Problem solving and search

Chapter 3
Reminders

Assignment 0 due midnight Thursday 9/8

Assignment 1 posted, due 9/20 (online or in box in 283)
Outline

◊ Problem-solving agents
◊ Problem types
◊ Problem formulation
◊ Example problems
◊ Basic search algorithms
Problem-solving agents

function **Simple-Problem-Solving-Agent** *(percept)* returns an action

static:  *seq*, an action sequence, initially empty

  *state*, some description of the current world state

  *goal*, a goal, initially null

  *problem*, a problem formulation

*state* ← **Update-State**(state, percept)

if *seq* is empty then

  *goal* ← **Formulate-Goal**(state)

  *problem* ← **Formulate-Problem**(state, goal)

  *seq* ← **Search**(problem)

  *action* ← **First**(seq); *seq* ← **Rest**(seq)

return *action*

Note: this is offline problem solving; solution executed “eyes closed.”

Online problem solving involves acting without complete knowledge.

Problems formulated in terms of **atomic** states
Example: Romania

On holiday in Romania; currently in Arad.
Flight leaves tomorrow from Bucharest

Formulate goal:
be in Bucharest

Formulate problem:
states: various cities
actions: drive between cities

Find solution:
sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest
Example: Romania
Problem types

Deterministic, fully observable $\implies$ single-state problem
- Agent knows exactly which state it will be in; solution is a sequence

Non-observable $\implies$ sensorless problem (a.k.a. conformant)
- Agent may have no idea where it is; solution (if any) is a sequence

Nondeterministic and/or partially observable $\implies$ contingency problem
- Percepts provide new information about current state
- Solution is a contingent plan or a policy
- Often interleave search, execution

Unknown state space $\implies$ exploration problem ("online")
Example: vacuum world

Single-state, start in #5. Solution??

1
2
3
4
5
6
7
8
Example: vacuum world

Single-state, start in #5. **Solution**??

*Right, Suck*

Sensorless, start in \(\{1, 2, 3, 4, 5, 6, 7, 8\}\)
e.g., *Right* goes to \(\{2, 4, 6, 8\}\). **Solution**??
Example: vacuum world

Single-state, start in #5. Solution??
[Right, Suck]

Sensorless, start in \{1, 2, 3, 4, 5, 6, 7, 8\}
e.g., Right goes to \{2, 4, 6, 8\}. Solution??
[Right, Suck, Left, Suck]

Contingency, start in #5
Murphy’s Law: Suck can dirty a clean carpet
Local sensing: dirt, location only.
Solution??
Example: vacuum world

Single-state, start in #5. Solution?? [Right, Suck]

Sensorless, start in \{1, 2, 3, 4, 5, 6, 7, 8\} e.g., Right goes to \{2, 4, 6, 8\}. Solution?? [Right, Suck, Left, Suck]

Contingency, start in #5
Murphy’s Law: Suck can dirty a clean carpet
Local sensing: dirt, location only.
Solution??
Initial belief state is \{5, 7\} [Right, if dirt then Suck]
A problem is defined by four items:

**initial state** e.g., “at Arad”

**successor function** $S(x) =$ set of action–state pairs
e.g., $S(Arad) = \{\langle Arad \rightarrow Zerind, Zerind \rangle, \ldots \}$

**goal test**, can be
explicit, e.g., $x =$ “at Bucharest”
implicit, e.g., $NoDirt(x)$

**path cost** (additive)
e.g., sum of distances, number of actions executed, etc.
$c(x, a, y)$ is the step cost, assumed to be $\geq 0$

A solution is a sequence of actions leading from the initial state to a goal state
Selecting a state space

Real world is absurdly complex
⇒ state space must be abstracted for problem solving

(Abstract) state = set of real states

(Abstract) action = complex combination of real actions
e.g., “Arad → Zerind” represents a complex set
of possible routes, detours, rest stops, etc.

