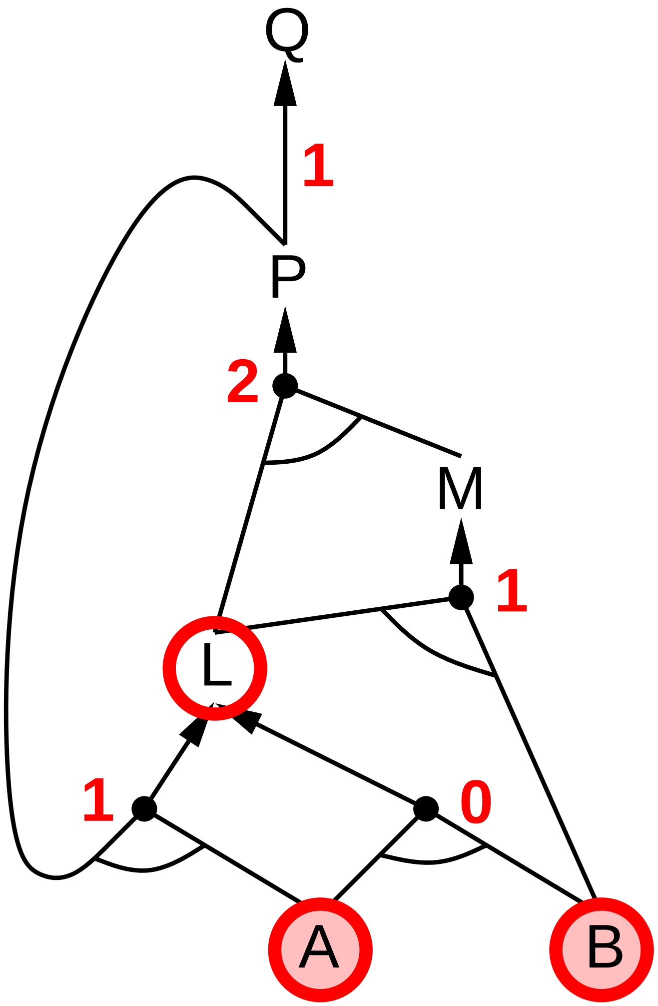
Forward chaining example



Forward chaining example



Forward chaining example



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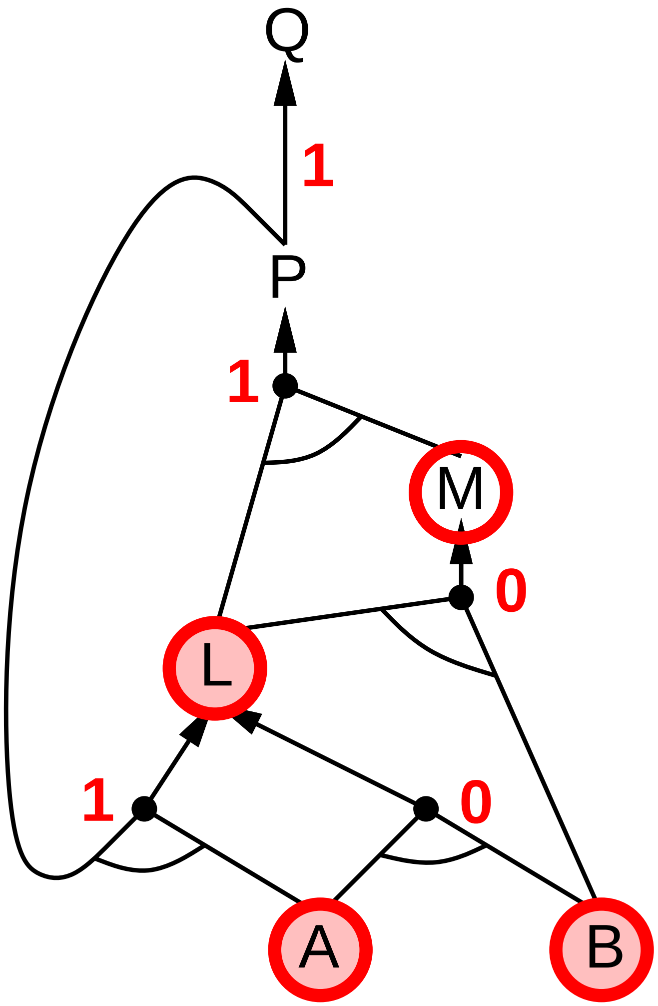

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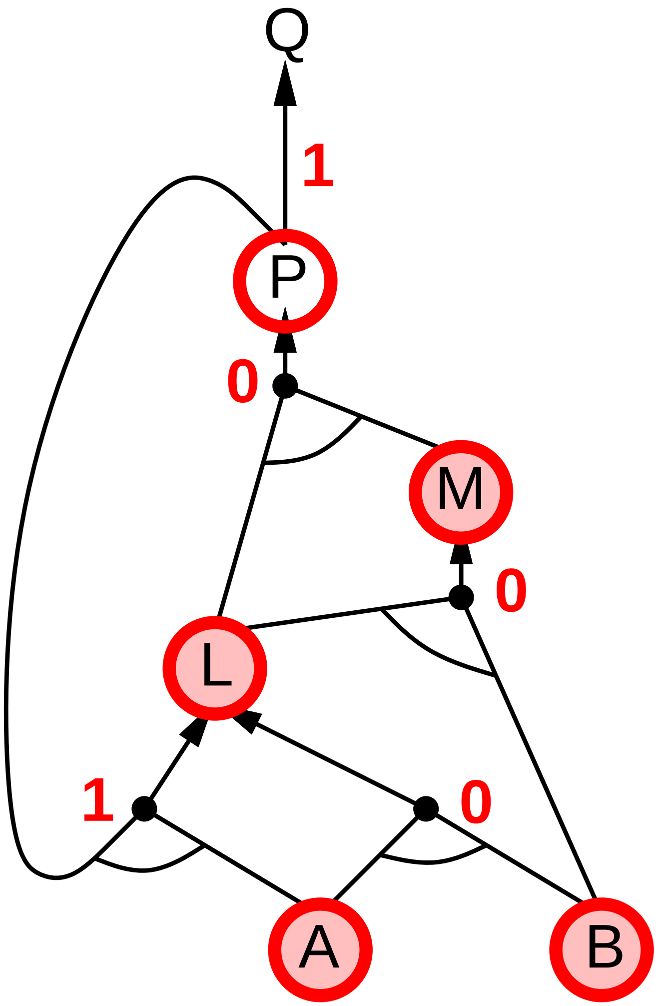

