Logic and propositional logic

Chapter 7.1–7.4

Outline

◇ Knowledge-based agents

◇ Wumpus world

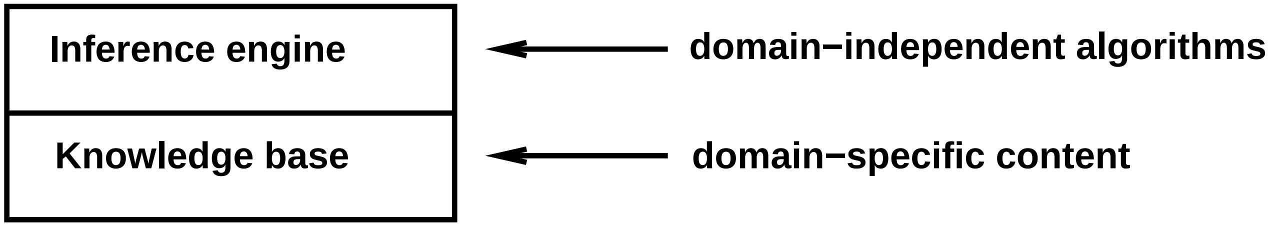
◇ Logic in general—models and entailment

◇ Propositional (Boolean) logic

◇ Equivalence, validity, satisfiability

◇ Inference by exhaustive model checking

Knowledge bases



Knowledge base = set of sentences in a **formal** language

Declarative approach to building an agent (or other system):
Tell it what it needs to know

Then it can Ask itself what to do—answers should follow from the KB

Agents can be viewed at the knowledge level
i.e., **what they know**, regardless of how implemented

Or at the implementation level
i.e., data structures in KB and algorithms that manipulate them

A simple knowledge-based agent

function KB-AGENT(*percept*) **returns** an *action*
  static: *KB*, a knowledge base
      t, a counter, initially 0, indicating time

  TELL(*KB*, MAKE-PERCEPT-SENTENCE(*percept*, *t*))
  action ← ASK(*KB*, MAKE-ACTION-QUERY(*t*))
  TELL(*KB*, MAKE-ACTION-SENTENCE(*action*, *t*))
  $t \leftarrow t + 1$
  return *action*

The agent must be able to:
- Represent states, actions, etc.
- Incorporate new percepts
- Update internal representations of the world
- Deduce hidden properties of the world
- Deduce appropriate actions

$\Rightarrow$ sound and complete reasoning with partial information states

Wumpus World PEAS description

Performance measure

gold +1000, death -1000

-1 per step, -10 for using the arrow

Environment

Squares adjacent to wumpus are smelly

Squares adjacent to pit are breezy

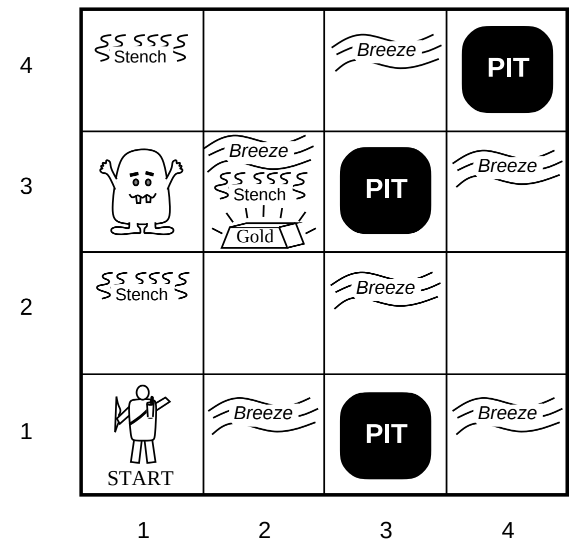
Glitter iff gold is in the same square

Shooting kills wumpus if you are facing it

Shooting uses up the only arrow

Grabbing picks up gold if in same square

Releasing drops the gold in same square



Actuators Left turn, Right turn,
Forward, Grab, Release, Shoot

Sensors Breeze, Glitter, Smell

Wumpus world characterization

Observable??

Wumpus world characterization

Observable?? No—only local perception

Deterministic??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete?? Yes

Single-agent??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

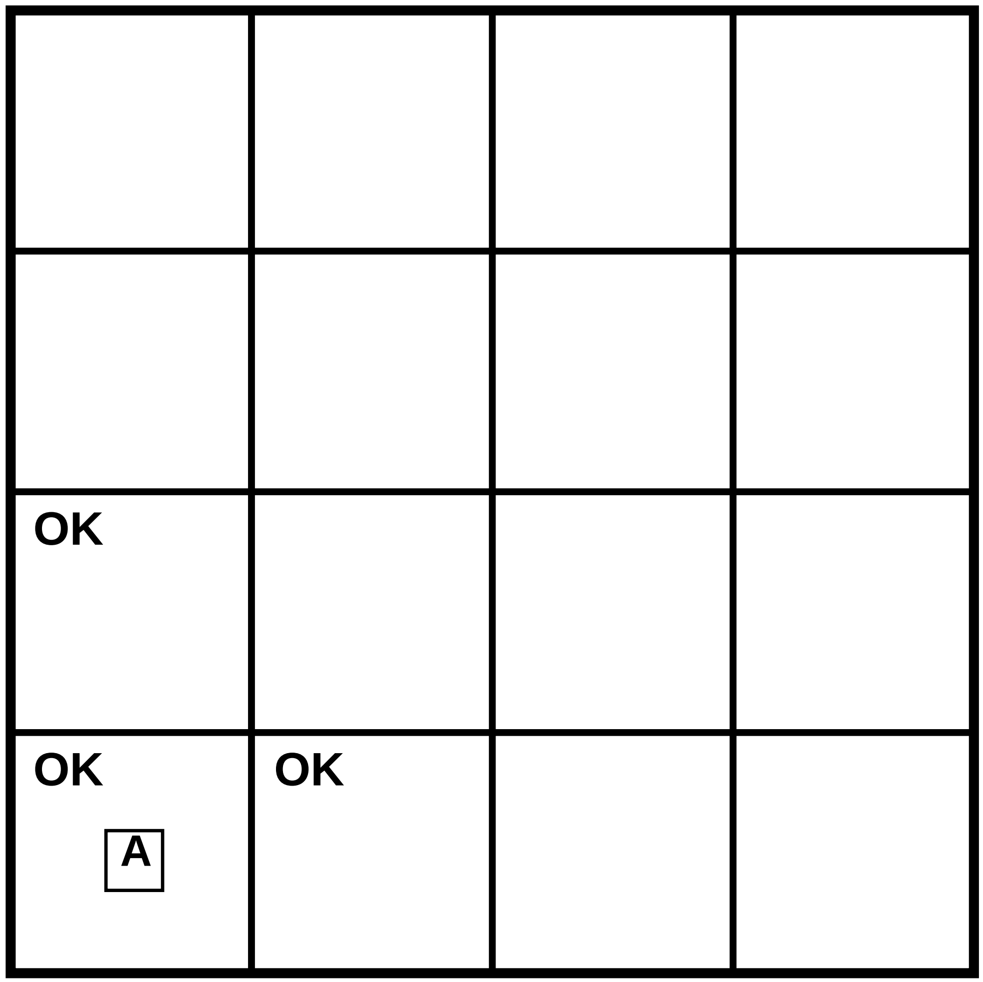
Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete?? Yes

