EE 147/247A Prof. Pister
Fall 2021

Homework Assignment #5

Due on bcourses Thursday 10/8/2021 (zero credit after 9 AM Sunday)

1. Fall 2017 Midterm question 2

[4] Four resistors are in a Wheatstone bridge with a 1V excitation. The resistors all have a temperature coefficient of resistance of 1%, and a gauge factor of 100.

a. If one resistor is stretched by 10 parts per million, what is the magnitude of the change in the bridge output.

$$\epsilon = \frac{10}{1{,}000{,}000} = 10^{-5}, \frac{\Delta R}{R} = G\epsilon = 10^{-3}, V_{out} = \frac{\Delta R}{4R} V_x$$

V_{out}= 250uV

b. If the electronics can detect signals down to 100nV, what is the minimum detectable strain?

$$V_{out\ min} = 100nV = \frac{\Delta R_{min}}{4R} V_x, \frac{\Delta R}{R} = 400 * 10^{-9} = G\epsilon, \epsilon = 4 * 10^{-9}$$

ε_{min}= 4 nano strains

c. What is the temperature change in a single resistor which will give a 100nV bridge output?

$$V_{out\ min} = 100nV = \frac{\Delta R_{min}}{4R} V_x, \frac{\Delta R}{R} = 400 * 10^{-9} = TCR * \Delta T, \Delta T = 400 * 10^{-7}$$

ΔT= $40 * 10^{-6} C$

2. Fall 2020 Midterm question 3

[8] For a comb drive resonator, if all dimensions scale by a factor S, how do the following parameters scale?

Parameter	Scales as (e.g. S, S^2, sqrt(S), 1, etc.)
mass	wlt: S^3
Spring constant	$\frac{a^3 b}{L^3}$: S
damping	$\frac{A}{g}$: S
Electrostatic force	$\frac{A}{g^2}$ or $\frac{t}{g}$: 1
Low frequency deflection	$\frac{1}{k}$: S^-1
Quality factor Q	$\frac{k}{w_n b}$: S
Resonant deflection	Qx_{DC}: 1

3. Fall 2020 Midterm question 4

[8] In the polyMUMPs process, list the thin film layers that will be present on the substrate (starting at the substrate and working up, in order) before the release etch, in regions with the following masking layers

2 Points per scenario

-1 points for each additional layer and -1 for each layer missed

a. POLY0, POLY2, METAL

Sub, nitride, Poly0, Ox1, Ox2, Poly2, Metal

b. ANCHOR2, POLY2

Sub, nitride, poly2

c. POLY0, ANCHOR2, POLY2, METAL

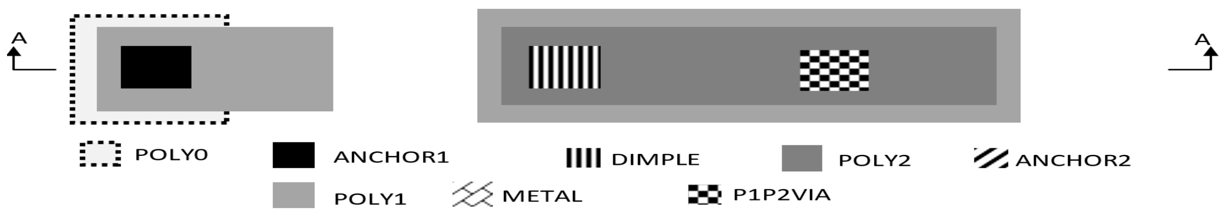
Sub, nitride, poly0, poly2, metal

d. POLY1, PIP2VIA, POLY2

Sub, nitride, ox1, poly1, poly2

4. Fall 2020 Midterm question 5

. [10] The layout below is to be made in the polyMUMPs process. Assume that the contacts are 2um square and draw a cross-section through AA of what the structure will look like before the final HF etch. Assume that all of the contacts are 2x2um2, and the grid below is also 2um squares.