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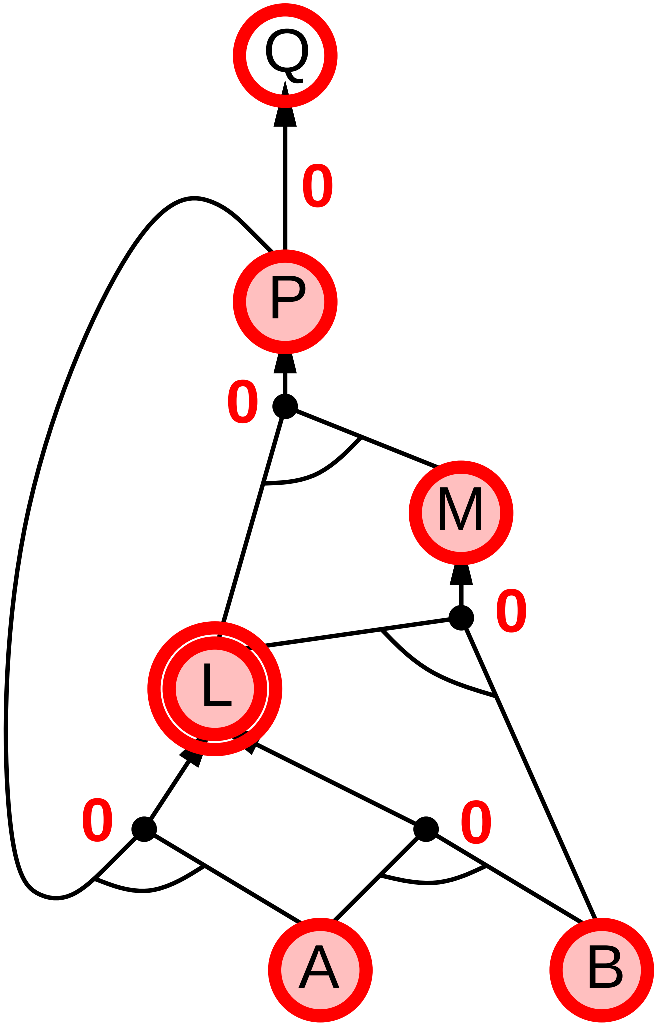
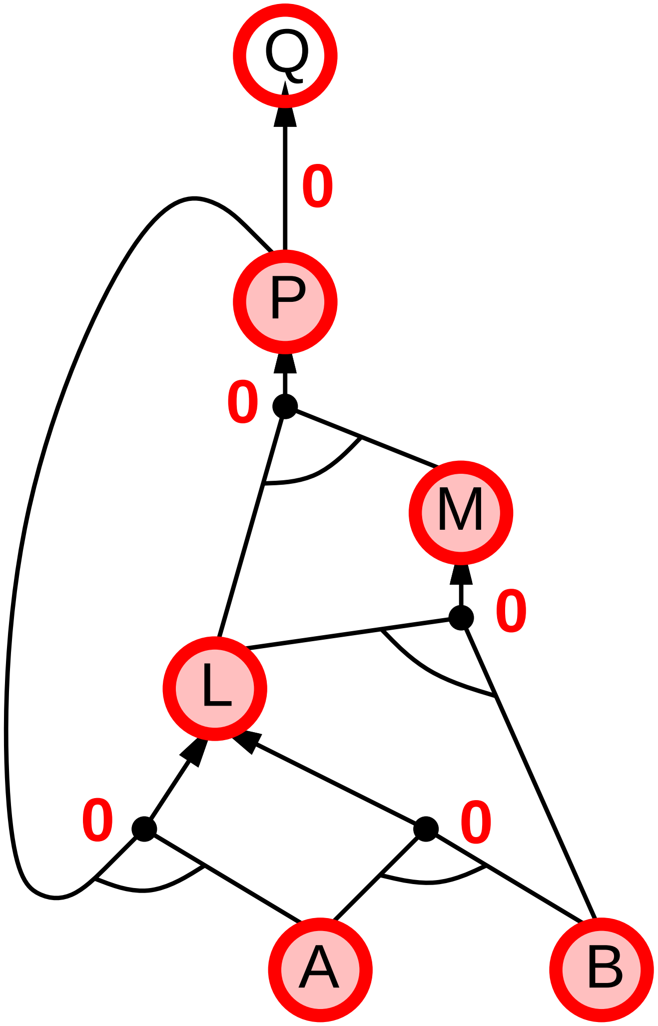


Forward chaining example



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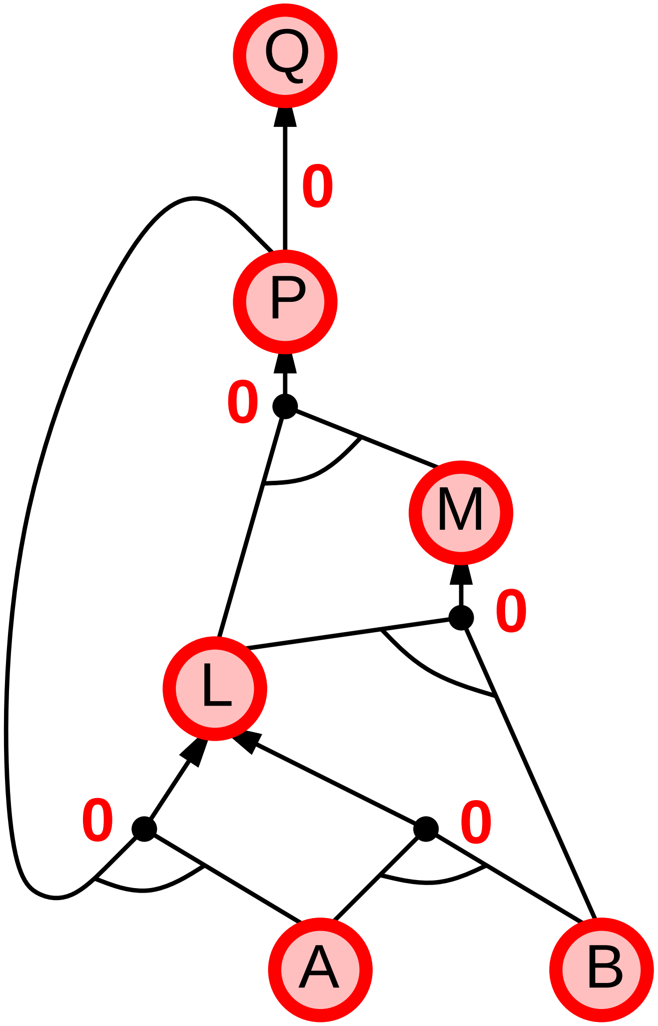


Proof of completeness

FC derives every atomic sentence that is entailed by KB

1. FC reaches a fixed point where no new atomic sentences are derived

2. Consider the final state as a model m, assigning true/false to symbols

3. Every clause in the original KB is true in m
   Proof: Suppose a clause $a_1 \land \ldots \land a_k \Rightarrow b$ is false in m
   Then $a_1 \land \ldots \land a_k$ is true in m and b is false in m
   Therefore the algorithm has not reached a fixed point!