Single-agent?? Yes—Wumpus is essentially a natural feature

Exploring a wumpus world



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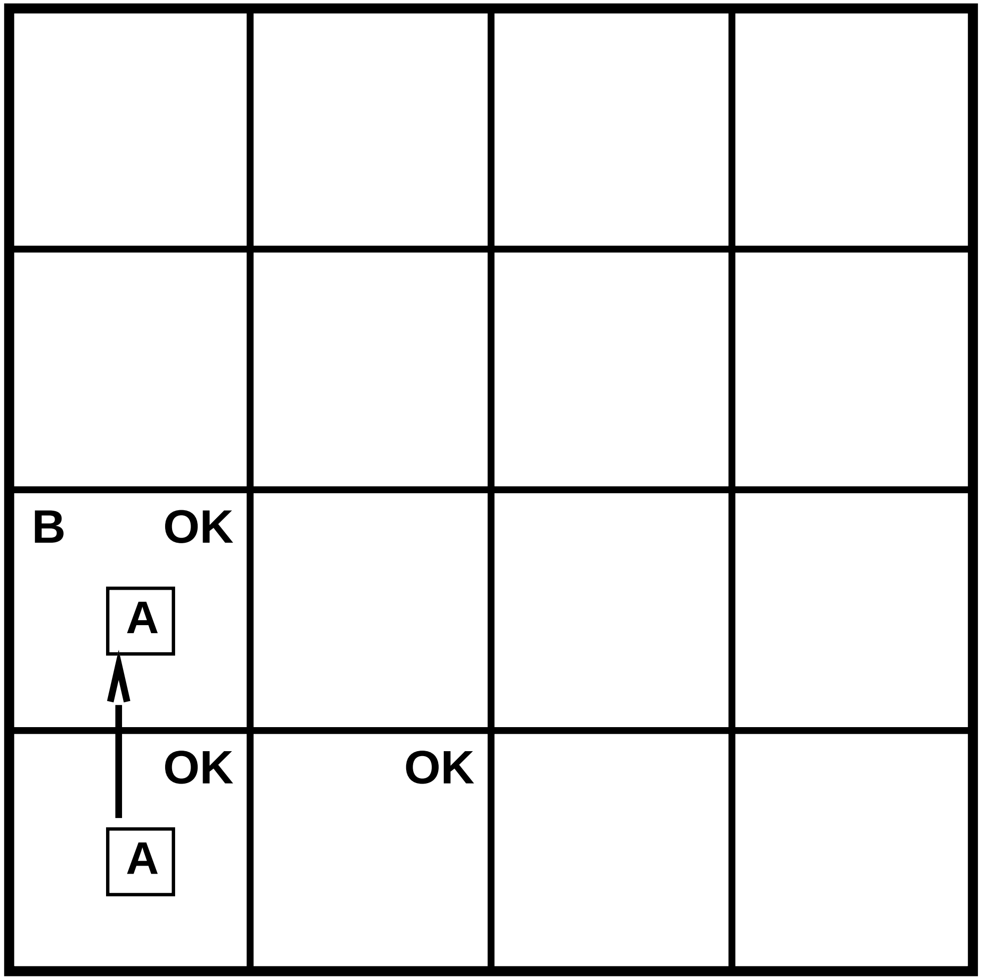

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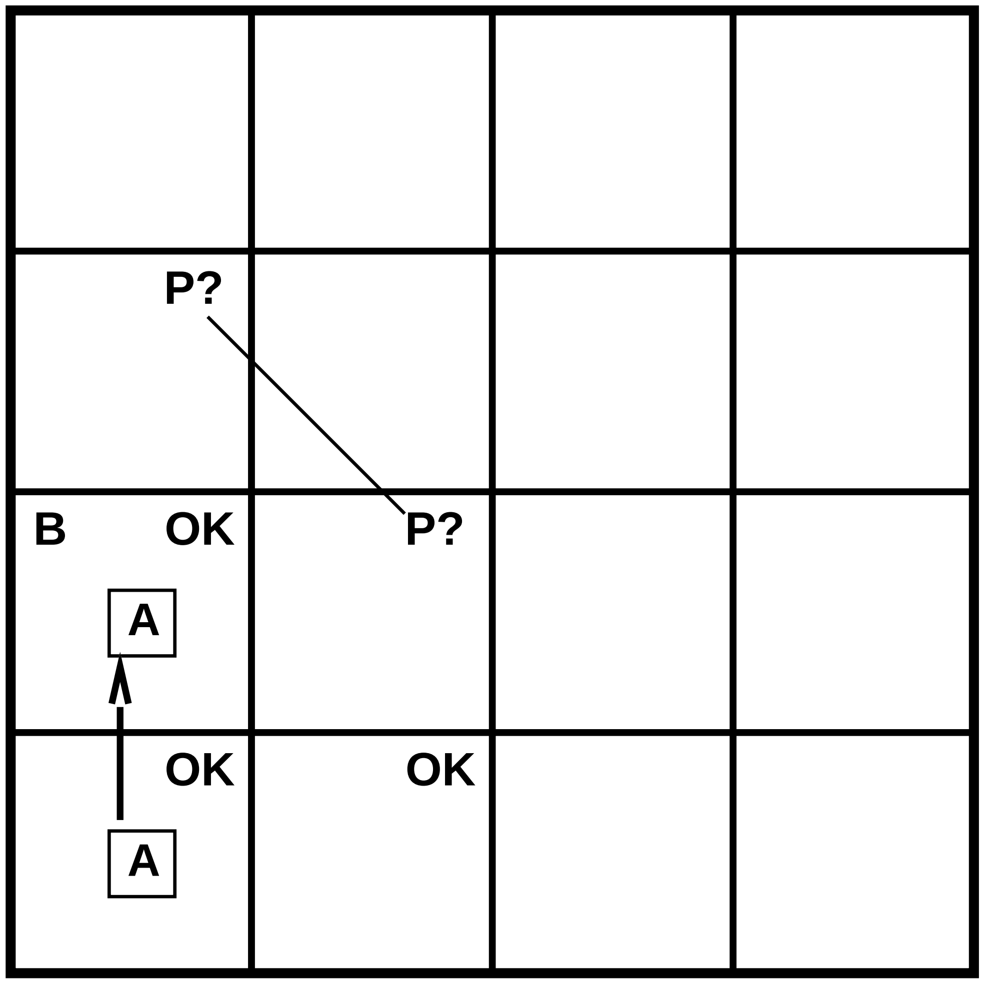

