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# A Near Field Propagation Law & A Novel Fundamental Limit to Antenna Gain Versus Size

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**ABSTRACT**

This paper presents a theoretical analysis of the near field channel in free space. Then this paper validates the theoretical model by comparison to data measured in an open field. The results of this paper are important for low frequency RF systems, such as those operating at short range in the AM broadcast band. Finally this paper establishes a novel fundamental limit for antenna gain versus size.

## 1. INTRODUCTION

The "leading edge" of RF practice moves to increasingly higher and higher frequency in lock step with advances in electronics technology. The most commercially significant RF systems are those operating at microwave frequencies and above, such as cellular telephones and wireless data networks. Microwave frequencies have the advantage of short wavelengths, making antenna design relatively straightforward, and vast expanses of spectrum, making large bandwidth, high data rate transmissions possible.

There are many applications, however, that do not require large bandwidths. These include real time locating systems (RTLS) and low data rate communications systems, such as hands free wireless mikes or other voice or low data rate telemetry links. For applications like these, lower frequencies have great utility.

- Lower frequencies tend to be more penetrating than higher frequencies.
- Lower frequencies tend to diffract around objects that would block higher frequencies.
- Lower frequencies are less prone to multipath.

An amazing and often overlooked world of RF phenomena lies within about a half wavelength of an electrically small antenna. This realm is known as the "near field zone." The near field zone is usually neglected by RF scientists and engineers because typical RF links operate at distances of many wavelengths where near field effects are utterly insignificant. "Near field" means different things in different contexts. Fortunately there is an excellent article available that sorts through the various definitions of near field and provides some guidance [1]. The present discussion, takes the near field zone as the region within about a half wavelength of an electrically small antenna.

The aim of the present paper is to derive a near field propagation equation. This paper compares the near field propagation equation to data obtained in an open field environment. Finally, this near field propagation equation is used to derive a fundamental limit for antenna size versus gain.

## 2. PATH GAIN IN FAR AND NEAR FIELD

This section will discuss the path gain for traditional far field links and summarize the differences between far field and near field links. Then, this section will present a recently introduced a near field link equation that provides path loss for low frequency near field links [2].

The path gain ($P$) defines the relationship between transmitted power ($P_{TX}$) and received power ($P_{RX}$) in a far-field RF link. This relation was first given by Harald Friis [3]:

$$P(f,d)=\frac{P_{RX}}{P_{TX}}=\frac{G_{TX}G_{RX}\lambda^2}{(4\pi)^2 d^2}=\frac{G_{TX}G_{RX}}{4}\frac{1}{(kd)^2} \tag{1}$$

In this formula, $G_{RX}$ and $G_{TX}$ are the receive and transmit antenna gains (respectively), $d$ is the distance between the antennas, $\lambda$ is wavelength, and $k = 2\pi/\lambda$ is the wave number. The reason for writing Friis's law in a non-standard way (using wave number) will become clear momentarily. The upshot of Friis's Law is that the far-field power rolls off as the inverse square of the distance

($1/d^2$). Near-field links do not obey this relationship. Near field power rolls off as powers higher than inverse square, typically inverse fourth ($1/d^4$) or higher.

This near field behavior has several important consequences. First, the available power in a near field link will tend to be much higher than would be predicted from the usual far-field, Friis's Law relationship. This means a higher signal-to-noise ratio (SNR) and a better performing link. Second, because the near-fields have such a rapid roll-off, range tends to be relatively finite and limited. Thus, a near-field system is less likely to interfere with another RF system outside the operational range of the near-field system.

Electric and magnetic fields behave differently in the near field, and thus require different link equations. Reception of an electric field signal requires an electric antenna, like a whip or a dipole. Reception of a magnetic field signal requires a magnetic antenna, like a loop or a loopstick. The received signal power from a co-polarized electric antenna is proportional to the time average value of the incident electric field squared. For the case of a small electric dipole transmit antenna radiating in the azimuthal plane and being received by a vertically polarized electric antenna, the received power is:

$$P_{RX(E)} \sim \left\langle |\mathbf{E}|^2 \right\rangle \sim \left( \frac{1}{(kd)^2} - \frac{1}{(kd)^4} + \frac{1}{(kd)^6} \right) \qquad (2)$$

Similarly, the received signal power from a co-polarized magnetic antenna is proportional to the time average value of the incident magnetic field squared:

$$P_{RX(H)} \sim \left\langle |\mathbf{H}|^2 \right\rangle \sim \left( \frac{1}{(kd)^2} + \frac{1}{(kd)^4} \right) \qquad (3)$$