4. Hence m is a model of KB

5. If $KB \models q$, q is true in **every** model of KB, including m

General idea: construct any model of KB by sound inference, check α

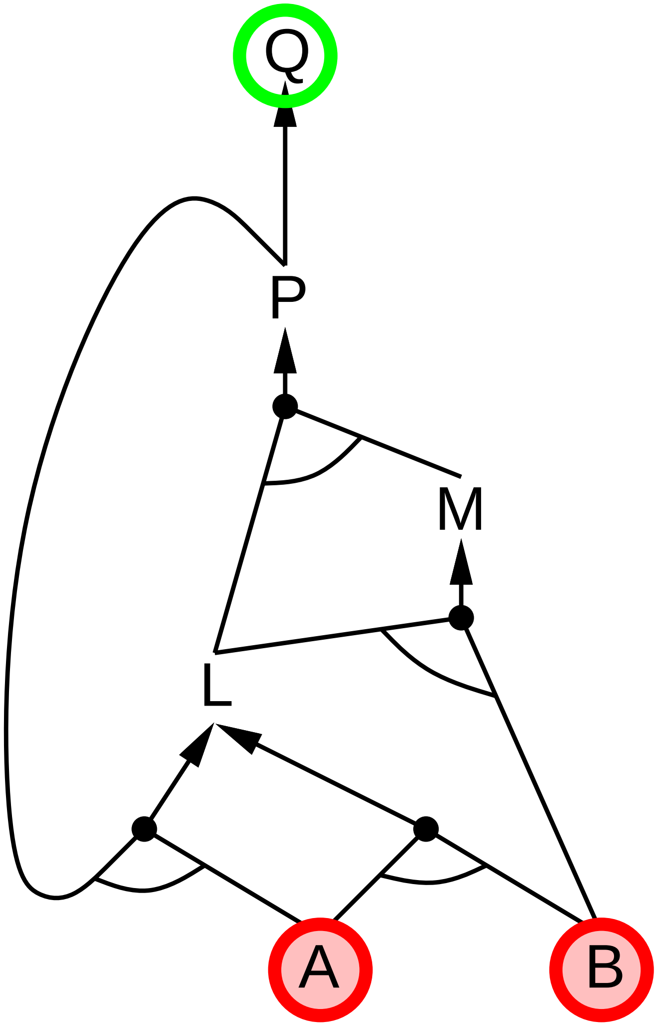
Backward chaining

Idea: work backwards from the query q:
  to prove q by BC,
    check if q is known already, or
    prove by BC all premises of some rule concluding q

Avoid loops: check if new subgoal is already on the goal stack

Avoid repeated work: check if new subgoal
  1) has already been proved true, or
  2) has already failed

Backward chaining example



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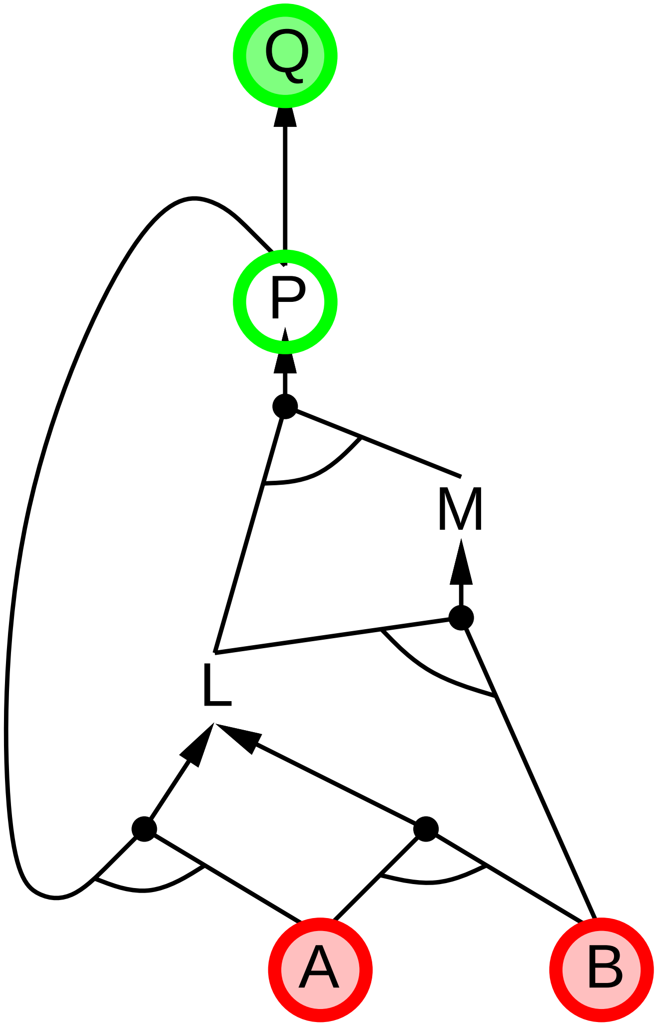

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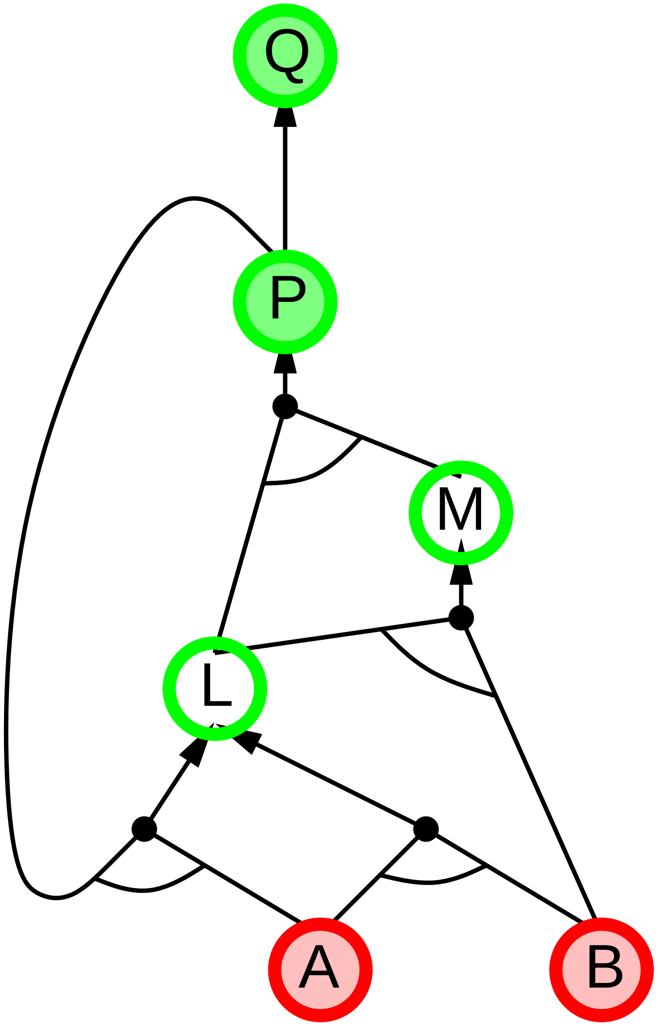