For guaranteed realizability, any real state “in Arad”
must get to some real state “in Zerind”

(Abstract) solution
= sequence of abstract actions
= set of real paths that are solutions in the real world

Each abstract action should be “easier” than the original problem!
Example: vacuum world state space graph

states??
actions??
goal test??
path cost??
Example: vacuum world state space graph

**states??**: integer dirt and robot locations (ignore dirt amounts etc.)
**actions??**
**goal test??**
**path cost??**
Example: vacuum world state space graph

states??: integer dirt and robot locations (ignore dirt amounts etc.)
actions??: Left, Right, Suck, NoOp
goal test??
path cost??
**Example: vacuum world state space graph**

**States??**: integer dirt and robot locations (ignore dirt amounts etc.)

**Actions??**: *Left*, *Right*, *Suck*, *NoOp*

**Goal test??**: no dirt

**Path cost??**
**Example: vacuum world state space graph**

**States**: integer dirt and robot locations (ignore dirt **amounts** etc.)

**Actions**: *Left*, *Right*, *Suck*, *NoOp*

**Goal Test**: no dirt

**Path Cost**: 1 per action (0 for *NoOp*)
Example: The 8-puzzle

Start State

Goal State

states??
actions??
goal test??
path cost??
Example: The 8-puzzle

<table>
<thead>
<tr>
<th>Start State</th>
<th>Goal State</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

**states??**: integer locations of tiles (ignore intermediate positions)

**actions??**

**goal test??**

**path cost??**
Example: The 8-puzzle

**states**: integer locations of tiles (ignore intermediate positions)
**actions**: move blank left, right, up, down (ignore unjamming etc.)
**goal test**
**path cost**
Example: The 8-puzzle

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: = goal state (given)
path cost??
Example: The 8-puzzle

- **States??**: integer locations of tiles (ignore intermediate positions)
- **Actions??**: move blank left, right, up, down (ignore unjamming etc.)
- **Goal Test??**: = goal state (given)
- **Path Cost??**: 1 per move

[Note: optimal solution of $n$-Puzzle family is NP-hard]
Example: robotic assembly

**states**: real-valued coordinates of robot joint angles and parts of the object to be assembled

**actions**: continuous motions of robot joints

**goal test**: complete assembly *with no robot included!*

**path cost**: time to execute
Tree search algorithms

Basic idea:
offline, simulated exploration of state space
by generating successors of already-explored states
(a.k.a. expanding states)

function Tree-Search(problem, strategy) returns a solution, or failure
initialize the search tree using the initial state of problem
loop do
  if there are no candidates for expansion then return failure
  choose a leaf node for expansion according to strategy
  if the node contains a goal state then return the corresponding solution
  else expand the node and add the resulting nodes to the search tree
end
Tree search example
Tree search example

Arad

Sibiu

Timisoara

Zerind

Arad

Fagaras

Oradea

Rimnicu Vilcea

Arad

Lugoj

Oradea
Tree search example
A **state** is a (representation of) a physical configuration. A **node** is a data structure constituting part of a search tree, includes parent, children, depth, path cost $g(x)$. States do not have parents, children, depth, or path cost!

The **Expand** function creates new nodes, filling in the various fields and using the **SuccessorFn** of the problem to create the corresponding states.
Implementation: general tree search

function Tree-Search(problem, fringe) returns a solution, or failure
  fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← Remove-Front(fringe)
    if Goal-Test(problem, State(node)) then return Solution(node)
    fringe ← InsertAll(Expand(node, problem), fringe)
  end loop

function Expand(node, problem) returns a set of nodes
  successors ← the empty set; state ← State[node]
  for each action, result in Successor-Fn(problem, state) do
    s ← a new Node
    Parent-Node[s] ← node; Action[s] ← action; State[s] ← result
    Path-Cost[s] ← Path-Cost[node]+Step-Cost(state, action, result)
    Depth[s] ← Depth[node] + 1
    add s to successors
  end for
  return successors
Search strategies

A strategy is defined by picking the **order of node expansion**

Strategies are evaluated along the following dimensions:
- **completeness**—does it always find a solution if one exists?
- **time complexity**—number of nodes generated/expanded
- **space complexity**—maximum number of nodes in memory
- **optimality**—does it always find a least-cost solution?