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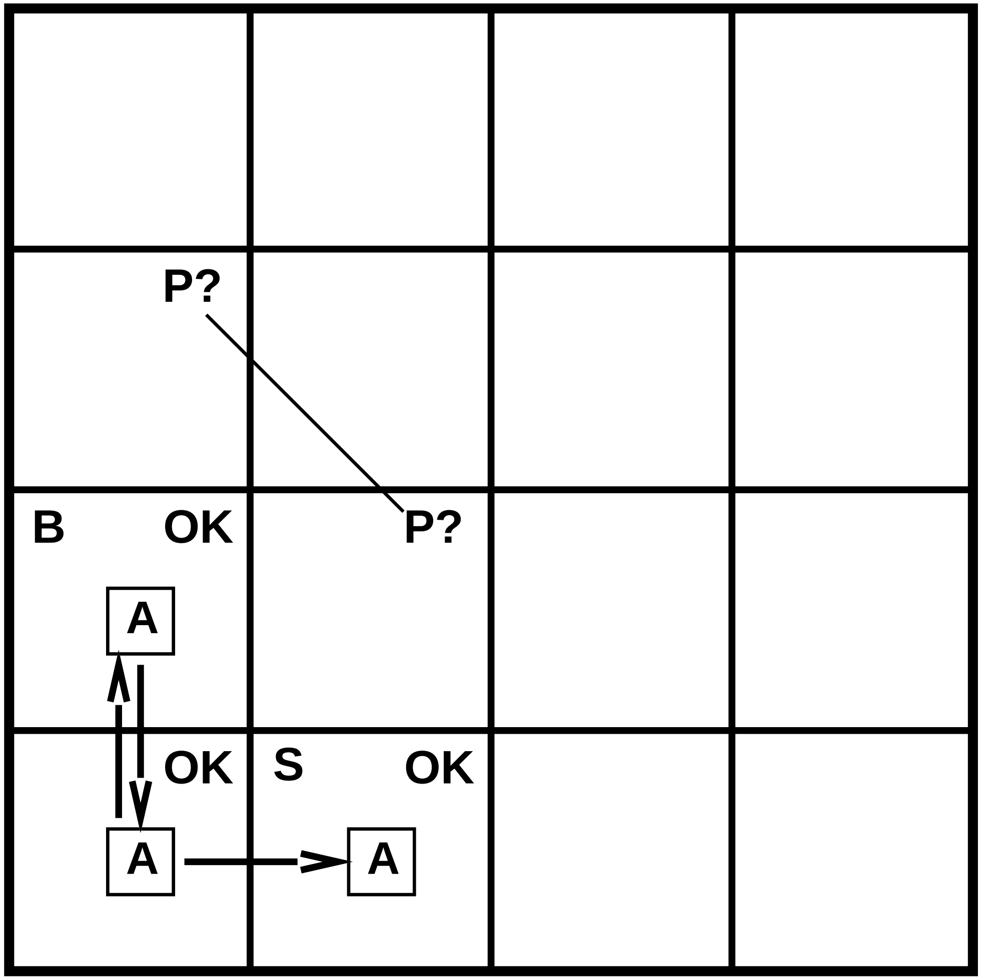

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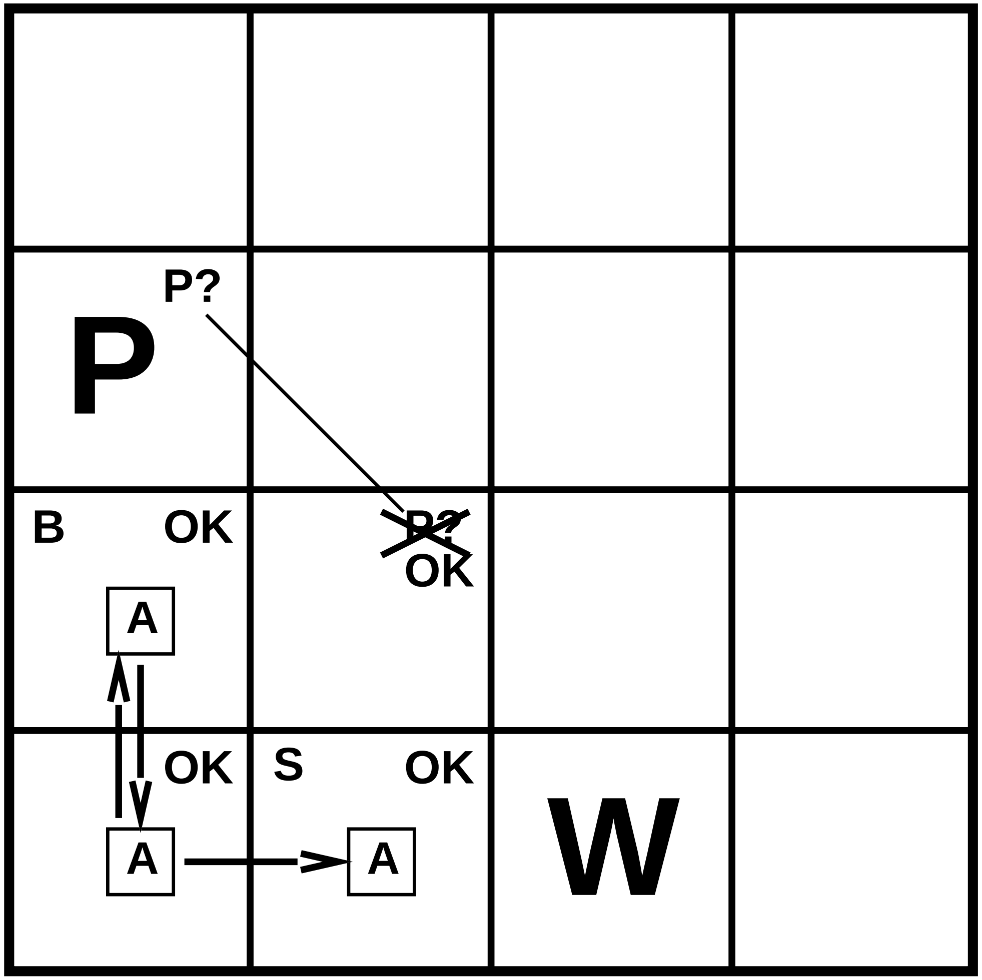