Thus, the "near field" path gain formulas are:

$$\begin{aligned} P_E(d,f) &= \frac{P_{RX(E)}}{P_{TX}} \\ &= \frac{G_{TX} G_{RX(E)}}{4} \left( \frac{1}{(kd)^2} - \frac{1}{(kd)^4} + \frac{1}{(kd)^6} \right) \end{aligned} \qquad (4)$$

for the electric field signal, and:

$$P_H(d,f) = \frac{P_{RX(H)}}{P_{TX}} = \frac{G_{TX} G_{RX}}{4} \left( \frac{1}{(kd)^2} + \frac{1}{(kd)^4} \right) \qquad (5)$$

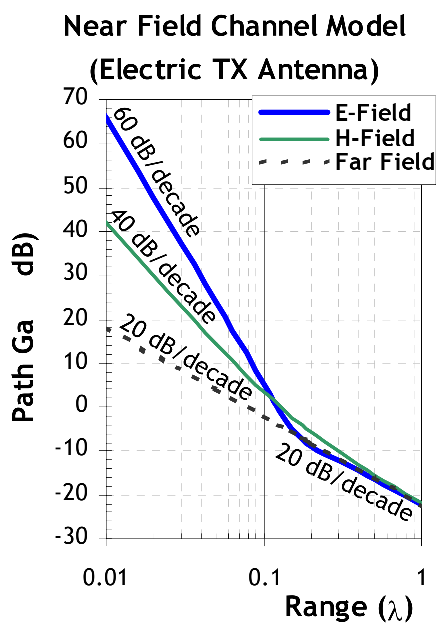



**Figure 1** Path gain of a typical near field channel

for the magnetic field signal. Equation 4 is the propagation law for like antennas (electric to electric or magnetic to magnetic) and Equation 5 is the near field propagation law for unlike antennas (magnetic to electric or electric to magnetic). Typical path gain in a near field channel is on the order of –6 dB. At very short ranges, path gain may be on the order of +60 dB or more. At an extreme range of about one wavelength the path gain may be about –18 dB. This behavior is summarized in Figure 1.

## 3. PROPAGATION DATA

The Q-Track Corporation has pioneered a novel low frequency tracking technology known as "Near Field Electromagnetic Ranging," or ***NFER***™ technology. ***NFER***™ technology operates in the AM broadcast band (525-1715 kHz) on an unlicensed basis under the authority of the FCC's Part 15 regulations [4]. Q-Track's 1295 kHz ($\lambda$ = 213.5 m) prototypes have a demonstrated accuracy of about 30 cm at ranges of up to 70 m outdoors, and an accuracy of about 4 m at ranges of up to 70 m indoors. For more information on Q-Track's ***NFER***™ technology, please see the Q-Track website [5]. Table 1 shows parameters for a prototype ***NFER***™ tracking system.

| Parameter: | Value: |
|---|---|
| Transmit Gain $G_{TX}$ = | -51 dB |
| E Receive Antenna Gain $G_{RXE}$ = | -53 dB |
| H Receive Antenna Gain $G_{RXH}$ = | -71 dB |
| Transmit Power $P_{TX}$ = | +20 dBm |

**Table 1:** Parameters for a prototype ***NFER***™ tracking system.


(a) (b)

**Figure 2 (a)** Q-Track's prototype beacon transmitter with whip antenna. **(b)** Q-Track prototype locator receiver with three element array [both figures courtesy Q-Track; © 2004].