Time and space complexity are measured in terms of
- $b$—maximum branching factor of the search tree
- $d$—depth of the least-cost solution
- $C^*$—path cost of the least-cost solution
- $m$—maximum depth of the state space (may be $\infty$)
Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

Iterative deepening search
Breadth-first search

Expand shallowest unexpanded node

**Implementation:**

*fringe* is a FIFO queue, i.e., new successors go at end
Breadth-first search

Expand shallowest unexpanded node

**Implementation:**

*fringe* is a FIFO queue, i.e., new successors go at end

```
  A
 / \  \\
B   C
 /   /
D   E  F  G
```
Breadth-first search

Expand shallowest unexpanded node

**Implementation:**

*fringe* is a FIFO queue, i.e., new successors go at end
Breadth-first search

Expand shallowest unexpanded node

Implementation: 
*fringe* is a FIFO queue, i.e., new successors go at end
Properties of breadth-first search

Complete??
Properties of breadth-first search

**Complete**?
Yes (if $b$ is finite)

**Time**?

Properties of breadth-first search

**Complete**? Yes (if \( b \) is finite)

**Time**? \[ 1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1}), \text{ i.e., exp. in } d \]

**Space**??
Properties of breadth-first search

Complete??  Yes (if \( b \) is finite)

Time??  \( 1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1}) \), i.e., exp. in \( d \)

Space??  \( O(b^{d+1}) \) (keeps every node in memory)

Optimal??
Properties of breadth-first search

**Complete?** Yes (if \( b \) is finite)

**Time?** \( 1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1}) \), i.e., exp. in \( d \)

**Space?** \( O(b^{d+1}) \) (keeps every node in memory)

**Optimal?** No, unless step costs are constant

**Space** is the big problem; can easily generate nodes at 100MB/sec so 24hrs = 8640GB.
Uniform-cost search

Expand least-cost unexpanded node

**Implementation:**

\( \text{fringe} = \text{queue ordered by path cost, lowest first} \)

Equivalent to breadth-first if step costs all equal

**Complete??** Yes, if step cost \( \geq \epsilon \)

**Time??** \# of nodes with \( g \leq \) cost of optimal solution, \( O(b^{C^*/\epsilon}) \)

where \( C^* \) is the cost of the optimal solution

**Space??** \# of nodes with \( g \leq \) cost of optimal solution, \( O(b^{C^*/\epsilon}) \)

**Optimal??** Yes—nodes expanded in increasing order of \( g(n) \)
Depth-first search

Expand deepest unexpanded node

**Implementation:**

fringe = LIFO queue, i.e., put successors at front

![Diagram of a tree starting with node A, and its successors B, C, D, E, F, G, H, I, J, K, L, M, N, O. Each node represents a search node, and the tree structure illustrates the depth-first search process.]
Depth-first search

Expand deepest unexpanded node

**Implementation:**

*fringe* = LIFO queue, i.e., put successors at front
Depth-first search

Expand deepest unexpanded node

Implementation:

\textit{fringe} = \text{LIFO queue}, i.e., put successors at front
Depth-first search

Expand deepest unexpanded node

**Implementation:**

\[ fringe = \text{LIFO queue, i.e., put successors at front} \]
Depth-first search

Expand deepest unexpanded node

**Implementation:**

\( fringe \) = LIFO queue, i.e., put successors at front
Depth-first search

Expand deepest unexpanded node

**Implementation:**

*fringe* = LIFO queue, i.e., put successors at front
Depth-first search

Expand deepest unexpanded node

**Implementation:**

*fringe* = LIFO queue, i.e., put successors at front
Depth-first search

Expand deepest unexpanded node

**Implementation:**

fringe = LIFO queue, i.e., put successors at front
Depth-first search

Expand deepest unexpanded node

**Implementation:**

$fringe = \text{LIFO queue, i.e., put successors at front}$
Depth-first search

Expand deepest unexpanded node

Implementation:

*fringe* = LIFO queue, i.e., put successors at front
Depth-first search

Expand deepest unexpanded node

**Implementation:**

*fringe* = LIFO queue, i.e., put successors at front
Depth-first search

Expand deepest unexpanded node

**Implementation:**

*fringe* = LIFO queue, i.e., put successors at front
Properties of depth-first search

Complete??
Properties of depth-first search

**Complete**
No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
⇒ complete in finite spaces

**Time**
Properties of depth-first search

**Complete**

No: fails in infinite-depth spaces, spaces with loops
  Modify to avoid repeated states along path
    ⇒ complete in finite spaces

**Time**

$O(b^m)$: terrible if $m$ is much larger than $d$
  but if solutions are dense, may be much faster than breadth-first

**Space**
Properties of depth-first search

**Complete??** No: fails in infinite-depth spaces, spaces with loops
   Modify to avoid repeated states along path
   ⇒ complete in finite spaces

**Time??** $O(b^m)$: terrible if $m$ is much larger than $d$
   but if solutions are dense, may be much faster than breadth-first