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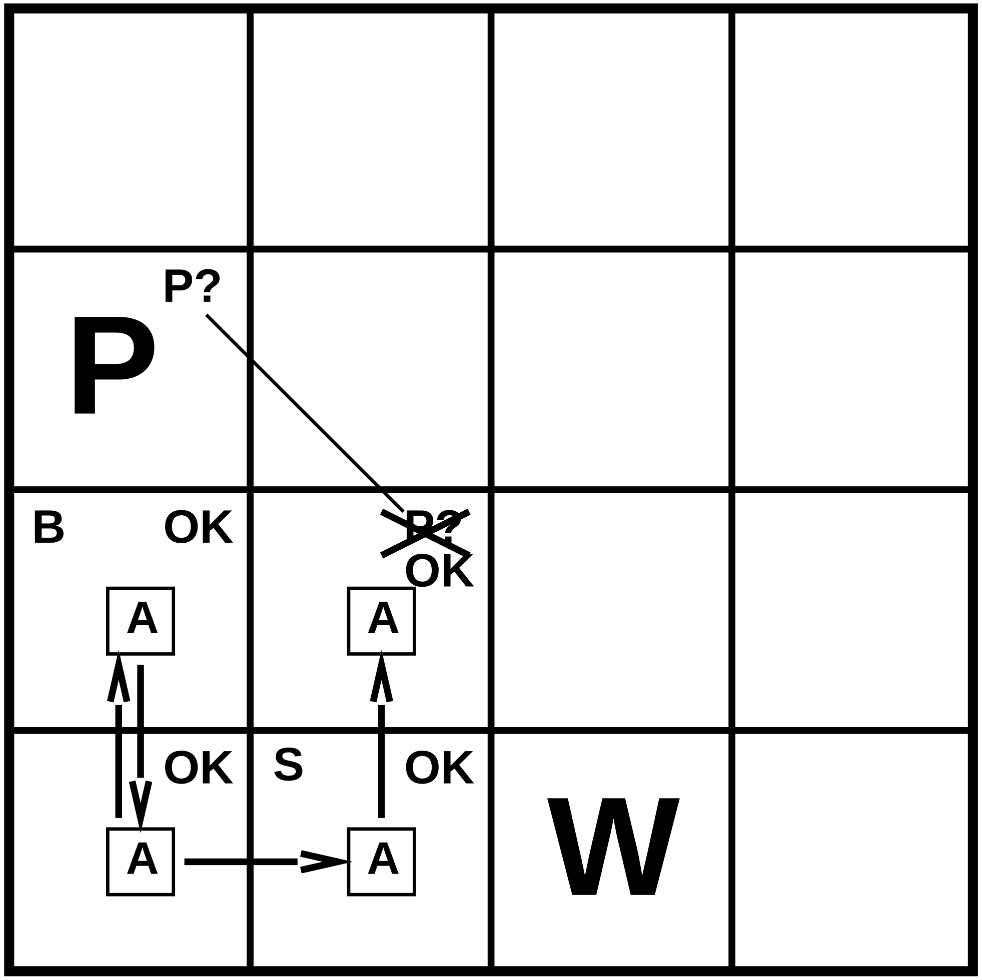

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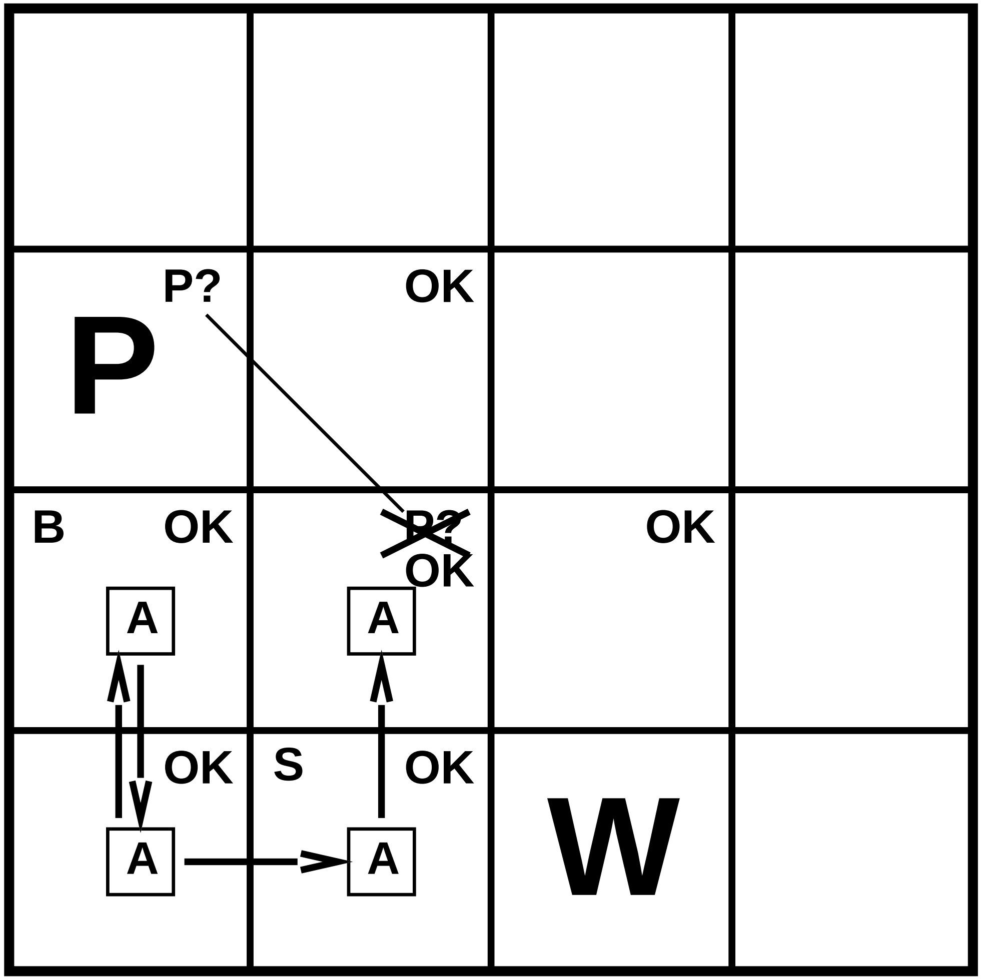

