## Problem solving and search

Chapter 3



## Reminders

**Assignment 0 due midnight Thursday 9/8**

**Assignment 1 posted**, due 9/20 (online or in box in 283)



## Outline

◇ Problem-solving agents

◇ Problem types

◇ Problem formulation

◇ Example problems

◇ Basic search algorithms



## Problem-solving agents

```
function Simple-Problem-Solving-Agent( percept) returns an action
   static: seq, an action sequence, initially empty
           state, some description of the current world state
           goal, a goal, initially null
           problem, a problem formulation
   state ← Update-State(state, percept)
   if seq is empty then
       goal ← Formulate-Goal(state)
       problem ← Formulate-Problem(state, goal)
       seq ← Search( problem)
   action ← First(seq); seq ← Rest(seq)
   return action
```

Note: this is offline problem solving; solution executed "eyes closed."
Online problem solving involves acting without complete knowledge.

Problems formulated in terms of **atomic** states



## Example: Romania

On holiday in Romania; currently in Arad.
Flight leaves tomorrow from Bucharest

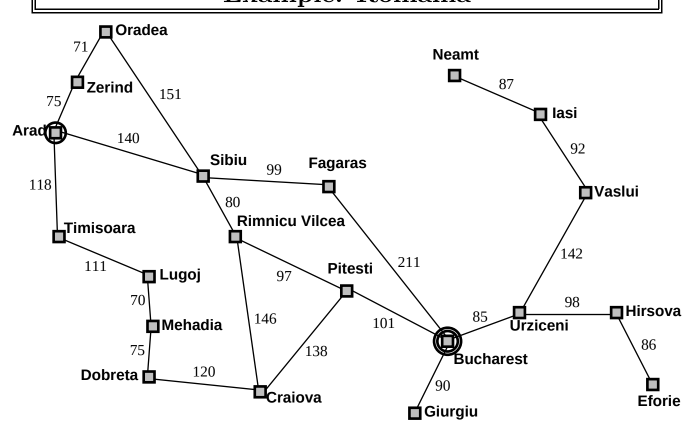
Formulate goal:
be in Bucharest

Formulate problem:
states: various cities
actions: drive between cities

Find solution:
sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest



## Example: Romania



## Problem types

Deterministic, fully observable $\Longrightarrow$ single-state problem
- Agent knows exactly which state it will be in; solution is a sequence

Non-observable $\Longrightarrow$ sensorless problem (a.k.a. conformant)
- Agent may have no idea where it is; solution (if any) is a sequence

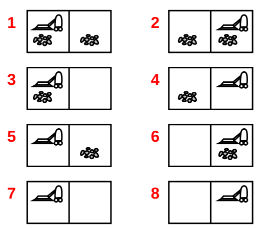
Nondeterministic and/or partially observable $\Longrightarrow$ contingency problem
- Percepts provide **new** information about current state
- Solution is a contingent plan or a policy
- Often **interleave** search, execution

Unknown state space $\Longrightarrow$ exploration problem ("online")



## Example: vacuum world

Single-state, start in #5. Solution??

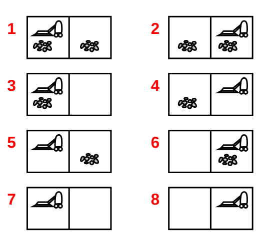





## Example: vacuum world

Single-state, start in #5. Solution??
$[Right, Suck]$

Sensorless, start in $\{1, 2, 3, 4, 5, 6, 7, 8\}$
e.g., $Right$ goes to $\{2, 4, 6, 8\}$. Solution??



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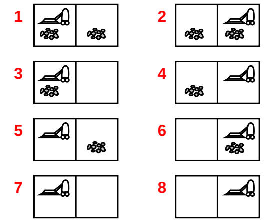
$[Right, Suck, Left, Suck]$

Contingency, start in #5
Murphy's Law: $Suck$ can dirty a clean carpet
Local sensing: dirt, location only.
Solution??