**Space??** $O(bm)$, i.e., linear space!

**Optimal??**
Properties of depth-first search

**Complete**? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
⇒ complete in finite spaces

**Time**? $O(b^m)$: terrible if $m$ is much larger than $d$
but if solutions are dense, may be much faster than breadth-first

**Space**? $O(bm)$, i.e., linear space!

**Optimal**? No
Depth-limited search

= depth-first search with depth limit \(l\), returns \textit{cutoff} if any path is cut off by depth limit

\textbf{Recursive implementation:}

\begin{verbatim}
function \textsc{Depth-Limited-Search}( problem, limit) returns soln/fail/cutoff

Recursive-DLS(Make-Node(Initial-State[problem]), problem, limit)

function Recursive-DLS( node, problem, limit) returns soln/fail/cutoff

cutoff-occurred? ← false
if Goal-Test(problem, State[node]) then return node
else if Depth[node] = limit then return cutoff
else for each successor in Expand(node, problem) do
    result ← Recursive-DLS(successor, problem, limit)
    if result = cutoff then cutoff-occurred? ← true
    else if result ⬇ failure then return result
if cutoff-occurred? then return cutoff else return failure
\end{verbatim}
Iterative deepening search

**function** Iterative-Deepening-Search(*problem*) **returns** a solution

**inputs:** *problem*, a problem

for depth ← 0 to ∞ do
    result ← Depth-Limited-Search(*problem*, depth)
    if result ≠ cutoff then return result
end
Iterative deepening search \( l = 0 \)
Iterative deepening search $l = 1$

Limit = 1
Iterative deepening search $l = 2$
Iterative deepening search \( l = 3 \)

Limit = 3
Properties of iterative deepening search

Complete??
Properties of iterative deepening search

Complete?? Yes

Time??
Properties of iterative deepening search

Complete? Yes

Time? \((d + 1)b^0 + db^1 + (d - 1)b^2 + \ldots + b^d = O(b^d)\)

Space??
Properties of iterative deepening search

**Complete**? Yes

**Time**? \((d + 1)b^0 + db^1 + (d - 1)b^2 + \ldots + b^d = O(b^d)\)

**Space**? \(O(bd)\)

**Optimal**?
Properties of iterative deepening search

**Complete??** Yes

**Time??** \((d + 1)b^0 + db^1 + (d - 1)b^2 + \ldots + b^d = O(b^d)\)

**Space??** \(O(bd)\)

**Optimal??** No, unless step costs are constant

Can be modified to explore uniform-cost tree

Numerical comparison for \(b = 10\) and \(d = 5\), solution at far right leaf:

\[
N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450
\]

\[
N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100
\]

IDS does better because other nodes at depth \(d\) are not expanded

BFS can be modified to apply goal test when a node is **generated**
## Summary of algorithms

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Breadth-First</th>
<th>Uniform-Cost</th>
<th>Depth-First</th>
<th>Depth-Limited</th>
<th>Iterative Deepening</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete?</td>
<td>Yes*</td>
<td>Yes*</td>
<td>No</td>
<td>Yes, if $l \geq d$</td>
<td>Yes</td>
</tr>
<tr>
<td>Time</td>
<td>$b^{d+1}$</td>
<td>$b^{[C^*/\varepsilon]}$</td>
<td>$b^m$</td>
<td>$b^l$</td>
<td>$b^d$</td>
</tr>
<tr>
<td>Space</td>
<td>$b^{d+1}$</td>
<td>$b^{[C^*/\varepsilon]}$</td>
<td>$bm$</td>
<td>$bl$</td>
<td>$bd$</td>
</tr>
<tr>
<td>Optimal?</td>
<td>No*</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No*</td>
</tr>
</tbody>
</table>
Repeated states

Failure to detect repeated states can cause \textit{exponentially} more work!
Graph search

function GRAPH-SEARCH(\textit{problem}, \textit{fringe}) returns a solution, or failure

\begin{itemize}
\item \textit{closed} $\leftarrow$ an empty set
\item \textit{fringe} $\leftarrow$ \textsc{Insert(Make-Node(Initial-State[\textit{problem}]), \textit{fringe})}
\item loop do
\begin{itemize}
\item if \textit{fringe} is empty then return failure
\item \textit{node} $\leftarrow$ \textsc{Remove-Front(\textit{fringe})}
\item if \textsc{Goal-Test(\textit{problem}, State[\textit{node}])} then return \textit{node}
\item if \text{State[\textit{node}]} is not in \textit{closed} then
\begin{itemize}
\item add \text{State[\textit{node}]} to \textit{closed}
\item \textit{fringe} $\leftarrow$ \textsc{InsertAll(Expand(\textit{node, problem}), \textit{fringe})}
\end{itemize}
\end{itemize}
end
\end{itemize}
**Graph search**