1 point for Nitride and Substrate shown below Poly0 and Ox1

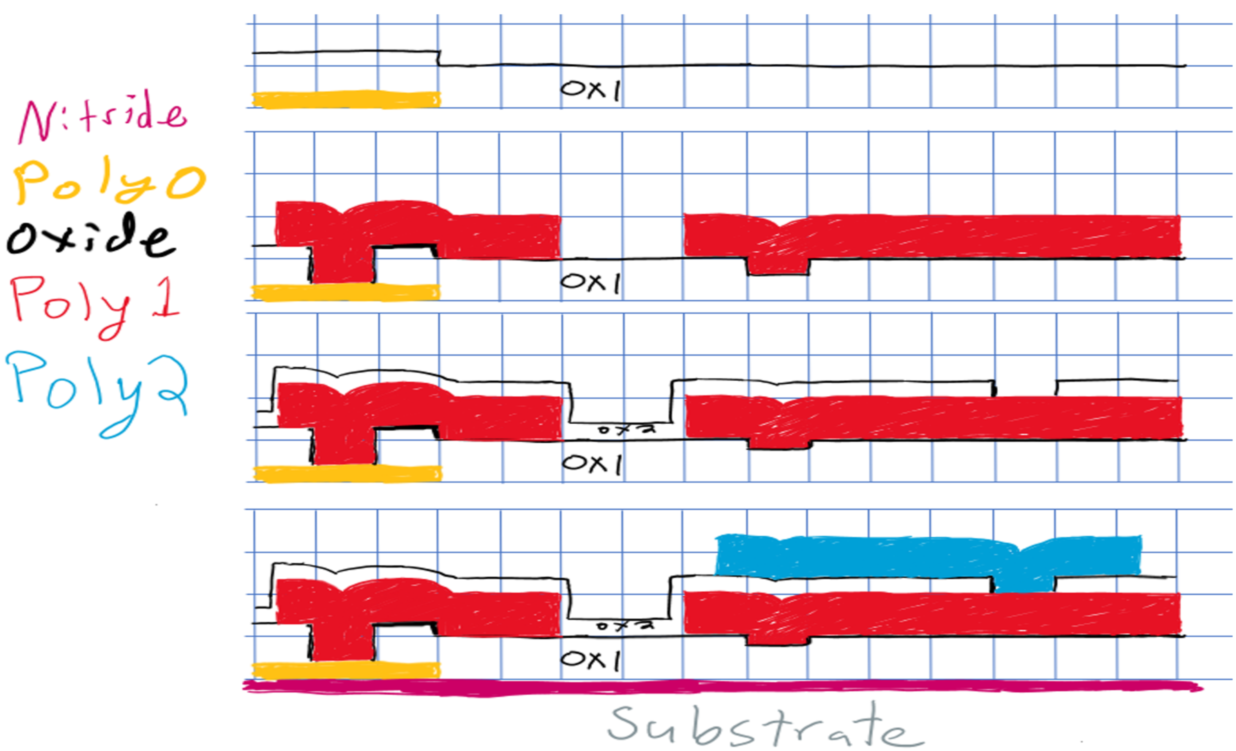
1 point for Anchor1

1 point for Dimple (pointy or flat is fine but can't be very deep)

1 point for via

1 point for ~right scale for each layer (5 layers=5 points)

1 point for shape



5. [6 pts] A resonator with a spring constant k, mass m, and a Q of 100 is driven by an external force to have an amplitude of motion of x_0. There are two frequencies at which the magnitude of the sine terms (sum of spring and mass) is equal to the magnitude of the cosine term (damping). The difference between these is the 3dB bandwidth.
   a. [4 pts] Estimate those two frequencies. You should be able to do this by solving a quadratic equation in w. A useful relationship: $km=b^2Q^2$

$$(k - m\omega^2) = +/-b\omega$$

$$\omega^2 +/- \frac{b\omega}{m} - \frac{k}{m} = 0$$

$$\omega = \frac{-/+\frac{b}{m} +/- \sqrt{\left(\frac{b}{m}\right)^2 + 4\frac{k}{m}}}{2}$$

$$b = \frac{k}{w_n Q} \; so \; \frac{b}{m} = \frac{w_n}{Q}$$

$$\omega = \frac{-/+\frac{w_n}{Q} +/- \sqrt{\frac{w_n}{Q}^2 + 4w_n^2}}{2}$$

Only care about positive solutions so

$$\frac{-/+\frac{w_n}{Q} + \sqrt{\frac{w_n}{Q}^2 + 4w_n^2}}{2} \approx w_n -/+ \frac{w_n}{2Q}$$

1 pt for at least one correct quadratic equation. 1 pt for some attempt to solve it. 1 pt each for correct solution.