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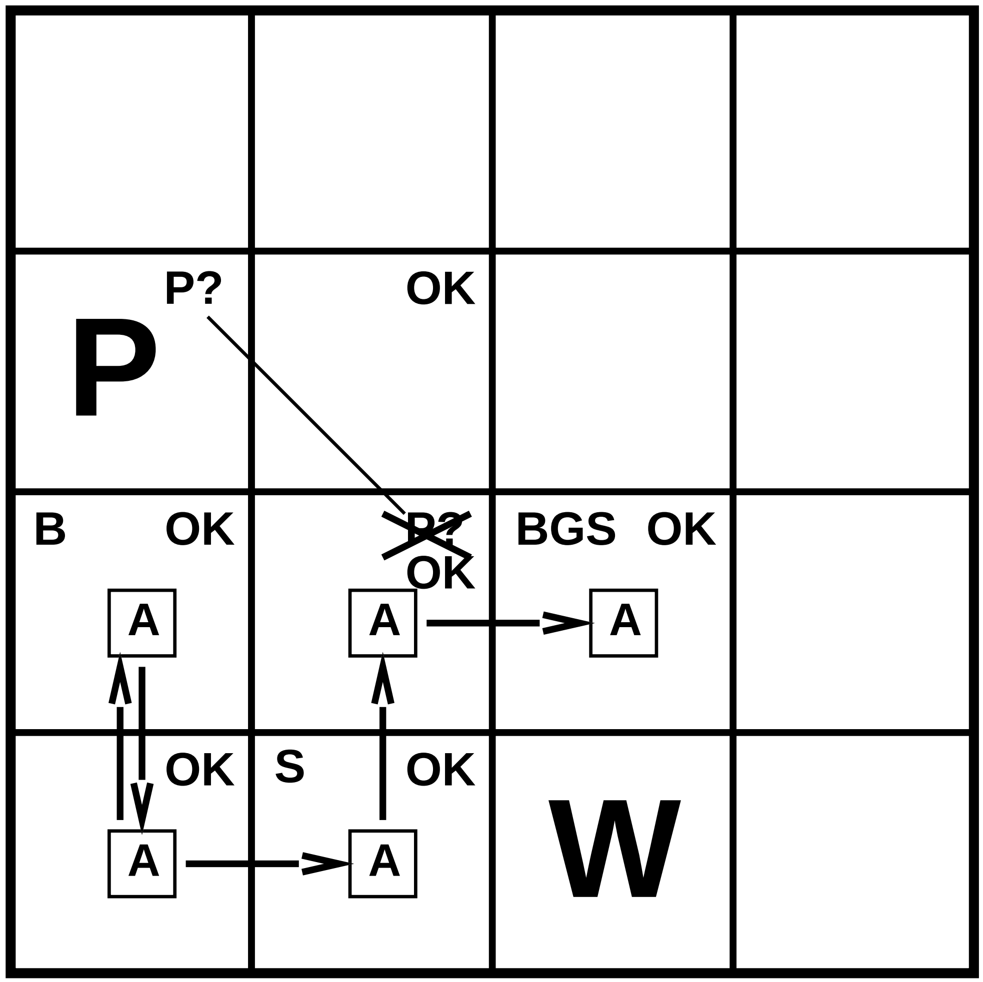
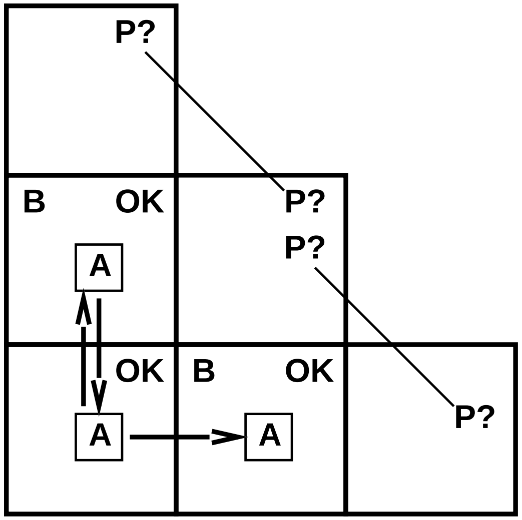
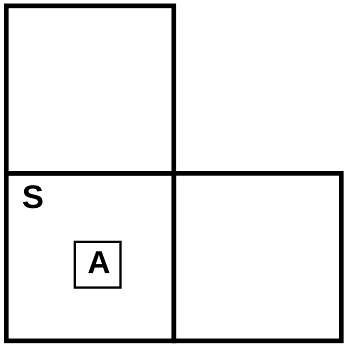


Other tight spots



Breeze in (1,2) and (2,1)
$\Rightarrow$ no safe actions

Assuming pits uniformly distributed,
(2,2) has pit w/ prob 0.86, vs. 0.31



Smell in (1,1)
$\Rightarrow$ cannot move
Can use a strategy of coercion:
shoot straight ahead
wumpus was there $\Rightarrow$ dead $\Rightarrow$ safe
wumpus wasn't there $\Rightarrow$ safe

Logic in general

Logics are formal languages for representing information
such that conclusions can be drawn

Syntax defines the sentences in the language

Semantics define the "meaning" of sentences;
i.e., define truth of a sentence in a world

E.g., the language of arithmetic

$x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence

$x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number y

$x + 2 \geq y$ is true in a world where $x = 7,\ y = 1$
$x + 2 \geq y$ is false in a world where $x = 0,\ y = 6$

Entailment

Entailment means that one thing **follows from** another:

$KB \models \alpha$

Knowledge base KB entails sentence α
if and only if
α is true in all worlds where KB is true

E.g., the KB containing "the Giants won" and "the Reds won"
entails "Either the Giants won or the Reds won"

E.g., $x+y=4$ entails $4=x+y$

Entailment is a relationship between sentences (i.e., **syntax**)
that is based on **semantics**

Note: brains process **syntax** (of some sort)

Models

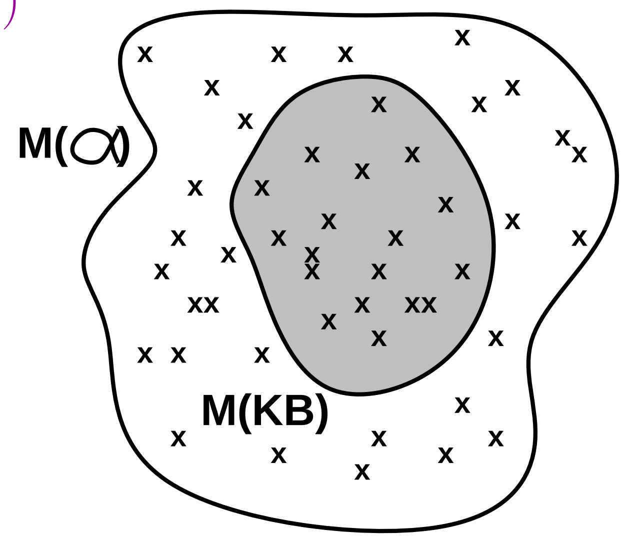
Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated

We say m is a model of a sentence α if α is true in m

$M(\alpha)$ is the set of all models of α

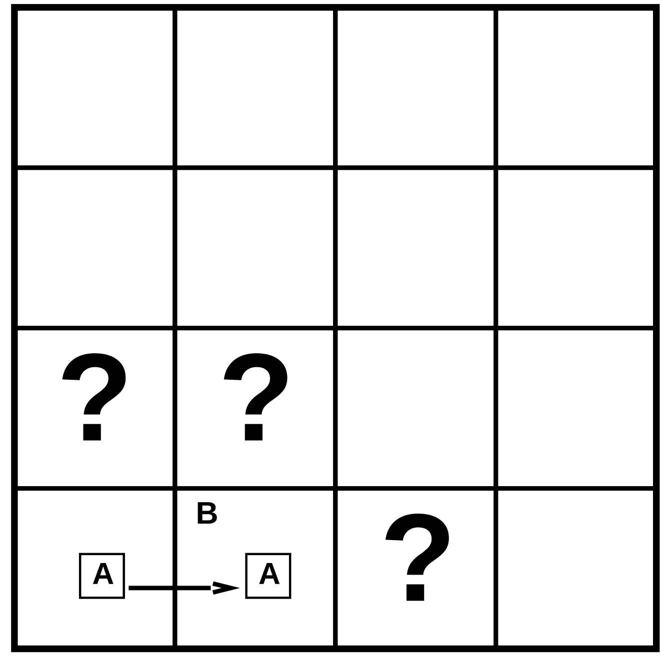
Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