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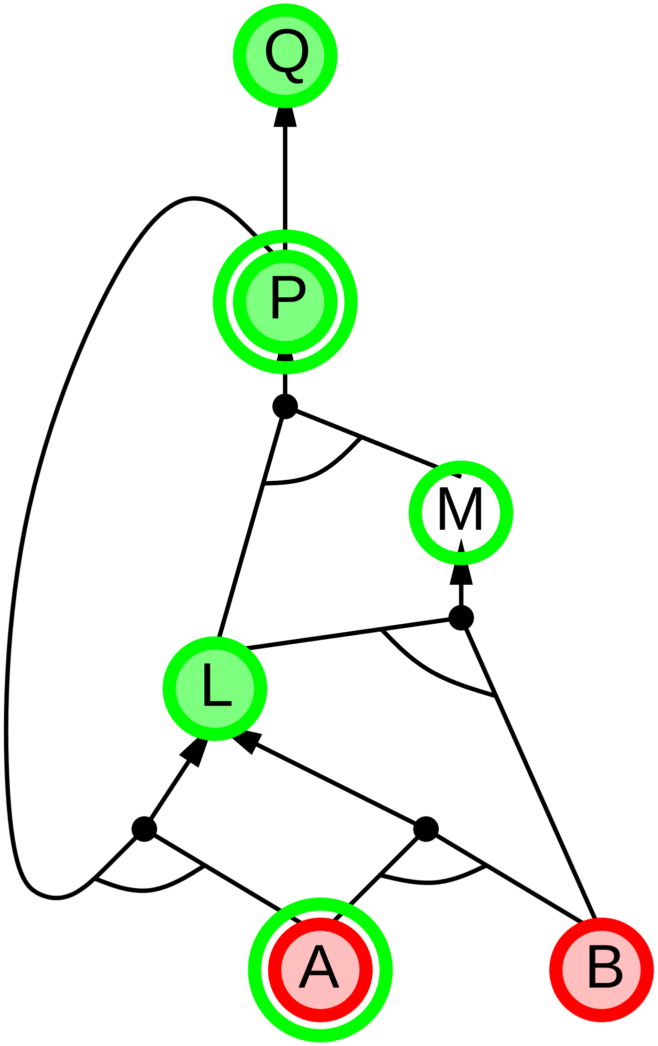

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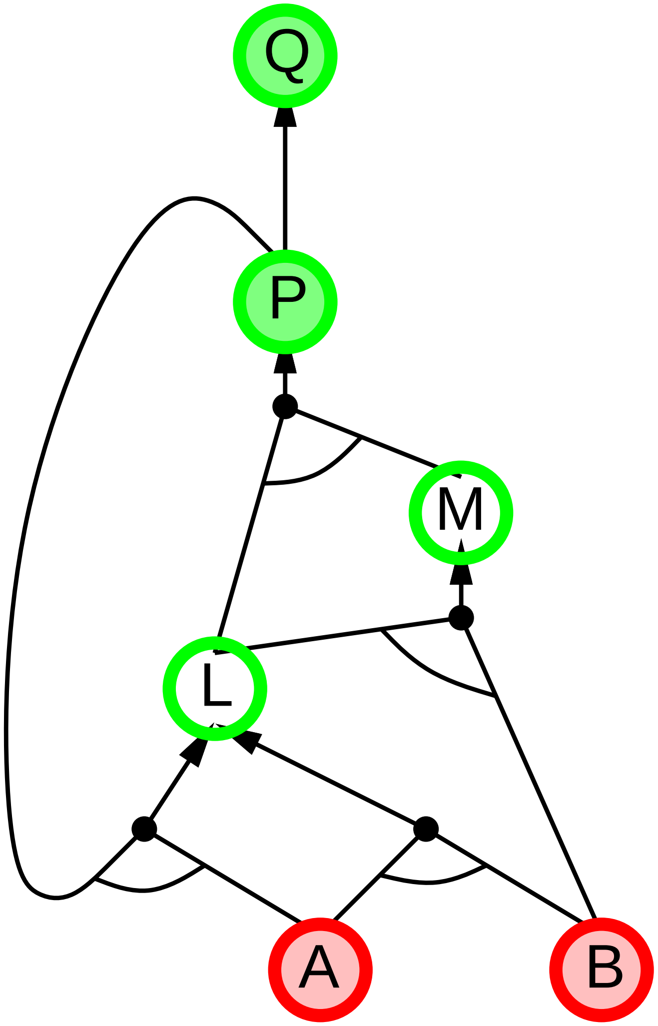

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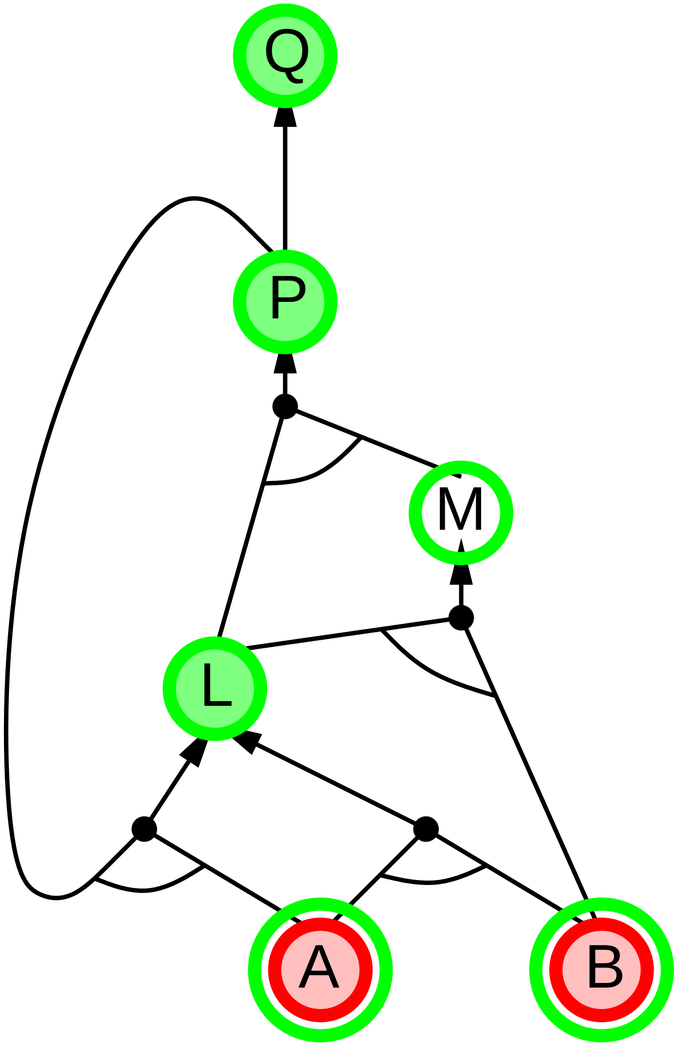