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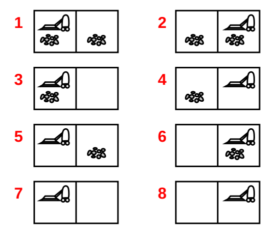
$[Right, Suck, Left, Suck]$

Contingency, start in #5
Murphy's Law: $Suck$ can dirty a clean carpet
Local sensing: dirt, location only.
Solution??
Initial belief state is $\{5, 7\}$
$[Right, \textbf{if } dirt \textbf{ then } Suck]$






## Single-state problem formulation

A problem is defined by four items:

initial state e.g., "at Arad"

successor function $S(x)$ = set of action–state pairs
e.g., $S(Arad) = \{\langle Arad \rightarrow Zerind, Zerind\rangle, \ldots\}$

goal test, can be
explicit, e.g., $x$ = "at Bucharest"
implicit, e.g., $NoDirt(x)$

path cost (additive)
e.g., sum of distances, number of actions executed, etc.
$c(x, a, y)$ is the step cost, assumed to be $\geq 0$

A solution is a sequence of actions
leading from the initial state to a goal state

## Selecting a state space

Real world is absurdly complex
⇒ state space must be **abstracted** for problem solving

(Abstract) state = set of real states

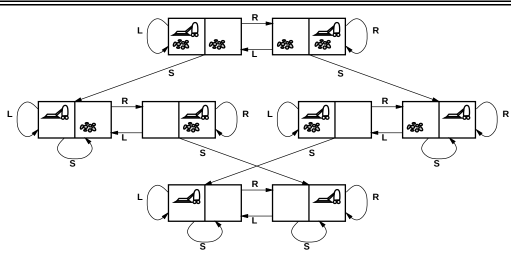
(Abstract) action = complex combination of real actions
e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.
For guaranteed realizability, **any** real state "in Arad" must get to **some** real state "in Zerind"

(Abstract) solution
= sequence of abstract actions
= set of real paths that are solutions in the real world

Each abstract action should be "easier" than the original problem!



## Example: vacuum world state space graph



states??
actions??
goal test??
path cost??



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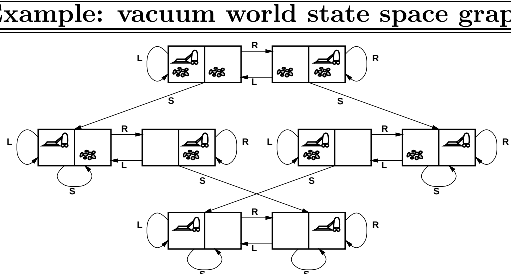

states??: integer dirt and robot locations (ignore dirt amounts etc.)
actions??
goal test??
path cost??



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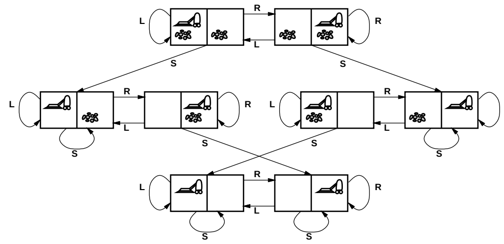

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actions??: *Left*, *Right*, *Suck*, *NoOp*
goal test??
path cost??



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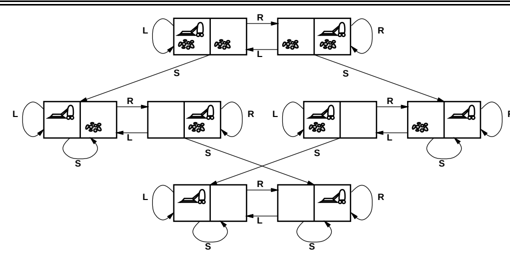

states??: integer dirt and robot locations (ignore dirt amounts etc.)
actions??: *Left*, *Right*, *Suck*, *NoOp*
goal test??: no dirt
path cost??



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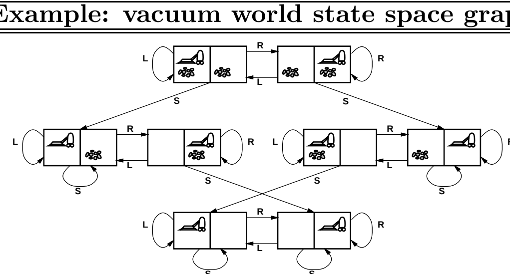

states??: integer dirt and robot locations (ignore dirt amounts etc.)
actions??: *Left*, *Right*, *Suck*, *NoOp*
goal test??: no dirt
path cost??: 1 per action (0 for *NoOp*)

## Example: The 8-puzzle

| | | |
|---|---|---|
| 7 | 2 | 4 |
| 5 | | 6 |
| 8 | 3 | 1 |

Start State

| | | |
|---|---|---|
| | 1 | 2 |
| 3 | 4 | 5 |
| 6 | 7 | 8 |

Goal State

states??
actions??
goal test??
path cost??



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Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??
goal test??
path cost??



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Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??
path cost??



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actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: = goal state (given)
path cost??