Difference between the two solutions = $\frac{w_n}{Q}$

   b. [1 pt] Is the difference roughly the resonant frequency divided by the Q?

Yes.

   c. [1 pt] What is the difference in the phase at these two frequencies?

90 degrees

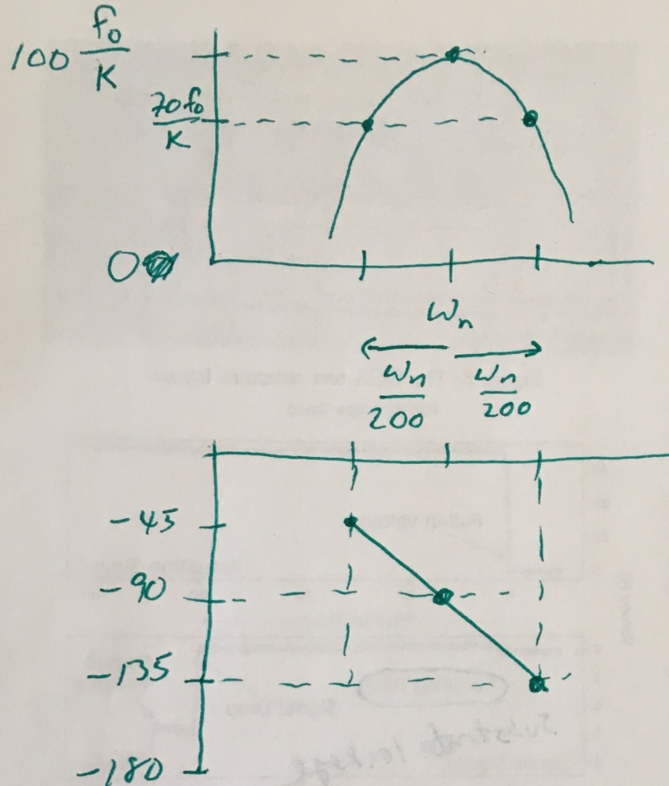
6. [18 pts total] Using the results above, plot the magnitude and phase of the resonator displacement in response to a sine wave $f(t)=f_0\sin(\omega t)$ a few different ways. Note that you will be plotting the magnitude and phase of the complex number $H(jw) = 1/[\omega_n^2-\omega^2 + (\omega_n/Q)j\omega]$
   a. Use a linear axis for frequency, magnitude, and phase, and zoom in on the frequency range ω_n +/- ω_n/Q
   [6 pts] 1 pt for each of the six dots below. You should really have both the frequency and the value correct for each dot, since that's what you just calculated. But as long as you get something close you can take the point.

   b. Use a linear axis for frequency, magnitude, and phase. The range of frequency should be 0 to 2 ω_n. The range for magnitude should include 0 and the peak magnitude. If you do this accurately, your plots will look like a couple of straight lines – that's ok.

   [6 pts] 1 pt each for a flat line very close to zero for the magnitude and phase below resonance, 1 pt for a flat line very close to zero for the magnitude above resonance, and 1 pt for a flat line very close to -180 for the phase above resonance. 1 pt for the resonant peak having the right value. 1 pt for the phase have the right value at resonance.

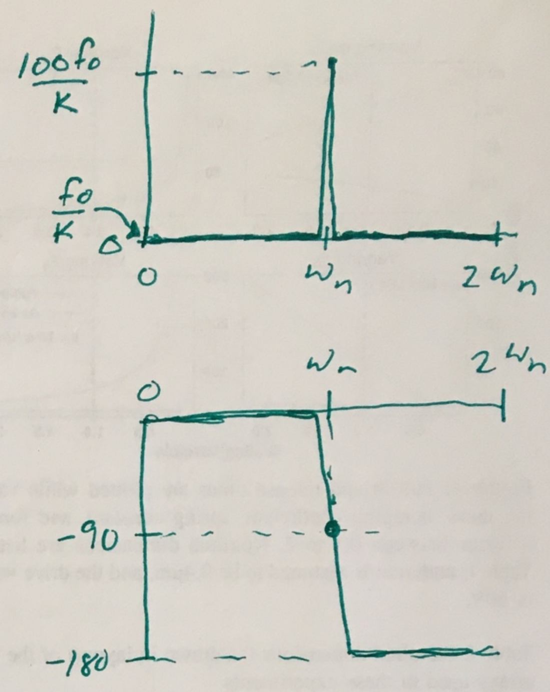
   c. Use a log/log axis for magnitude vs. frequency plot, and a linear phase, log frequency axis for the phase plot. This is called a Bode plot.
   [6 pts] 1 pt for each phase line segment. 1 pt for magnitude flat before wn, peaked 100x higher, and falling with slope -2 above resonance