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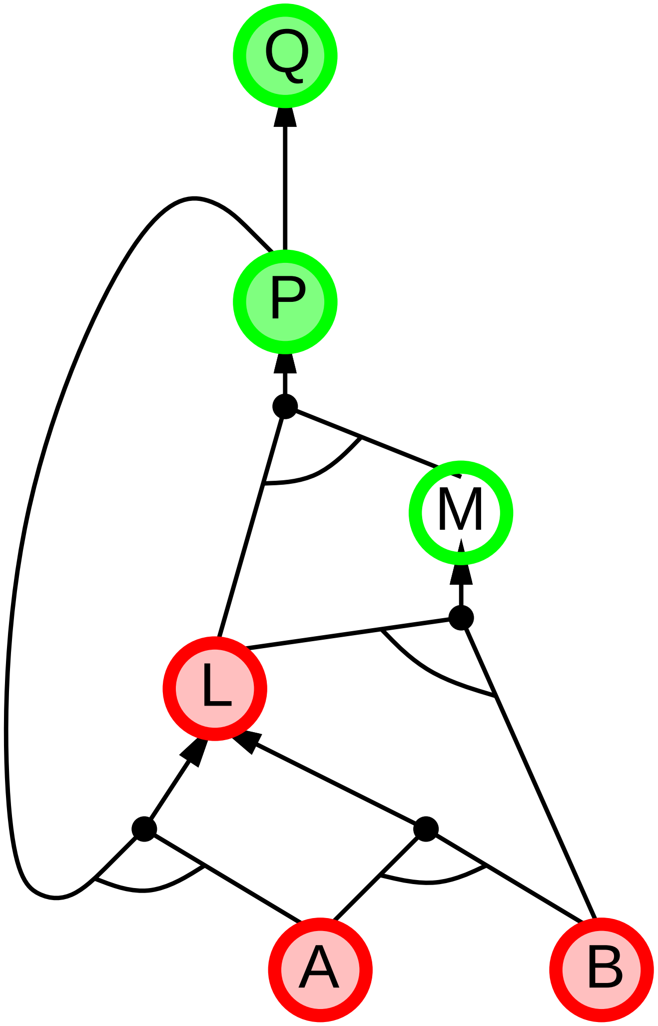

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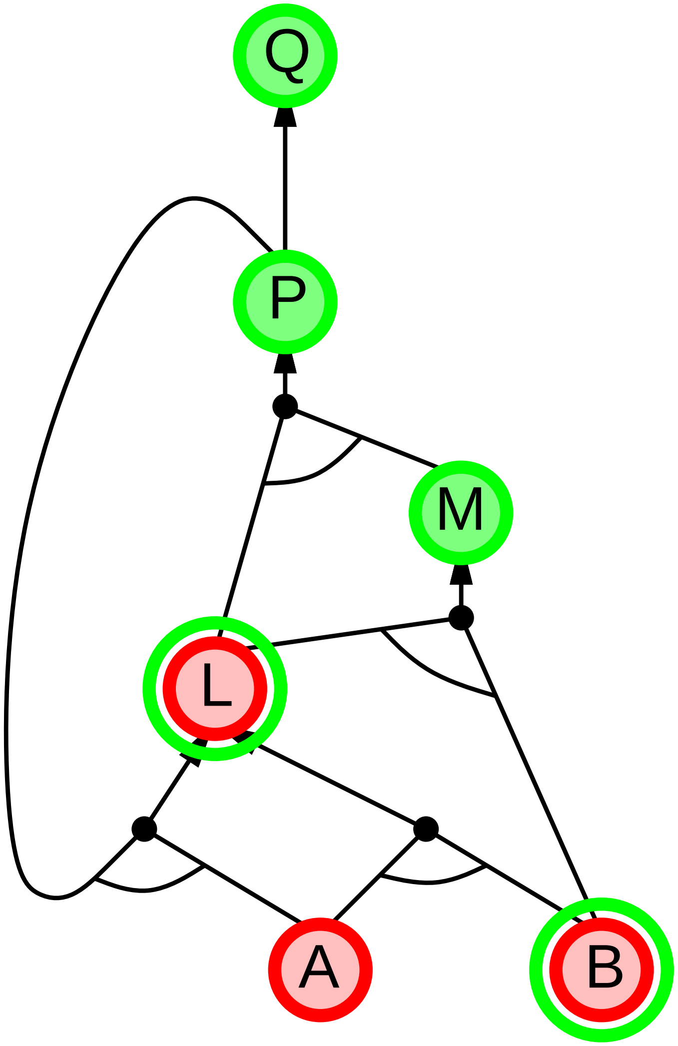

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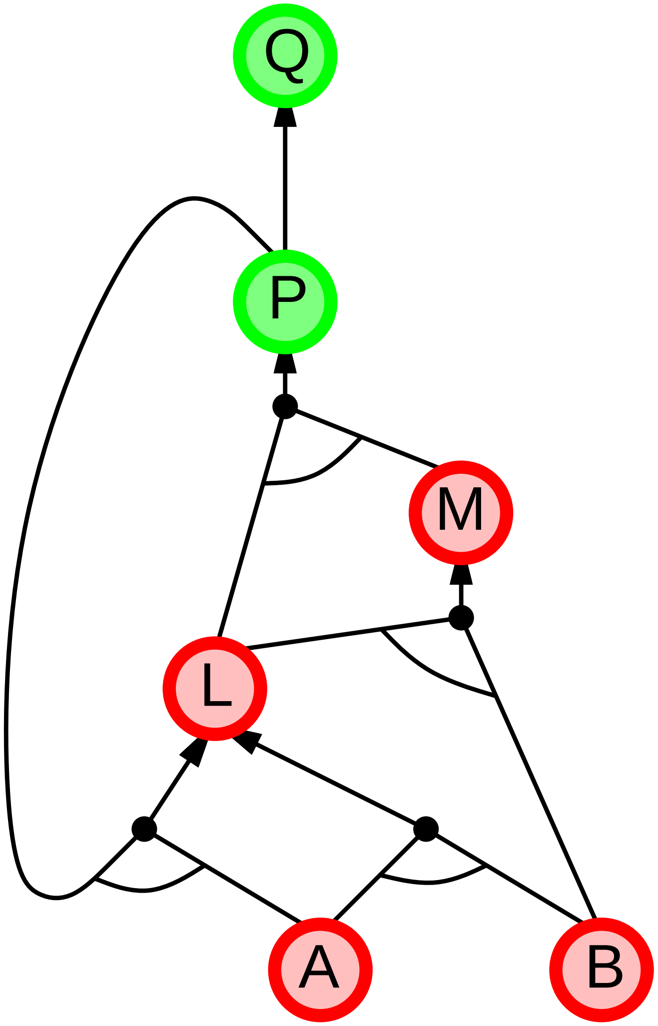