E.g. $KB =$ Giants won and Reds won
$\alpha =$ Giants won



Entailment in the wumpus world

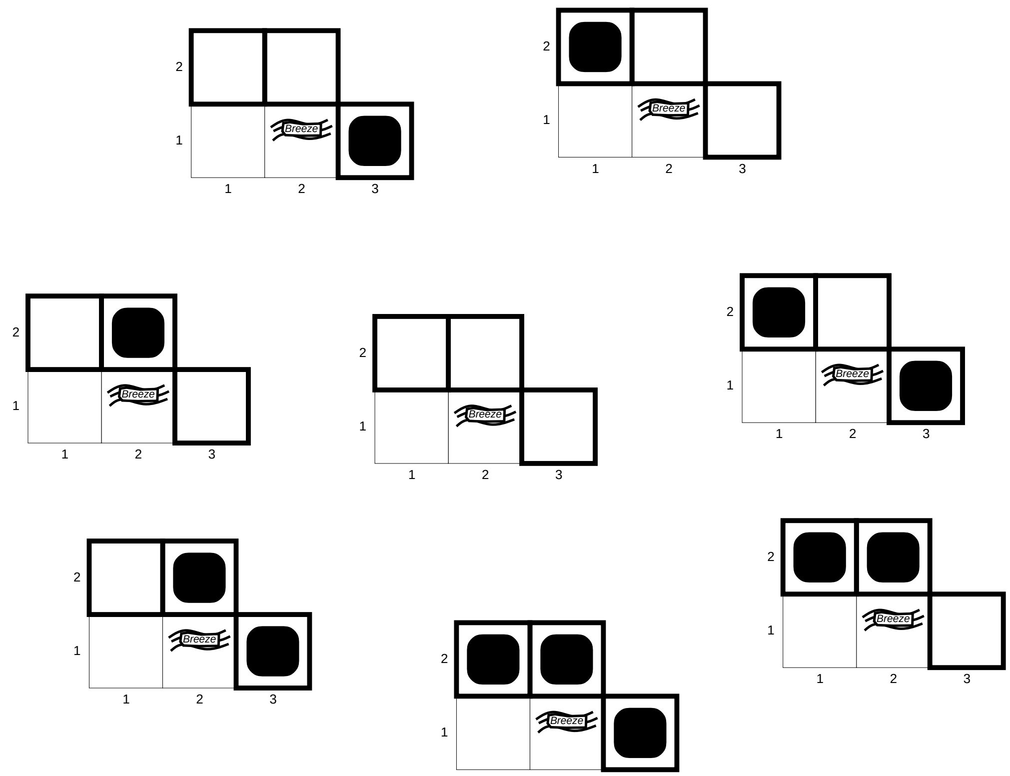
Situation after detecting nothing in [1,1], moving right, breeze in [2,1]



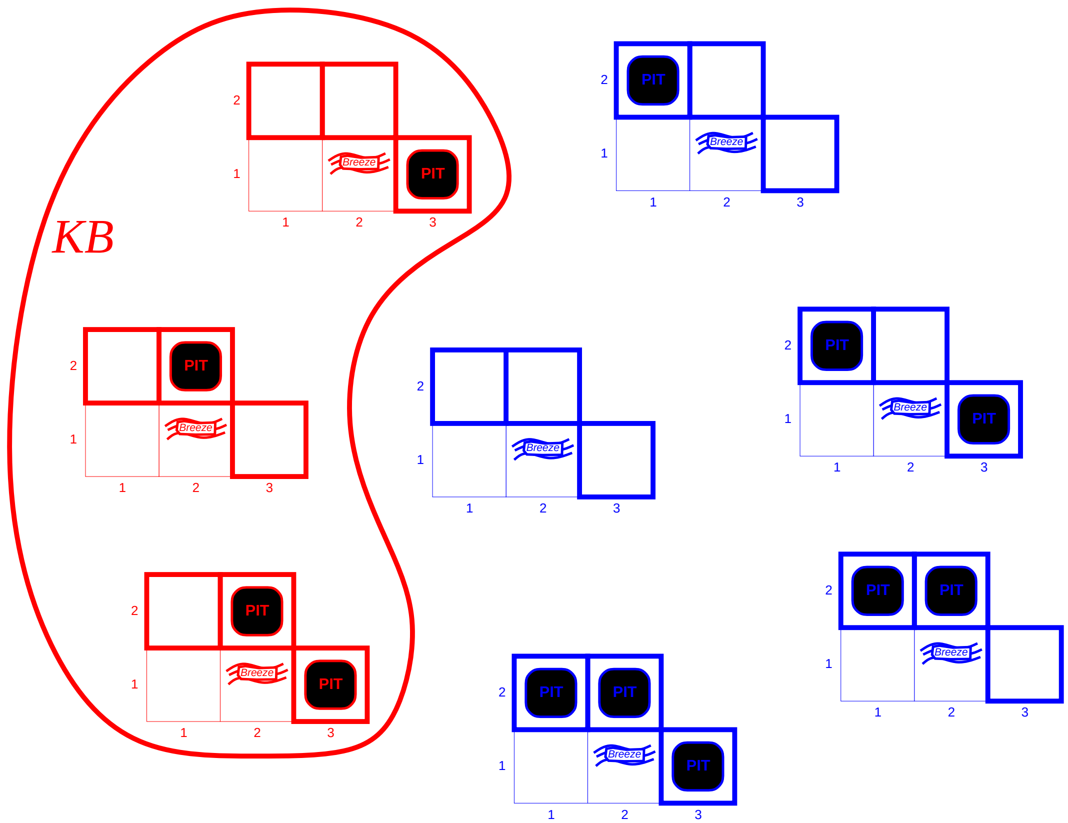
Consider possible models for ?s assuming only pits

