CS188 Spring 1993 — Intro to AI — Stuart Russell

Midterm 1

You have exactly 1 hour. The exam is open-book, open-notes.
You will not necessarily finish all questions, so do your best ones first.
Write your answers in blue books. Hand them all in.

60 points total (1 point per minute). Panic not.

1. **(15 pts.) Definitions**
   Provide brief, *precise* definitions of the following:
   (a) (3 pts) Dynamic environment
   (b) (3 pts) Admissible heuristic
   (c) (3 pts) Minimax decision
   (d) (3 pts) Complete inference procedure
   (e) (3 pts) Rational agent

2. **(17 pts.) Heuristic search**

   Suppose you're trying to solve the following puzzle. The puzzle involves numbers from 100 to 999.

   You're given two numbers called S and G. You're also given a set of numbers called *bad*. A move consists of transforming one number into another by adding 1 to one of its digits or subtracting 1 from one of its digits; for instance, a move can take you from 678 to 679; or from 234 to 134. Moves are subject to the following constraints:

   - You cannot add to the digit 9 or subtract from the digit 0. That is to say, no "carries" are allowed and the digits must remain in the range from 0 to 9.
   - You cannot make a move which transforms your current number into one of the numbers in the set *bad*.
   - You cannot change the same digit twice in two successive moves.

   Since the numbers have only 3 digits, there are at most 6 possible moves at the start. And since all moves except the first are preceded by another move which uses one of the digits, after the start there are at most 4 possible moves per turn. You solve the puzzle by getting from S to G in the fewest possible moves. Your task is to use A* search to find a solution to the puzzle.

   (a) (2 pts) Briefly list the information needed in the state description (not the node description) in order to apply A* to this problem.
   (b) (3 pts) Find a heuristic for use with A* search in this problem which is admissible and which does not require extensive mathematical calculation (that is, you should be able to use it in solving the second part of this problem without needing a calculator!). Explain clearly why your heuristic is admissible. Try to find a heuristic which is as powerful as possible while still remaining admissible.
   (c) (2 pts) Give a domain-independent description of what f, g, and h values represent in A* search in general.

(d) (10 pts) Use your heuristic to carry out an A* search to find a solution when $S = 567$, $G = 777$, and $bad = \{666, 667\}$. Draw the search tree as you go, showing for each node both the state it represents (include all information needed to distinguish this state from other states) and the node's f, g, and h values. Also, whenever you expand a node (generate its successors), label it with a number indicating the order of expansion (the start node would be labeled 1, the next node to be expanded would be 2, etc.). Include in your tree *all legal successors* of each node you expand.

Hint: for nodes that tie for best-node-to-expand, it's probably best to give the search a slightly "depth-first" character by choosing the node with higher g value.

3. (12 pts.) Logic and knowledge representation

Translate each of the following English sentences into the language of situation calculus and unrestricted first-order logic. ("Unrestricted", as opposed to clausal form or any other highly regimented logical notation.) All quantifiers should be shown explicitly (that is, don't use unquantified variables and assume that the reader will know where the quantifier should go and what kind of quantifier it should be). You will need to use situations because some facts about the world being described change over time, and in some cases you're being asked to describe those changes. (Never mind the truth or falsity of any of these sentences in any world; that's not the point!).

(a) (3 pts) There can't be two different robots at the same vertex at the same time.

(b) (3 pts) At any given time, there's a robot at precisely one vertex.

(c) (3 pts) If a robot at one vertex tries to move to another visible vertex and there's no robot at the other vertex, then the robot ends up at the other vertex.

(d) (3 pts) If a robot at one vertex tries to move to another visible vertex and there's a robot at the other vertex, then the first robot stays put.

Briefly define in English each predicate or function you use.

4. (16 pts.) Logical transformation and inference

Here are two sentences in the language of first-order logic:

- (A) $\forall x \exists y\ (x \geq y)$
- (B) $\exists y \forall x\ (x \geq y)$

(a) (2 pts) Assume that the variables range over all the natural numbers $0, 1, 2 \ldots \infty$, and that the "$\geq$" predicate means "greater than or equal to". Under this interpretation, translate these sentences into English.

(b) (1 pt) Is (A) true under this interpretation?

(c) (1 pt) Is (B) true under this interpretation?

(d) (1 pt) Does (A) logically entail (B)? (Just say "yes" or "no"; the question of proof will be dealt with below.)

(e) (1 pt) Does (B) logically entail (A)?

(f) (5 pts) Try to prove that (A) follows from (B), using resolution. Do this even if you think that (B) does not logically entail (A); continue until the proof breaks down and you cannot proceed (if it does break down). The procedure is the following:

- Negate the goal sentence;
- Move negation inwards, using quantifier equivalence rules.
- Skolemize the sentences and put them into conjunctive normal form;
- Perform resolution until the empty clause is derived.

Show the unifying substitution for each resolution step. If the proof fails, explain exactly where, how, and why it breaks down.

(g) (5 pts) Now try to prove that (B) follows from (A). The instructions are exactly the same as in the previous part.