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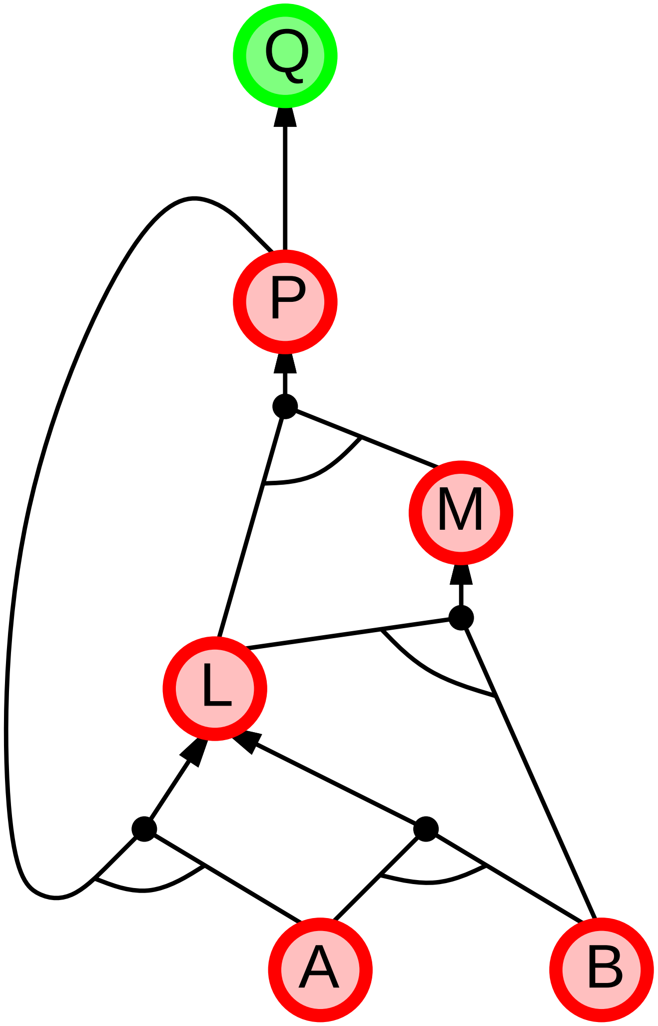

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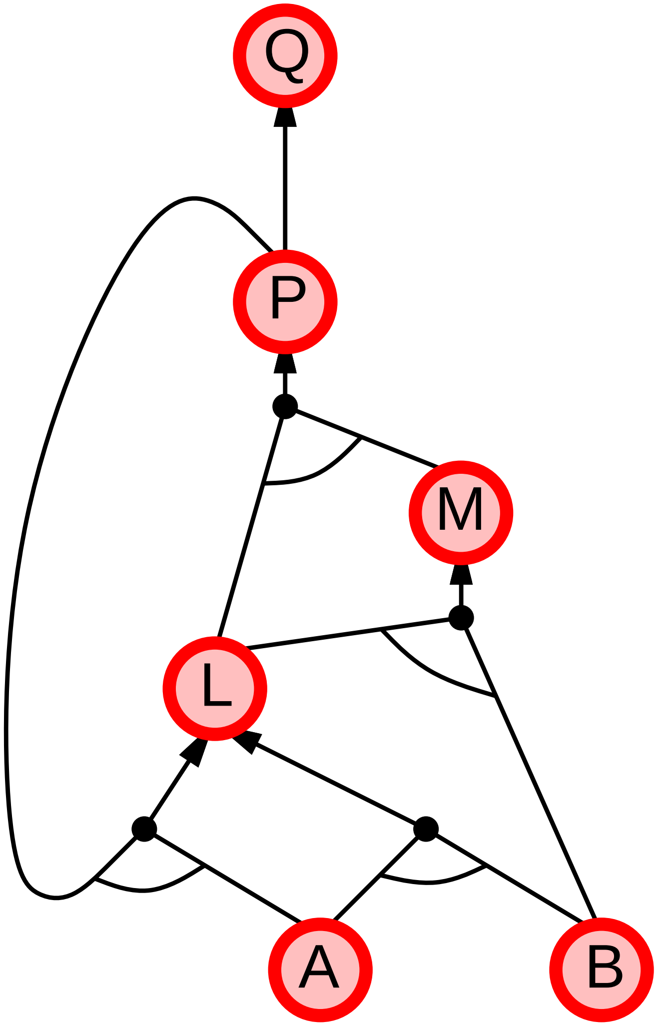


Forward vs. backward chaining

FC is data-driven, cf. automatic, unconscious processing,
e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is goal-driven, appropriate for problem-solving,
e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be **much less** than linear in size of KB

Resolution

Conjunctive Normal Form (CNF—universal)

$$\textbf{conjunction} \text{ of } \underbrace{\textbf{disjunctions} \text{ of } \textbf{literals}}_{\textbf{clauses}}$$

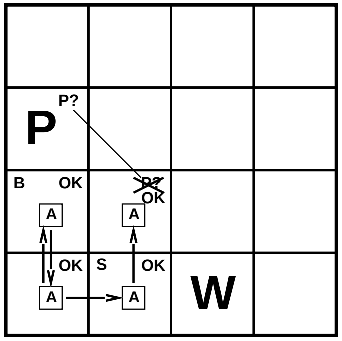
E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

Resolution inference rule (for CNF):

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$



Resolution is sound and complete for propositional logic

Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \;\Rightarrow\; \beta) \land (\beta \;\Rightarrow\; \alpha)$.

   $(B_{1,1} \;\Rightarrow\; (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \;\Rightarrow\; B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \lor \beta$.

   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg(P_{1,2} \lor P_{2,1}) \lor B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land ((\neg P_{1,2} \land \neg P_{2,1}) \lor B_{1,1})$