3 Boolean choices $\Rightarrow$ 8 possible models

Wumpus models

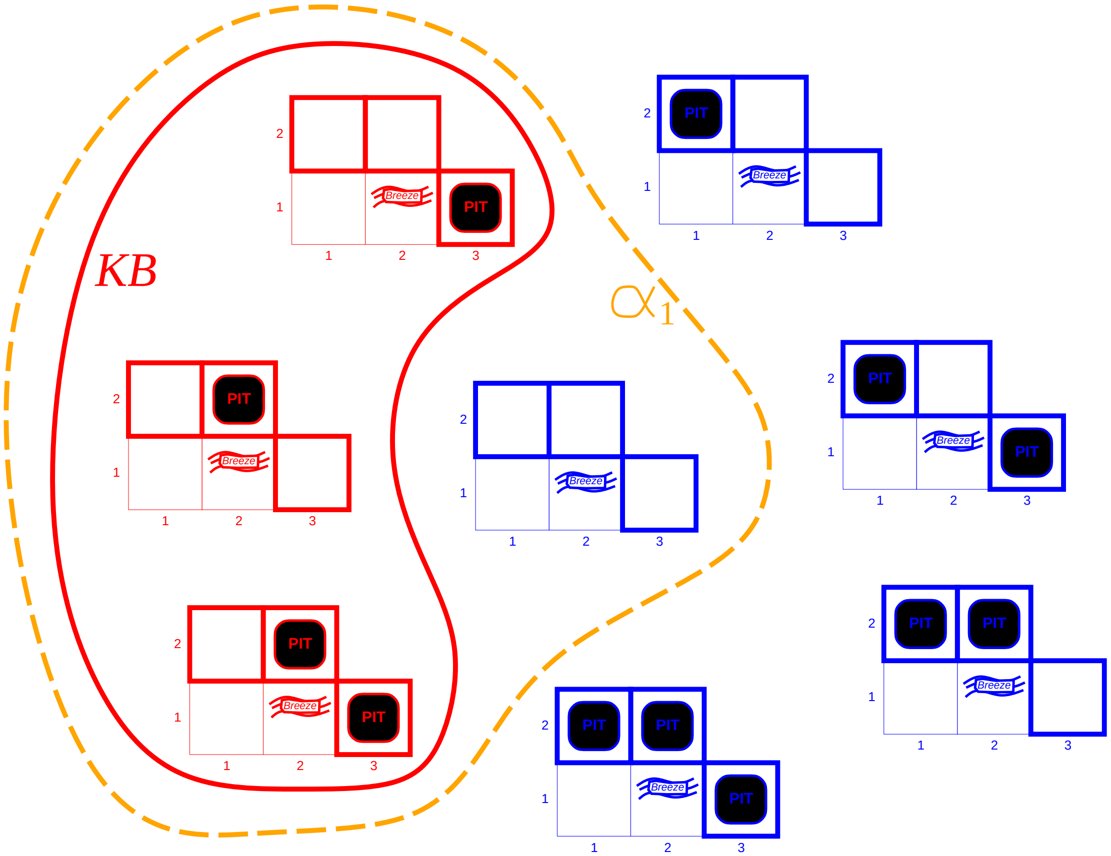


Wumpus models



KB = wumpus-world rules + observations

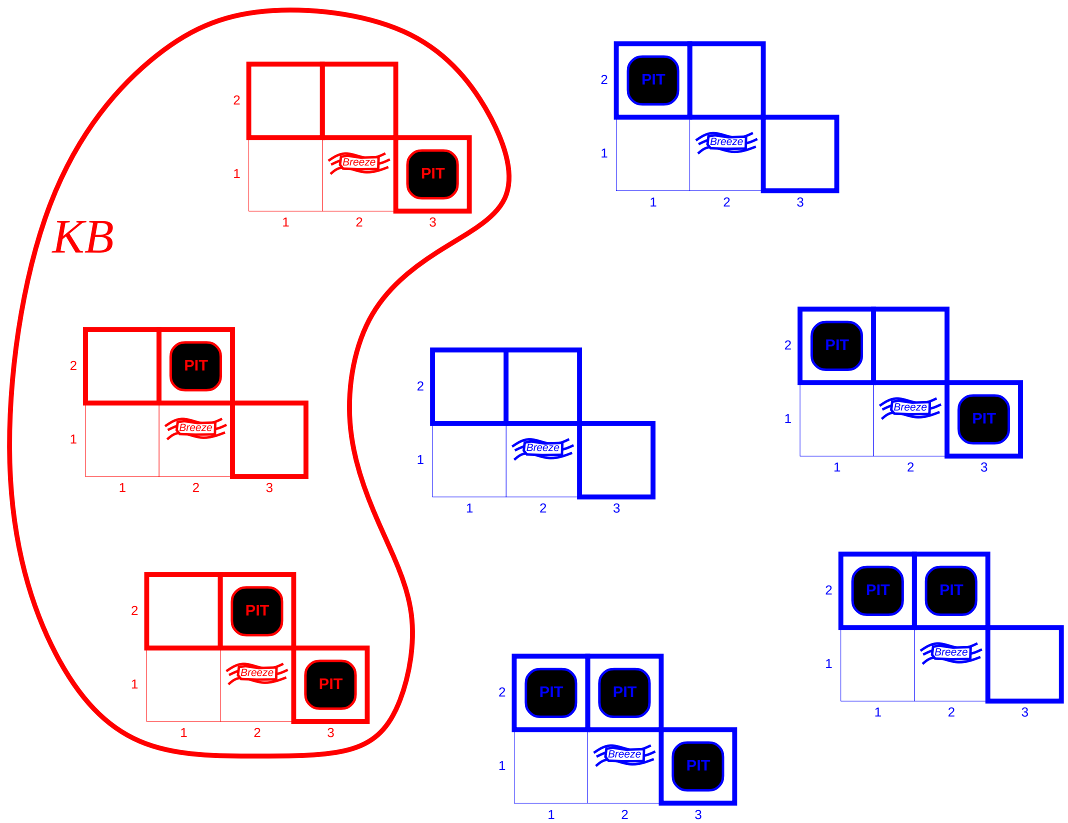
Wumpus models



KB = wumpus-world rules + observations

α_1 = "[1,2] is safe", $KB \models \alpha_1$, proved by model checking

Wumpus models



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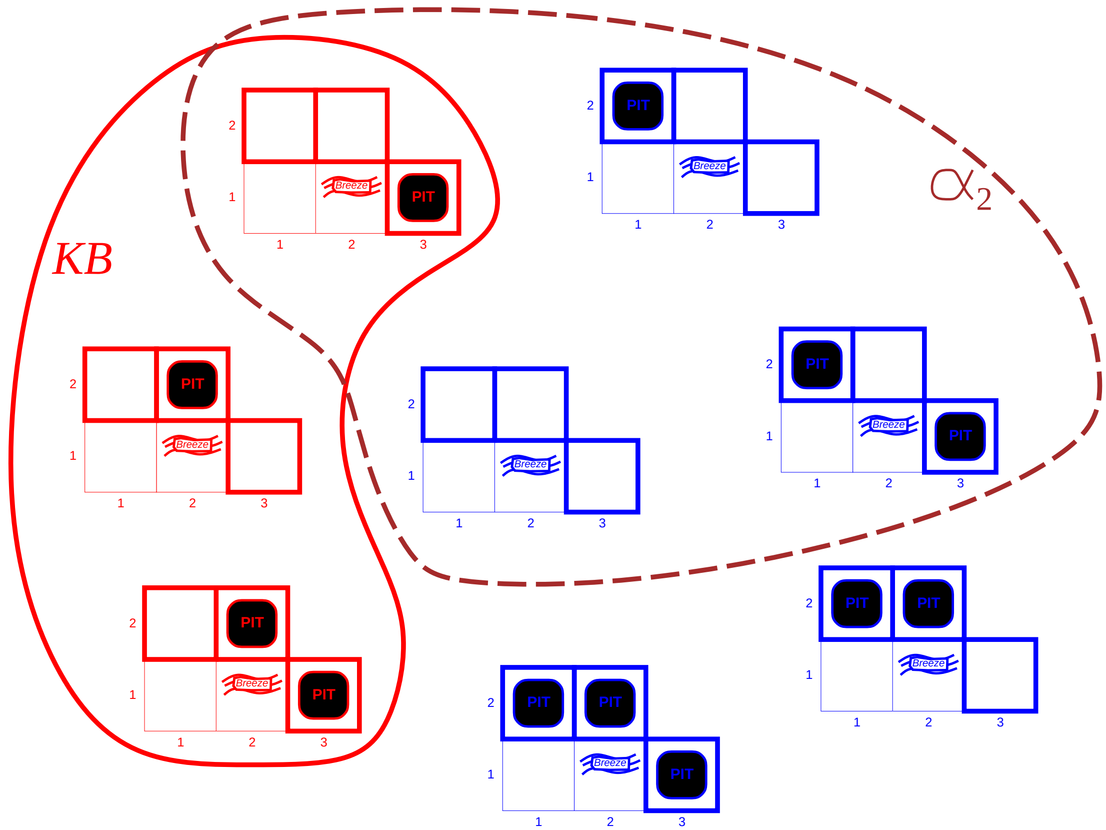
Wumpus models



KB = wumpus-world rules + observations

α_2 = "[2,2] is safe", $KB \not\models \alpha_2$

Inference

$KB \vdash_i \alpha$ means "sentence α can be derived from KB by procedure i"

Consequences of KB are a haystack; α is a needle.
Entailment = needle in haystack; inference = finding it

Soundness: i is sound if
whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness: i is complete if
whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure

That is, the procedure will answer any question whose answer follows from what is known

Propositional logic: Syntax

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols P_1, P_2 etc are sentences

If S is a sentence, $\neg S$ is a sentence (negation)

If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)

If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)

If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g. $P_{1,2}$ $P_{2,2}$ $P_{3,1}$ (3 symbols $\Rightarrow$ $2^3 = 8$ models)
$\quad\;\; true \;\; true \;\; false$

Rules for evaluating truth with respect to a model m:

$\neg S$	is true iff	S	is false		
$S_1 \wedge S_2$	is true iff	S_1	is true **and**	S_2	is true
$S_1 \vee S_2$	is true iff	S_1	is true **or**	S_2	is true
$S_1 \Rightarrow S_2$	is true iff	S_1	is false **or**	S_2	is true
i.e.,	is false iff	S_1	is true **and**	S_2	is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$	is true **and**	$S_2 \Rightarrow S_1$	is true

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\begin{aligned} \neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) &= \neg true \wedge (true \vee false) \\ &= false \wedge true = false \end{aligned}$$

Wumpus world sentences: symbols

$P_{i,j}$ has intended meaning "there is a pit in $[i,j]$"
$B_{i,j}$ has intended meaning "there is a breeze in $[i,j]$"

This means we connect sensors and actuators to symbols, and write axioms, in such a way that this meaning is respected

$R_1: \quad \neg P_{1,1}$ (given)
$R_2: \quad \neg B_{1,1}$ (percept)
$R_3: \quad B_{2,1}$ (percept)

Wumpus world sentences: axioms

An axiom is just a sentence asserted to be true about the domain
(typically **general** rather than specific to a particular situation)

E.g., "Pits cause breezes in adjacent squares"

Wumpus world sentences: axioms

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$$R_4: \quad B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$$
$$R_5: \quad B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$$

I.e., "A square is breezy **if and only if** there is an adjacent pit"

