K = 1 N/m  \quad V_{SI} = 15 V  \quad g_0 = 3 \text{ mm}  \quad Q = 100

1. How far can we decently use a DC voltage? 1 mm
2. What is the range of stable DC voltages? ±15 V
3. What is the force to pull in? (1 mm)(1 N/m) = 1 N
4. What is the actuator area? \( A = \frac{1}{2} \pi \cdot \frac{g_0}{Q} \)
5. What is the minimum DC step to cause pull-in?
6. What is the minimum AC voltage to cause pull-in?
7. What is the resonant frequency change at \( V = 10 V \)?

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Step response of spin axis, underdamped

\[ \frac{1}{2} F_L \frac{L}{(\frac{g_0}{Q})^2} \quad S_{DC} = g_0 - \frac{F_L}{K} \]

pick \( g_{DC} = g_0 - \frac{1}{6} g_0 + \text{a little bit} \)

\[ F = \frac{1}{2} F_{puln} = \frac{1}{2} \left( \frac{1}{2} \frac{g_0}{Q} \right) = \frac{1}{2} \frac{6 V_0^2}{(\frac{5}{6} g_0)^2} \]

\[ V_{\frac{1}{2}} = \frac{1}{2} V_p \pi \left( \frac{5/6}{2/3} \right)^2 \]

\[ V_{\frac{1}{2}} \approx 0.9 V_p \quad \text{abrupt} \quad V_{\frac{1}{2}} \text{ causes pull-in} \]
Frequency tuning

\[ K_{\text{eff}} = K_{\text{spring}} - K_{\text{electrostaticic}} \]

\[ \omega_n = \sqrt{\frac{K_{\text{eff}}}{m}} \]

\[ K_{\text{eff}} = \begin{cases} K_{\text{spring}} & V = 0 \\ 0 & V = V_p,2 \end{cases} \]

Tagg-style resonator

process variation

\[ a = 2 \mu m \pm \Delta a \]

say 10% variation

\[ k = E \frac{a^3}{4L^2} \Delta a \approx 0.2 \mu m \]

\[ \omega_n = \sqrt{\frac{k}{m}} = \sqrt{(1 + \frac{4a}{a})} \]

\[ \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1 + \frac{4a}{a}}{1 + \frac{2K}{K}}} \]

\[ \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1 + \frac{2K}{K}}{1 + \frac{2K}{a}}} \]

\[ \Delta K = 3 \frac{4a}{a} \]

\[ \frac{dE}{dV} = (-2)(-\frac{1}{2} \varepsilon_0 V) \frac{A}{q^2} \]

\[ = -2 \frac{F_e}{q} \]

positive spring constant

(destabilization)

if \( K \) varies 10%, \( \omega \) varies 5%

need to design \( K \) to be too stiff by 30%.

then use electrostatic spring to tune

\( \Delta a = \frac{4a}{a} \)

could use this gap

pretty non-linear

more linear

larger regulation
rotary torsional spring

torsion radial gap

\[ \Theta_{01} = 0.42 \Theta_0 \]

Non-linear spring e.g. \[ f(x) \]

\[ F = \begin{cases} \frac{1}{2} \epsilon_0 V^2 A \frac{1}{(x_0 + x)^2} \quad & \text{for small } V \\ \frac{1}{2} \epsilon_0 V^2 A \frac{1}{(x_0 - x)^2} \quad & \text{for bi-symmetric } V \end{cases} \]

Positive spring constant

\[ f = -Kx + \left| \frac{dF_e}{dx} \right|_x \]

Softens, tunes spring \( K \) by varying \( V \)

\[ \omega_n = \sqrt{\frac{K_{eff}}{m}} \]

\( 3 \) equilibria,
1 stable

Comb drive instability \( +V \)
Electrostatic spring softens
Electrostatic tuning

\[ F_0 = -\frac{1}{2} \epsilon_0 V^2 A \]

\[ F_x = \frac{1}{2} \epsilon_0 V^2 \frac{A}{(x_0 - x)^2} \]

\[ F_z = \frac{1}{2} \epsilon_0 V^2 \frac{A}{(x_0 + x)^2} \]