4. Apply distributivity law ($\lor$ over $\land$) and flatten:

   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$

Resolution algorithm

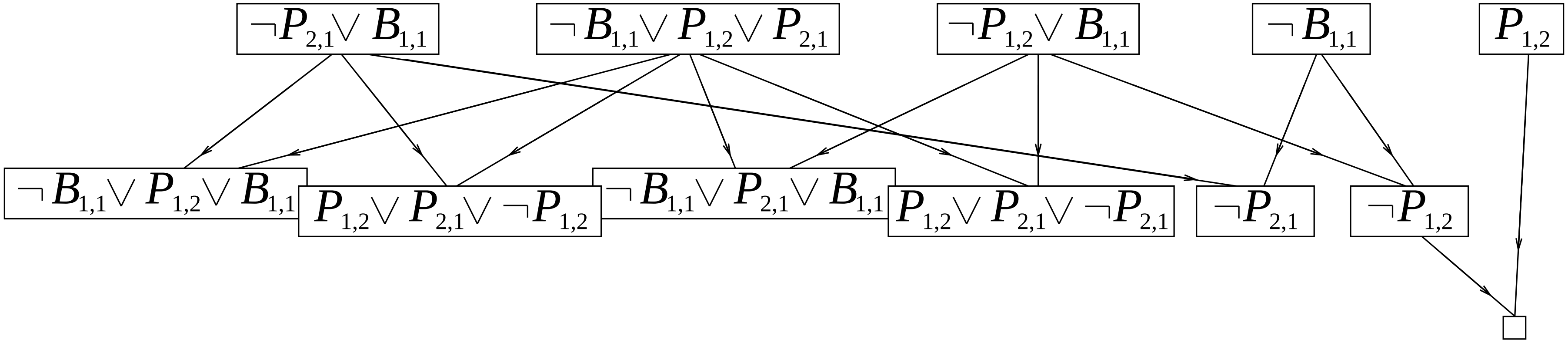
Proof by contradiction, i.e., show $KB \wedge \neg\alpha$ unsatisfiable

```
function PL-RESOLUTION($KB, \alpha$) returns true or false
    inputs: $KB$, the knowledge base, a sentence in propositional logic
            $\alpha$, the query, a sentence in propositional logic

    clauses ← the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
    new ← { }
    loop do
        for each $C_i$, $C_j$ in clauses do
            resolvents ← PL-RESOLVE($C_i, C_j$)
            if resolvents contains the empty clause then return true
            new ← new ∪ resolvents
        if new ⊆ clauses then return false
        clauses ← clauses ∪ new
```

Resolution example

$KB = (B_{1,1} \;\Leftrightarrow\; (P_{1,2} \lor P_{2,1})) \land \neg B_{1,1} \qquad \alpha = \neg P_{1,2}$



$\neg P_{2,1} \lor B_{1,1}$

$\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}$

$\neg P_{1,2} \lor B_{1,1}$

$\neg B_{1,1}$

$P_{1,2}$

$\neg B_{1,1} \lor P_{1,2} \lor B_{1,1}$

$P_{1,2} \lor P_{2,1} \lor \neg P_{1,2}$

$\neg B_{1,1} \lor P_{2,1} \lor B_{1,1}$

$P_{1,2} \lor P_{2,1} \lor \neg P_{2,1}$

$\neg P_{2,1}$

$\neg P_{1,2}$

$\square$

DPLL: backtracking++

Backtracking applied to SAT problems:
- variables are proposition symbols, clauses are constraints

Several key improvements:

1. Early termination: stop if all clauses true or any clause false
   e.g., $\{A = true\}$ satisfies $(A \lor B) \land (A \lor C)$

2. Pure symbols: symbol has same sign in all as-yet-unsatisfied clauses
   e.g., A and B are pure in $(A \lor \neg B) \land (\neg B \lor \neg C) \land (C \lor A)$
   $\Rightarrow$ assign symbol to make literals true

3. Unit clauses: clause has exactly one as-yet-unfalsified literal
   e.g., if $\{A = true\}$ already, $(\neg A \lor \neg B)$ is a unit clause
   $\Rightarrow$ assign symbol to make clause true (cf. forward chaining, MRV)

DPLL algorithm

```
function DPLL($clauses, symbols, model$) returns $true$ or $false$
    if every clause in $clauses$ is true in $model$ then return $true$
    if some clause in $clauses$ is false in $model$ then return $true$
    $P$, $value \leftarrow$ FIND-PURE-SYMBOL($symbols$, $clauses$, $model$)
    if $P$ is non-null then return DPLL($clauses$, $symbols$–$P$, $[P = value|model]$)
    $P$, $value \leftarrow$ FIND-UNIT-CLAUSE($clauses$, $model$)
    if $P$ is non-null then return DPLL($clauses$, $symbols$–$P$, $[P = value|model]$)
    $P \leftarrow$ FIRST($symbols$); $rest \leftarrow$ REST($symbols$)
    return DPLL($clauses$, $rest$, $[P = true|model]$) or
           DPLL($clauses$, $rest$, $[P = false|model]$)
```

Highly optimized implementation + caching unsolvable subassignments

⇒ modern solvers handle tens of millions of clauses

⇒ practical for large hardware and medium software verification

Propositions and time

Suppose the wumpus-world agent wants to keep track of its location

A sentence such as $L_{1,1} \wedge FacingRight \wedge Forward \;\Rightarrow\; L_{2,1}$
doesn't work: after one inference step, $L_{1,1}$ and $L_{2,1}$ are in KB!!

Changeable aspects of world need separate symbols for each time step
e.g., $L^1_{1,1}$ means "Agent is at [1,1] at time step 1", and

$$L^1_{1,1} \wedge FacingRight^1 \wedge Forward^1 \;\Rightarrow\; L^2_{2,1}$$

Reflex rules: for every t, we have, e.g., $Glitter^t \;\Rightarrow\; Grab^t$

☹Need copies of all axioms involving temporal symbols
for **every** time step (might be infinitely many!)

Tracking changes in the world

State estimation is the general task of keeping track of environment state given a stream of percepts

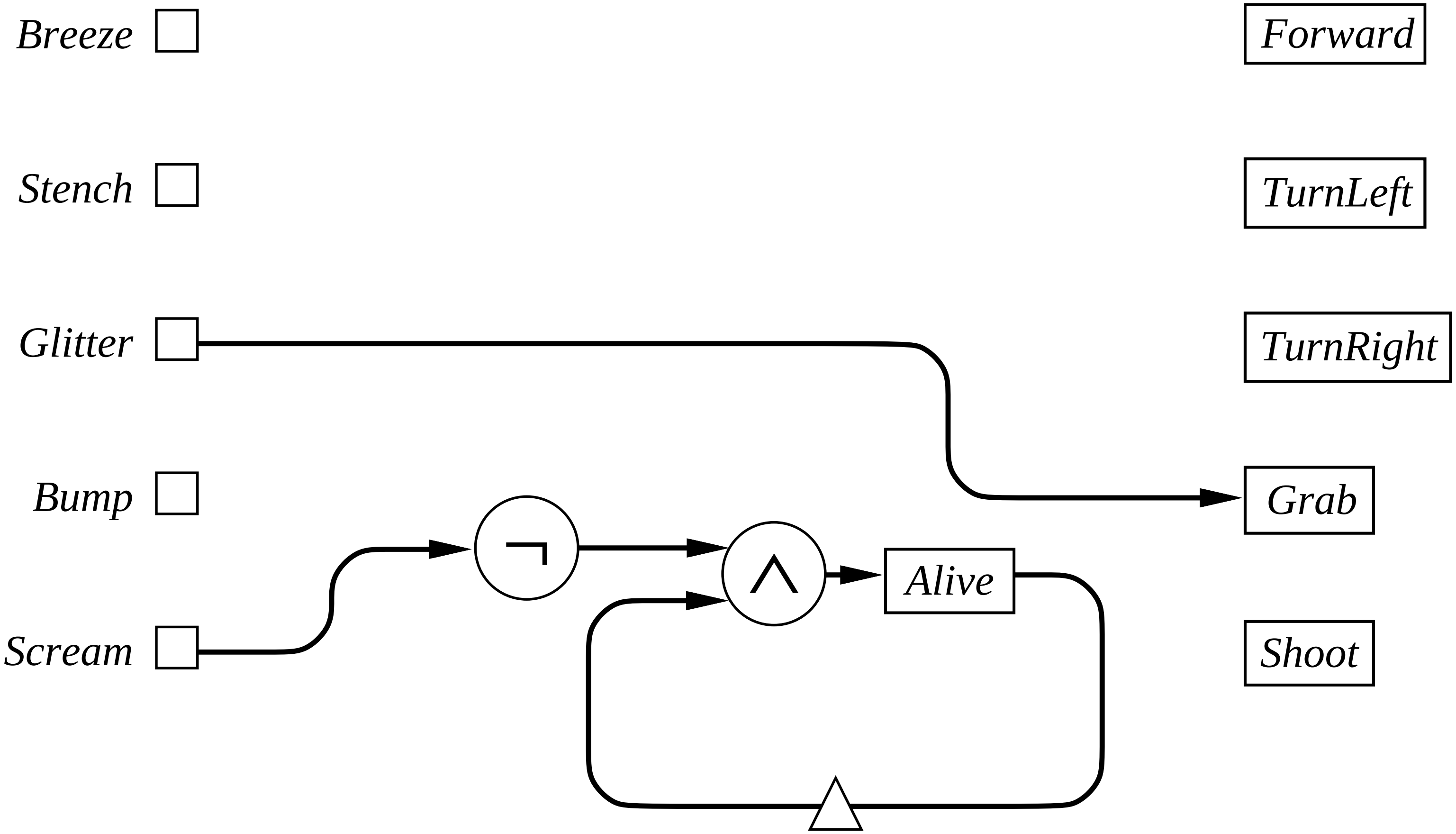
For logic-based systems: maintain a representation of the set of all logically possible world states, given axioms and percepts