Notice that we need one such sentence for every square!

For shooting, movement, etc., we need axiom sets for every time step!!!!

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Enumerate rows (different assignments to symbols),
if KB is true in row, check that α is too

Inference by enumeration

Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS?(KB, α) returns true or false
   inputs: KB, the knowledge base, a sentence in propositional logic
           α, the query, a sentence in propositional logic

   symbols ← a list of the proposition symbols in KB and α
   return TT-CHECK-ALL(KB, α, symbols, [ ])
```

```
function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
   if EMPTY?(symbols) then
       if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
       else return true
   else do
       P ← FIRST(symbols); rest ← REST(symbols)
       return TT-CHECK-ALL(KB, α, rest, EXTEND(P, true, model)) and
                  TT-CHECK-ALL(KB, α, rest, EXTEND(P, false, model))
```

$O(2^n)$ for n symbols; problem is **co-NP-complete**

Logical equivalence

Two sentences are logically equivalent iff true in same models:

$\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

$(\alpha \wedge \beta)$	$\equiv$ $(\beta \wedge \alpha)$	commutativity of $\wedge$
$(\alpha \vee \beta)$	$\equiv$ $(\beta \vee \alpha)$	commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma)$	$\equiv$ $(\alpha \wedge (\beta \wedge \gamma))$	associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma)$	$\equiv$ $(\alpha \vee (\beta \vee \gamma))$	associativity of $\vee$
$\neg(\neg\alpha)$	$\equiv$ α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$ $(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$ $(\neg\alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$ $((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta)$	$\equiv$ $(\neg\alpha \vee \neg\beta)$	De Morgan
$\neg(\alpha \vee \beta)$	$\equiv$ $(\neg\alpha \wedge \neg\beta)$	De Morgan
$(\alpha \wedge (\beta \vee \gamma))$	$\equiv$ $((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma))$	$\equiv$ $((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of $\vee$ over $\wedge$

Validity and satisfiability

A sentence is valid if it is true in **all** models,

e.g., $True$, $A \lor \neg A$, $A \Rightarrow A$, $(A \land (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in **some** model

e.g., $A \lor B$, C

A sentence is unsatisfiable if it is true in **no** models

e.g., $A \land \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \land \neg\alpha)$ is unsatisfiable

i.e., prove α by *reductio ad absurdum*

Summary

Logical agents apply inference to a knowledge base
to derive new information and make decisions

Basic concepts of logic:
- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundness: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Propositional logic does all this (but lacks expressive power)

Inference by enumerating models: sound, complete, $O(2^n)$