6A



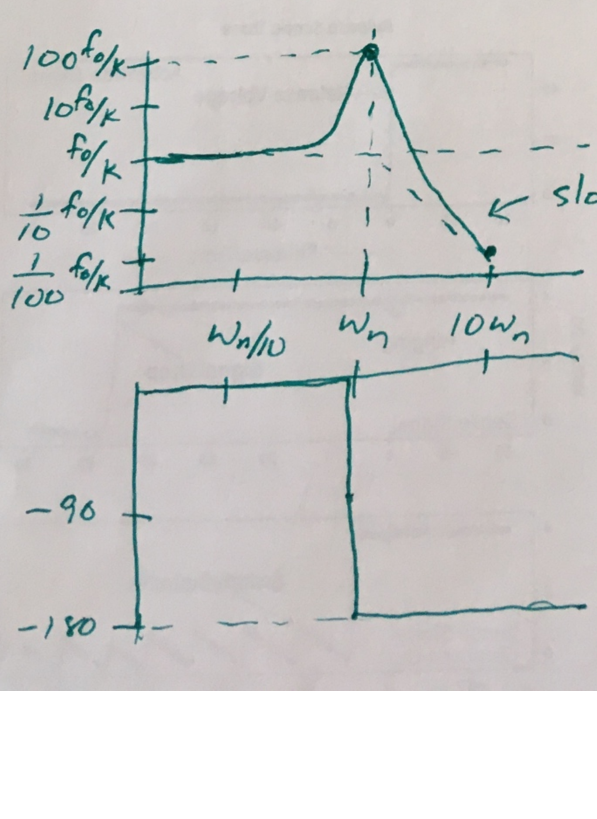
100 fo/K
70fo/K
0
Wn
Wn/200
Wn/200
-45
-90
-135
-180

6B



100fo/K
fo/K
0
0
Wn
2Wn
Wn
2Wn
0
-90
-180

6C



100fo/K
10fo/K
fo/K
1/10 fo/K
1/100 fo/K
slope = -2 decades/decade
Wn/10
Wn
10Wn
-90
-180

7. [18 pts total] A piezoresistive sensor is used with a digital voltmeter with +/-2V range and 0.1 mV accuracy. The gauge factor is 20, and the peak allowed strain is 0.5%

   a. [2 pts] The piezoresistor is used in a half bridge (voltage divider) with 2V excitation. What is the voltage output of the bridge as a function of strain?

$$\frac{\Delta R}{R} = G\epsilon$$

$$V_{out} = V_x\left(\frac{G\epsilon}{4} + \frac{1}{2}\right)$$

1 pt for each term

   b. [2 pts] What is the minimum and maximum value of the output voltage, given the allowed strain?

$$V_{out\ max} = V_x\left(\frac{G\epsilon}{4} + \frac{1}{2}\right) = 1.05\ since\ \epsilon_{max} = 0.5\%$$

$$V_{out\ min} = V_x\left(\frac{G\epsilon}{4} + \frac{1}{2}\right) = 0.95V\ since\ \epsilon_{min} = -0.5\%$$

   c. [2 pts] What is the resolvable strain?

$$V_{out\ \mathrm{min}\ detectable} = 0.1mV \rightarrow 1 + 0.1mV = V_x\left(\frac{G\epsilon}{4} + \frac{1}{2}\right) \rightarrow \epsilon_{\mathrm{min}\ detectable} = 10^{-5}$$