Basic trick: successor-state axioms **define** truth of proposition at $t+1$ from propositions at t

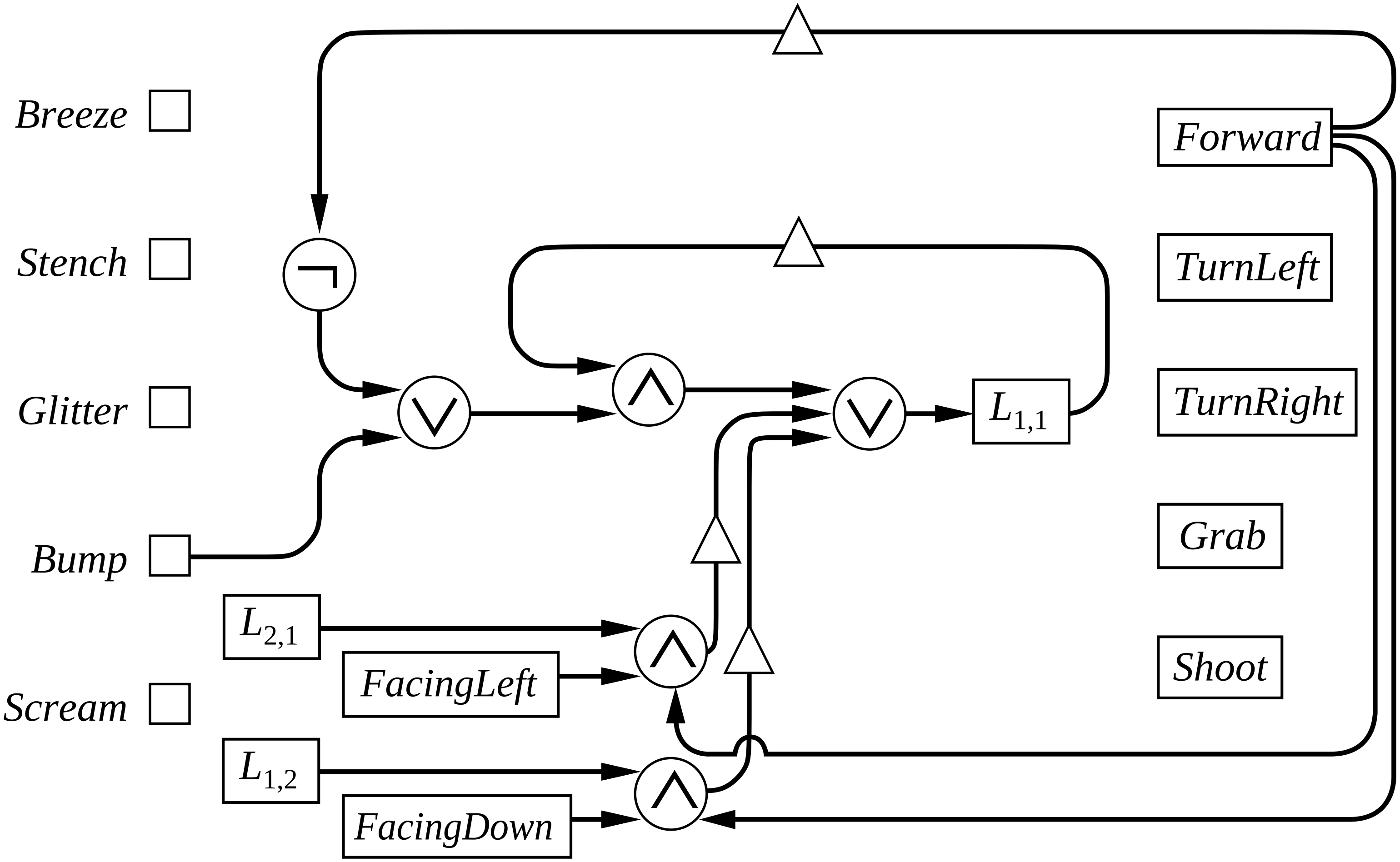
E.g., $Alive^t \;\Leftrightarrow\; \neg Scream^t \wedge Alive^{t-1}$

$$\begin{aligned} L_{1,1}^t \;\Leftrightarrow\; & (L_{1,1}^{t-1} \wedge (\neg Forward^{t-1} \vee Bump^t)) \\ & \vee (L_{1,2}^{t-1} \wedge (FacingDown^{t-1} \wedge Forward^{t-1})) \\ & \vee (L_{2,1}^{t-1} \wedge (FacingLeft^{t-1} \wedge Forward^{t-1})) \end{aligned}$$

Boolean circuit agents



Boolean circuit agents contd.



Summary

Inference methods work by theorem proving or model checking

Forward, backward chaining are linear-time, complete for Horn clauses

Resolution is complete for propositional logic

DPLL is an efficient, complete model checker;
WalkSAT is incomplete but often very fast in practice

Circuit-based agents provide a simple way to handle time
but are usually less complete than inference-based agents