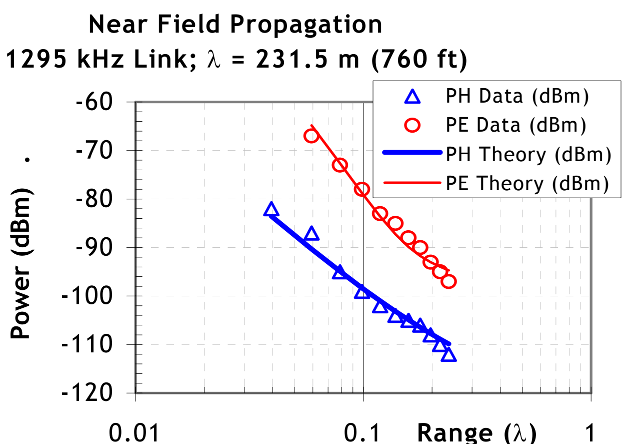


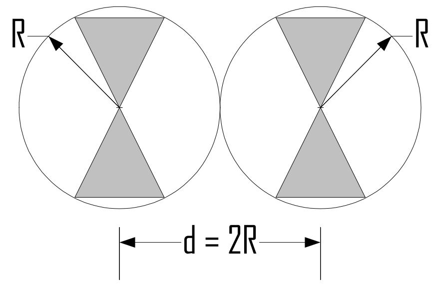

**Figure 3** Theory vs. experiment for a 1295 kHz link [Courtesy Q-Track; © 2004].

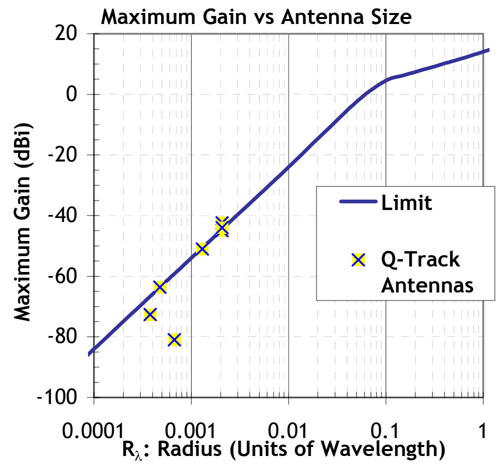
Q-Track's prototype beacon transmitter operates at 1295 kHz with a transmit power at the FCC limit of 100 mW. The beacon transmitter uses a 60 cm (2 ft) whip with a gain of approximately –51 dBi. Figure 2(a) shows Q-Track's beacon transmitter with whip antenna. Q-Track's prototype locator receiver uses a three antenna array to receive both electric and magnetic field components. The electric receive antenna is similar to the electric transmit antenna and has a gain of –53 dBi. The magnetic receive antenna is a box loop with a gain of about –71 dBi. Figure 2(b) shows Q-Track's locator receiver with three element antenna array. Figure 3 compares near field propagation results for a prototype ***NFER***™ tracking system to the theoretical predictions of Equations 4 and 5. Agreement is generally within a few dB.

## 4. LIMITS TO ANTENNA SIZE AND GAIN

The near field link equations define the path gain as a function of the transmit and receive antenna gains. Figure 1 appears to indicate that under some circumstances path gain may be greater than 0 dB. This means that the receive power could theoretically be greater than the transmit power. Since conservation of energy must apply to RF links, antenna gain cannot be arbitrarily large. There necessarily exists a limit to antenna gain as a function of antenna size.

The treatment of this section borrows on the concept of an antenna "boundary sphere" introduced by Wheeler and extended upon by Chu [6, 7]. A boundary sphere is the smallest sphere within which an antenna may be enclosed. Thus the radius of the boundary sphere defines the characteristic size of an antenna. A matched pair of antennas with boundary spheres of radius $R$ may be no closer than $d = 2\ R$ without overlapping, as shown in Figure 4(a). Taking this as the limit, one can apply the near field propagation equation for like antennas (Eq. 4) to establish a limit for antenna gain versus size:




**(a)** **(b)**

**Figure 4(a)** A matched pair of antennas with boundary spheres of radius $R$ can be no closer than about $d = 2\ R$ without their boundary spheres overlapping. **(b)** Limit on antenna gain vs. size showing a variety of Q-Track antennas [Q-Track; ©2004].

$$P_E(d,f)=\frac{P_{RX(E)}}{P_{TX}}\leq 1\geq\frac{GG}{4}\left(\frac{1}{(2kR)^2}-\frac{1}{(2kR)^4}+\frac{1}{(2kR)^6}\right)\rightarrow$$

$$G\leq\sqrt{\frac{4}{\left(\frac{1}{(2kR)^2}-\frac{1}{(2kR)^4}+\frac{1}{(2kR)^6}\right)}}=\frac{2(2kR)^3}{\sqrt{1-(2kR)^2+(2kR)^4}}=\frac{2(4\pi R_\lambda)^3}{\sqrt{1-(4\pi R_\lambda)^2+(4\pi R_\lambda)^4}} \qquad (7)$$

Figure 4(b) shows the gain limit as a function of boundary sphere radius in units of wavelength. This figure also shows gain and size of a variety of Q-Track's electrically small antennas for comparison.

## 5. CONCLUSIONS

This paper derived near field propagation relations analogous to Friis's law and compared theory to experimental data. This paper further derived a fundamental limit to antenna gain versus size and compared the result to a variety of antennas designed by the Q-Track Corporation. The often neglected world of the near field is not only susceptible to mathematical analysis but also yields lessons applicable to antenna design in general.

## 6. ACKNOWLEDGEMENTS

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## 7. REFERENCES

[1] Charles Capps, "Near Field or Far Field," EDN, August 16, 2001, pp. 95-102. This excellent article is available online at: http://www.edn.com/contents/images/150828.pdf
[2] Hans Schantz, Near Field Channel Model, IEEE P802.15-04/0417r2, October 27, 2004.
[3] Harald Friis, "A Note on a Simple Transmission Formula," Proc. IRE, **34**, 1946, pp. 254-256.
[4] Section 15.219 authorizes a transmit power of 100 mW (+20 dBm) into an antenna less than 3 m in dimensions on an unlicensed basis in the AM broadcast band.
[5] For more information on ***NFER***™ technology see http://www.q-track.com.
[6] Harold A. Wheeler, "Fundamental Limitations of Small Antennas," Proc. IRE, **35**, (1947), pp. 1479-1484.
[7] L.J. Chu, "Physical Limitations of Omni-Directional Antennas," Journal of Applied Physics, **19**, December 1948, pp. 1163-1175