1 pt for some attempt, 1 pt for correct answer

   d. [2 pts] If there is 2 mV of noise on the power supply (excitation voltage), what is the noise-equivalent strain?

$$V_{out-noise} = 1mV: strain = 0\ then\ 2mV\ on\ V_x = 1mV\ at\ output. Find\ strain\ where\ V_{out} = \Delta 1mV$$

$$1mV = V_x\left(\frac{G\epsilon}{4}\right) \rightarrow \epsilon_{\mathrm{min}\ det} = 10^{-4}$$

1 pt for some attempt, 1 pt for correct answer

The sensor is now put into a Wheatstone bridge with the same 2V excitation, and is the only active resistor. An amplifier with a gain of 10 is used between the bridge output and the voltmeter.

   e. [2 pts] What is the minimum and maximum input voltages to the voltmeter?

$$V_{bridge\ out\ max} = \frac{\Delta R}{4R}V_x = 0.5G\epsilon = 0.05$$

$$V_{voltmeter\ max} = 0.5V$$

$$V_{voltmeter\ min} = -0.5V$$

   f. [2 pts] What is the resolvable strain?

$$V_{voltmeter\ \mathrm{min}\ detectable} = 0.1mV = gain * \frac{\Delta R}{4R}V_x \Longrightarrow \epsilon_{\mathrm{min}\ detectable} = 10^{-6}$$

   g. [4 pts] With the same supply noise as above, what is the noise-equivalent strain at ε=1ppm, and at ε =0.5%?

$$V_{voltmeter\ noise\ at\ 1ppm} = gain * \frac{\Delta R}{4R}V_{noise} = 100nV$$

$$gain * \frac{\Delta R}{4R}V_x = 100nV \rightarrow \epsilon_{equivalent\ at\ 1ppm} = 10^{-9}$$ (far below the resolvable strain above)

$$V_{voltmeter\ noise\ at\ 0.5\%} = gain * \frac{\Delta R}{4R}V_{noise} = 500uV$$

$$gain * \frac{\Delta R}{4R}V_x = 500uV \rightarrow \epsilon_{equivalent\ at\ 0.5\%} = 5 * 10^{-6}$$

(limited by supply noise now, not voltmeter resolution)

For each part, 1 pt for making some attempt, 1 pt for correct answer

h. [2 pts] What is the gain of the amplifier that matches the allowable strain range to the input voltage range of the voltmeter?

$$2V = gain * \frac{\Delta R}{4R}\ 2V = gain * 0.025 \rightarrow gain = 40$$

8. [12 pts] In the SOIMUMPs process with 25 um thick SOI, you fabricate a beam of length L=1 mm with a gap-closing electrostatic actuator on the end. Assume that E=170GPa, the beam is 2um wide and the gap is 2 um wide and 100um long. You may ignore fringing fields, and treat this as a 1D problem (ignore rotation of the tip of the beam, and the corresponding messy electrostatics).

   a. [4 pts] what is the pull-in voltage?

$$k_y = \frac{Etw^3}{4L^3} = \frac{170GPa(25um)(2um)^3}{4(1000um)^3} = 0.0085\frac{N}{m}$$

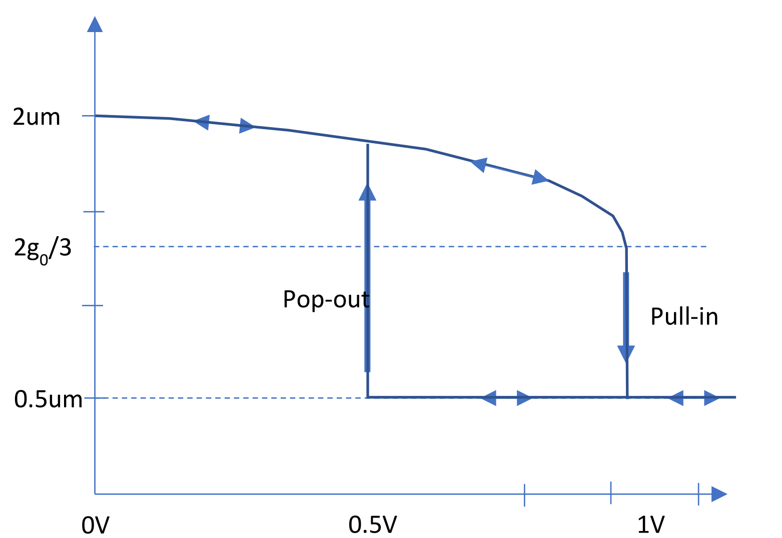
$$V_{PI} = \sqrt{\frac{8kg_0{}^3}{27\epsilon A}} = \sqrt{\frac{8(0.0085)(2um)^3}{27(8.854*10^{-12}\frac{F}{m})(100um*25um)}} = 0.95V$$