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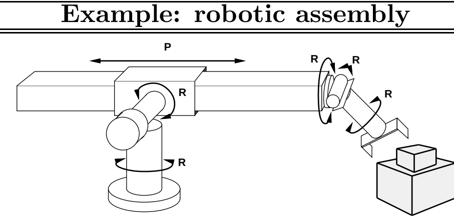
Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: = goal state (given)
path cost??: 1 per move

[Note: optimal solution of $n$-Puzzle family is NP-hard]



## Example: robotic assembly




states??: real-valued coordinates of robot joint angles
and parts of the object to be assembled

actions??: continuous motions of robot joints

goal test??: complete assembly **with no robot included!**

path cost??: time to execute

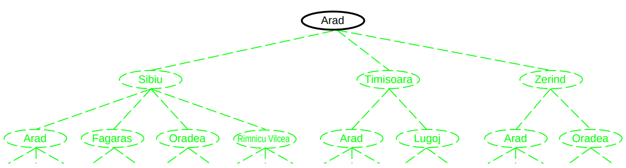
## Tree search algorithms

Basic idea:
offline, simulated exploration of state space
by generating successors of already-explored states
(a.k.a. expanding states)

```
function Tree-Search( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
   end
```



## Tree search example



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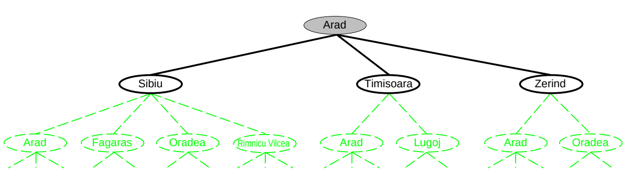




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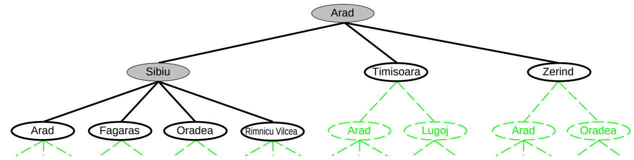
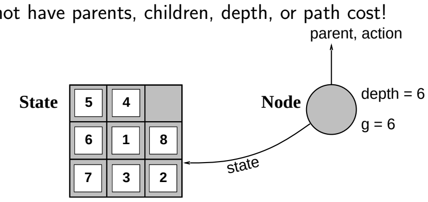





## Implementation: states vs. nodes

A state is a (representation of) a physical configuration
A node is a data structure constituting part of a search tree
includes parent, children, depth, path cost $g(x)$
States do not have parents, children, depth, or path cost!




The Expand function creates new nodes, filling in the various fields and using the SuccessorFn of the problem to create the corresponding states.



## Implementation: general tree search

```
function Tree-Search( problem, fringe) returns a solution, or failure
   fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
   loop do
      if fringe is empty then return failure
      node ← Remove-Front(fringe)
      if Goal-Test(problem, State(node)) then return Solution(node)
      fringe ← InsertAll(Expand(node, problem), fringe)

function Expand( node, problem) returns a set of nodes
   successors ← the empty set; state ← State[node]
   for each action, result in Successor-Fn(problem, state) do
      s ← a new Node
      Parent-Node[s] ← node;  Action[s] ← action;  State[s] ← result
      Path-Cost[s] ← Path-Cost[node]+Step-Cost(state, action, result)
      Depth[s] ← Depth[node] + 1
      add s to successors
   return successors
```

## Search strategies

A strategy is defined by picking the **order of node expansion**

Strategies are evaluated along the following dimensions:
- completeness—does it always find a solution if one exists?
- time complexity—number of nodes generated/expanded
- space complexity—maximum number of nodes in memory
- optimality—does it always find a least-cost solution?

Time and space complexity are measured in terms of
- $b$—maximum branching factor of the search tree
- $d$—depth of the least-cost solution
- $C^*$—path cost of the least-cost solution
- $m$—maximum depth of the state space (may be $\infty$)



## Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

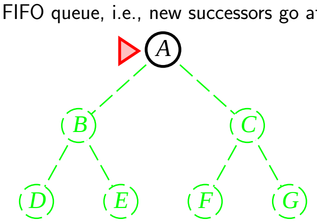
Iterative deepening search



## Breadth-first search

Expand shallowest unexpanded node

**Implementation**:
*fringe* is a FIFO queue, i.e., new successors go at end



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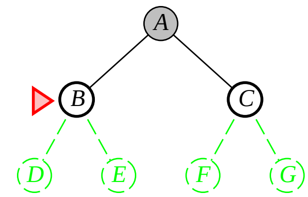




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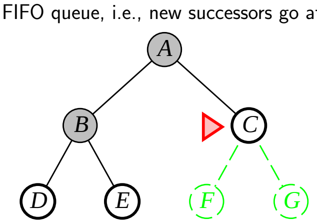




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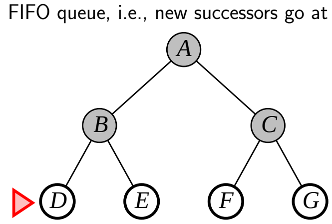

## Properties of breadth-first search

Complete??



