1. (12 pts.) True/False

(a) (2) True/False: There exists a task environment (PEAS) in which every agent is rational.

(b) (2) True/False: Suppose agent A selects its action uniformly at random from the set of possible actions. There exists a deterministic, fully observable task environment in which A is rational.

(c) (2) True/False: No logical agent can behave rationally in partially observable environment.

(d) (2) True/False: $\forall x, y \ x = y$ is satisfiable.

(e) (2) True/False: If $\theta$ unifies the atomic sentences $\alpha$ and $\beta$, then $\alpha \models \text{SUBST}(\theta, \beta)$.

(f) (2) True/False: In any finite state space, random-restart hillclimbing is an optimal algorithm.

2. (24 pts.) Search

Suppose there are two friends living in different cities on a map, such as the Romania map shown in Figure 3.2 of AIMA2e. On every turn, we can move each friend simultaneously to a neighboring city on the map. The amount of time needed to move from city $i$ to neighbor $j$ is equal to the road distance $d(i, j)$ between the cities, but on each turn the friend that arrives first must wait until the other one arrives (and calls the first on his/her cell phone) before the next turn can begin. We want the two friends to meet as quickly as possible. Let us formulate this as a search problem.

(a) (4) What is the state space? (You will find it helpful to define some formal notation here.)
(b) (4) What is the successor function?

(c) (2) What is the goal?

(d) (4) What is the step cost function?

(e) (6) Let $SLD(i,j)$ be the straight-line distance between any two cities $i$ and $j$. Which, if any, of the following heuristic functions are admissible? (If none, write NONE.)
   (i) $SLD(i,j)$  
   (ii) $2 \cdot SLD(i,j)$  
   (iii) $SLD(i,j)/2$

(f) (4) True/False: There are completely connected maps for which no solution exists.

3. (18 pts.) Propositional logic

(a) (9) Which of the following are entailed by the sentence $(A \lor B) \land (\neg C \lor \neg D \lor E)$?
   i. $(A \lor B)$
   ii. $(A \lor B \lor C) \land (B \land C \land D \Rightarrow E)$
   iii. $(A \lor B) \land (\neg D \lor E)$

(b) (3) True/False: Every nonempty propositional clause, by itself, is satisfiable.

(c) (6) True/False: Every set of five 3SAT clauses is satisfiable, provided that each clause mentions exactly three distinct variables.

4. (22 pts.) Logical knowledge representation

(a) (12) Which of the following are semantically and syntactically correct translations of “Everyone’s zipcode within a state has the same first digit”?
   i. $\forall x, s, z_1 \ [State(s) \land LivesIn(x, s) \land Zip(x) = z_1] \Rightarrow \forall y, z_2 \ [LivesIn(y, s) \land Zip(y) = z_2 \Rightarrow Digit(1, z_1) = Digit(1, z_2)]$.
   ii. $\forall x, s \ [State(s) \land LivesIn(x, s) \land \exists z_1 \ Zip(x) = z_1] \Rightarrow \forall y, z_2 \ [LivesIn(y, s) \land Zip(y) = z_2 \land Digit(1, z_1) = Digit(1, z_2)]$.
   iii. $\forall x, y, s \ State(s) \land LivesIn(x, s) \land LivesIn(y, s) \Rightarrow Digit(1, Zip(x) = Zip(y))$.
   iv. $\forall x, y, s \ State(s) \land LivesIn(x, s) \land LivesIn(y, s) \Rightarrow Digit(1, Zip(x)) = Digit(1, Zip(y))$. 
(b) (10) It was stated in lecture that a complete representation of the rules of chess in propositional logic would be unmanageably large—perhaps thousands of times larger than the first-order logic version. Which of the following are valid reasons for this?

i. The rules of chess are very complicated.

ii. A chess game can go on for hundreds of moves.

iii. There are several types of pieces.

iv. There are several pieces of each type.

v. There are 64 squares on the board.

5. (24 pts.) Game playing
Imagine that, in Q.2, one of the friends wants to avoid the other. The problem then becomes a two-player pursuit-evasion game. We assume now that the players take turns moving. The game ends only when the players are on the same node; the terminal payoff to the pursuer is minus the total time taken. (The evader “wins” by never losing.) Consider the following simple map, where the cost of every arc is 1 and initially the pursuer $P$ is at node $b$ and the evader $E$ is at node $d$.

Here is a partially constructed game tree for this map. Each node is labelled with the $P, E$ positions. $P$ moves first. The values of the leaves marked “?” are currently unknown.
(a) (3) Mark the values of the terminal nodes.

(b) (6) Inside each internal node, write the strongest fact you can infer about its value (either a number, one or more inequalities such as \( \geq 14 \), or a “?”).

(c) (6) Can shortest-path lengths on the map be used to bound the values of the “?” leaves. If so, why and how? If not, why not?

(d) (3) Mark inequalities on all the “?” leaves according to the method in (c). Remember the cost to get to each leaf as well as the cost to solve it.

(e) (6) Now suppose the tree as given, with the leaf bounds from (d), was evaluated left-to-right. CIRCLE those nodes “?” nodes that would not need to be expanded further, given the bounds from part (d), and CROSS OUT those that need not be considered at all.

(f) (10 extra credit) Can you say anything precise about who wins the game on a map that is a tree?