   b. [2pts] If there are gap-stops at 0.5 microns, what is the pull-out voltage?

$$V_{PO} = \sqrt{\frac{27}{4}\left(\frac{g_f}{g_o}\right)^2\left(1-\frac{g_f}{g_o}\right)}V_{PI} = \sqrt{\frac{81}{256}}0.95V = \frac{9}{16}0.95V = 0.5V$$

   c. [6 pts] Sketch the DC displacement vs. applied voltage, showing the hysteresis loop and clearly labeling "voltage increasing" and "voltage decreasing" parts of the curves.

1 pt each for: two vertical lines at pull-in and pull-out, curved line for stable deflection between 0 and VPI with voltage increasing, flat line at g_f=0.5um for voltage decreasing between PI and PO



9. [4 pts total] If you were to apply a sinusoidal voltage to this structure, could you get it to pull in at a lower voltage? Why/how? What frequency would you use if you were applying a pure sine wave with no DC bias?

Tricky! What happens if the frequency of the sinusoid was near resonance? We get a larger displacement! This larger displacement makes movable plate closer to the stationary plate, which means the gap is smaller.

$V_{PI}\ \alpha\ \sqrt{g_0{}^3}$ so if the gap decreases, then V_{PI} decreases.

To get a sinusoidal force at the resonant frequency you need to apply a pure sine wave at half that frequency, since it gets squared → frequency doubles

1 pt for saying that you can get it to pull in at a lower voltage.
1 pt for some explanation of why, having to do with resonance
2 pts if you said that the voltage should be at half the resonant frequency. 1 pt if you said at the resonance frequency.

10. [8 pts] In Spencer et al. (reference [1]), an electrostatic gap-closing relay is made with a gate/body overlap area A_{ov}, and initial gap g_o, a displacement g_d (the amount that it moves before contact), and a spring constant K (see table 1 for specific values of the "current devices" reported in the paper).
    a. Checkout the pictures of the people on the last page – you may recognize some of them

    Our Dean!

    For b and c use equations from #7 in this assignment and values from Table1 in the linked paper.

    $$A_{ov} = 450\ um^2$$
    $$g_0 = 180nm$$
    $$g_d = 90nm$$
    $$k = 62.5\frac{N}{m}$$

    b. [2 pts] calculate the expected pull-in voltage based on the specs for "current devices"

    $V_{PI} = 5$V (Not what's listed in Table1!)

    c. [2 pts] calculate the expected value for the pull-out voltage based on the same specs

    $V_{PO} = 4.75V$

    d. [2 pts] Compare your results from parts b and c to the measured results in Figure 11. Are their results close to what the theory says that should be? What do you think might explain any differences between theory and experiment?

    Pretty good approximation, especially for quick formula but not exact in part due to parasitic capacitance and resistance.

    e. [2 pts] How fast is the mechanical response of these devices today, and how fast do they hope to make them in the "scaled model" future? (see Table 1)

    "Current": 34us but scaled is only 0.02us-0.08us! Super fast for mechanical devices, but not very fast compared to transistors (which switch in a few picoseconds these days)

[1] Spencer, et al., "Demonstration of Integrated Micro-Electro-Mechanical Relay Circuits for VLSI Applications", IEEE JSSCV46N1, 2011.

11. [247] [10 points] As the beam gets shorter in the problem above, the parallel plate approximation for the capacitor becomes an increasingly poor model for the actuator, and the linear spring becomes a poor model for the beam, because both ignore rotation and torque. How would you go about calculating the actual pull-in voltage for this structure? Would the pull-out voltage be easier to calculate (assuming gap stops on both sides