## Properties of breadth-first search

Complete?? Yes (if $b$ is finite)

Time??



## Properties of breadth-first search

Complete?? Yes (if $b$ is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in $d$

Space??



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Complete?? Yes (if $b$ is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in $d$

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal??



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Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal?? No, unless step costs are constant

**Space** is the big problem; can easily generate nodes at 100MB/sec
so 24hrs = 8640GB.



## Uniform-cost search

Expand least-cost unexpanded node

**Implementation**:
*fringe* = queue ordered by path cost, lowest first

Equivalent to breadth-first if step costs all equal

Complete?? Yes, if step cost $\geq \epsilon$

Time?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$
where $C^*$ is the cost of the optimal solution

Space?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$

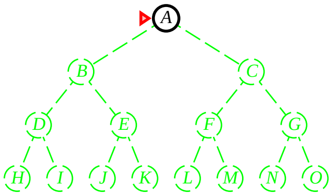
Optimal?? Yes—nodes expanded in increasing order of $g(n)$

## Depth-first search

Expand deepest unexpanded node

**Implementation**:

*fringe* = LIFO queue, i.e., put successors at front



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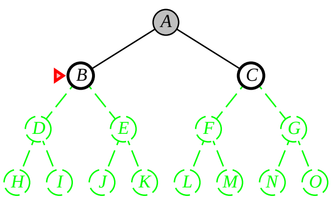




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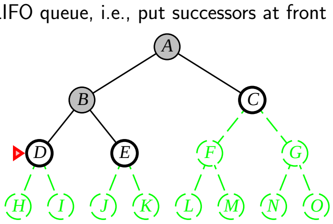




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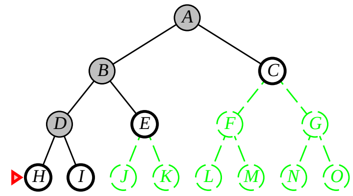




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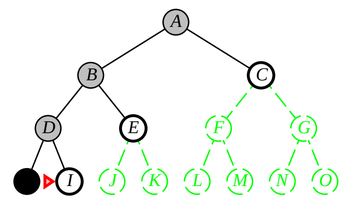




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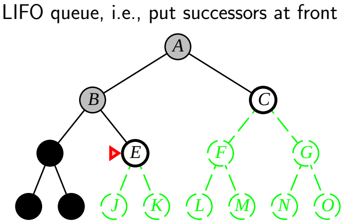

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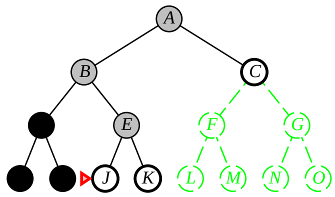




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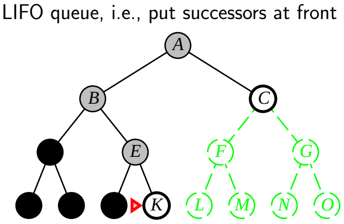




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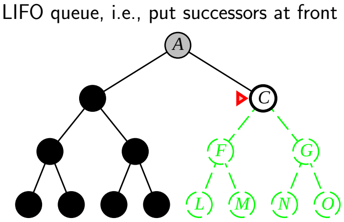




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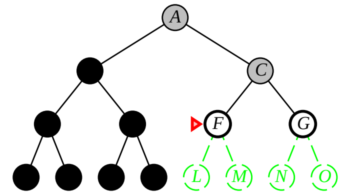




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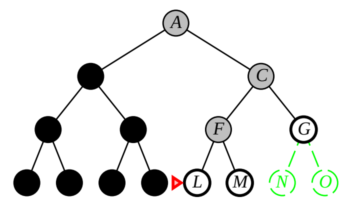




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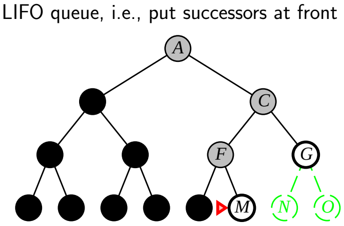

## Properties of depth-first search

Complete??



## Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time??



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but if solutions are dense, may be much faster than breadth-first

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Space?? $O(bm)$, i.e., linear space!

Optimal??



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but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal?? No



## Depth-limited search

= depth-first search with depth limit $l$,
returns *cutoff* if any path is cut off by depth limit

**Recursive implementation**:

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
   RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if GOAL-TEST(problem, STATE[node]) then return node
   else if DEPTH[node] = limit then return cutoff
   else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

## Iterative deepening search

**function** ITERATIVE-DEEPENING-SEARCH( *problem*) **returns** a solution
  **inputs**: *problem*, a problem

  **for** *depth* ← 0 **to** $\infty$ **do**
    *result* ← DEPTH-LIMITED-SEARCH( *problem*, *depth*)
    **if** *result* $\neq$ cutoff **then return** *result*
  **end**



## Iterative deepening search $l = 0$

Limit = 0



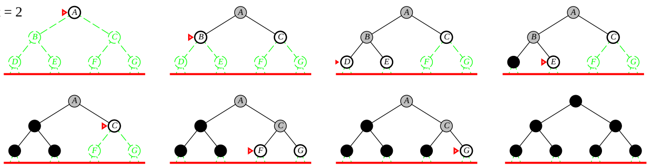
## Iterative deepening search $l = 1$

Limit = 1



## Iterative deepening search $l = 2$

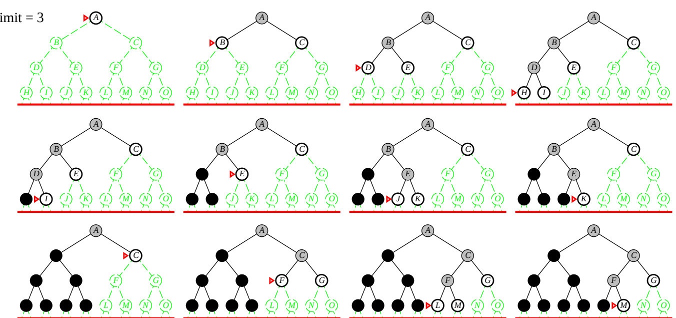
Limit = 2





## Iterative deepening search $l = 3$

Limit = 3





## Properties of iterative deepening search

Complete??

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Time??



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Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space??



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Optimal??



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Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal?? No, unless step costs are constant
Can be modified to explore uniform-cost tree

Numerical comparison for $b = 10$ and $d = 5$, solution at far right leaf:

$$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$$
$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100$$

IDS does better because other nodes at depth $d$ are not expanded

BFS can be modified to apply goal test when a node is **generated**



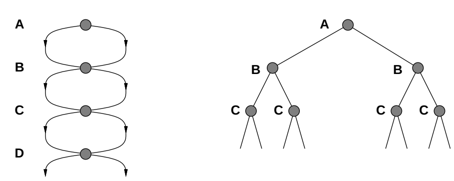
## Summary of algorithms

| Criterion | Breadth-First | Uniform-Cost | Depth-First | Depth-Limited | Iterative Deepening |
|---|---|---|---|---|---|
| Complete? | Yes$^*$ | Yes$^*$ | No | Yes, if $l \geq d$ | Yes |
| Time | $b^{d+1}$ | $b^{\lceil C^*/\epsilon \rceil}$ | $b^m$ | $b^l$ | $b^d$ |
| Space | $b^{d+1}$ | $b^{\lceil C^*/\epsilon \rceil}$ | $bm$ | $bl$ | $bd$ |
| Optimal? | No$^*$ | Yes | No | No | No$^*$ |



## Repeated states

Failure to detect repeated states can cause **exponentially** more work!



## Graph search

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
   closed ← an empty set
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
      if fringe is empty then return failure
      node ← REMOVE-FRONT(fringe)
      if GOAL-TEST(problem, STATE[node]) then return node
      if STATE[node] is not in closed then
         add STATE[node] to closed
         fringe ← INSERTALL(EXPAND(node, problem), fringe)
   end
```



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Use hash table for *closed* — constant-time lookup!



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Use hash table for *closed* — constant-time lookup!
Makes all algorithms complete in finite spaces!!
Makes all algorithms worst-case exponential space!!!
But size of graph often much less than $O(b^d)$!!!!



## Summary

Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored

Variety of uninformed search strategies

Iterative deepening search uses only linear space
and not much more time than other uninformed algorithms

Graph search can be exponentially more efficient than tree search