function **Graph-Search**(*problem, fringe*) returns a solution, or failure

1. \( \text{closed} \leftarrow \text{an empty set} \)
2. \( \text{fringe} \leftarrow \text{Insert(} \text{Make-Node(Initial-State[} \text{problem} \text{]), fringe} \text{)} \)
3. **loop do**
   - if \( \text{fringe} \) is empty then return failure
   - \( \text{node} \leftarrow \text{Remove-Front(} \text{fringe} \text{)} \)
   - if Goal-Test(*problem, State[node]*) then return node
   - if State[node] is not in \( \text{closed} \) then
     - add State[node] to \( \text{closed} \)
     - \( \text{fringe} \leftarrow \text{InsertAll(} \text{Expand(} \text{node, problem} \text{), fringe} \text{)} \)
4. **end**

✔️Use hash table for \( \text{closed} \) — constant-time lookup!
function **Graph-Search**(*problem, fringe*) *returns* a solution, or failure

\[
\begin{align*}
\text{closed} & \leftarrow \text{an empty set} \\
\text{fringe} & \leftarrow \text{Insert(Make-Node(Initial-State[problem]), fringe)} \\
\text{loop do} & \\
& \quad \text{if fringe is empty then return failure} \\
& \quad \text{node} \leftarrow \text{Remove-Front(fringe)} \\
& \quad \text{if Goal-Test(problem, State[node]) then return node} \\
& \quad \text{if State[node] is not in closed then} \\
& \quad & \quad \text{add State[node] to closed} \\
& \quad & \quad \text{fringe} \leftarrow \text{InsertAll(Expand(node, problem), fringe)} \\
\text{end}
\end{align*}
\]

Use hash table for *closed* — constant-time lookup!

Makes all algorithms complete in finite spaces!!
**Graph search**

function **Graph-Search**(*problem, fringe*) returns a solution, or failure

\[\text{closed} \leftarrow \text{an empty set}\]
\[\text{fringe} \leftarrow \text{INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)}\]

loop do
  if fringe is empty then return failure
  node \leftarrow \text{REMOVE-FRONT(fringe)}
  if \text{GOAL-TEST(problem, STATE[node])} then return node
  if STATE[node] is not in closed then
    add STATE[node] to closed
    fringe \leftarrow \text{INSERTALL(EXPAND(node, problem), fringe)}
  end


😊 Use hash table for *closed* — constant-time lookup!
😊 Makes all algorithms complete in finite spaces!!
😊 Makes all algorithms worst-case exponential space!!!
Graph search

**function** GRAPH-SEARCH \((\text{problem}, \text{fringe})\) **returns** a solution, or failure

- \(\text{closed} \leftarrow \text{an empty set}\)
- \(\text{fringe} \leftarrow \text{INSERT}(\text{MAKE-NODE}(\text{INITIAL-STATE}[\text{problem}]), \text{fringe})\)

**loop do**

- **if** \(\text{fringe} \) is empty **then return** failure
- \(\text{node} \leftarrow \text{REMOVE-FRONT}(\text{fringe})\)
- **if** \(\text{GOAL-TEST}(\text{problem}, \text{STATE}[\text{node}])\) **then return** \(\text{node}\)
- **if** \(\text{STATE}[\text{node}]\) is not in \(\text{closed}\) **then**
  - add \(\text{STATE}[\text{node}]\) to \(\text{closed}\)
- \(\text{fringe} \leftarrow \text{INSERT-ALL}(\text{EXPAND}(\text{node}, \text{problem}), \text{fringe})\)

**Use hash table for** \(\text{closed}\) — constant-time lookup!

- **Makes all algorithms complete in finite spaces!!**
- **Makes all algorithms worst-case exponential space!!!**
- **But size of graph often much less than** \(O(b^d)!!!!\)
Summary

Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored.

Variety of uninformed search strategies

Iterative deepening search uses only linear space and not much more time than other uninformed algorithms.

Graph search can be exponentially more efficient than tree search.