EE 147/247A Prof. Pister
Fall 2021

Homework Assignment #5

Due on bcourses Thursday 10/8/2021 (zero credit after 9 AM Sunday)

1. Fall 2017 Midterm question 2
2. Fall 2020 Midterm question 3
3. Fall 2020 Midterm question 4
4. Fall 2020 Midterm question 5
5. A resonator with a spring constant k, mass m, and a Q of 100 is driven by an external force to have an amplitude of motion of x_0. There are two frequencies at which the magnitude of the sine terms (sum of spring and mass) is equal to the magnitude of the cosine term (damping). The difference between these is the 3dB bandwidth.
    a. Estimate those two frequencies. You should be able to do this by solving a quadratic equation in w. A useful relationship: $km=b^2/Q^2$
    b. Is the difference roughly the resonant frequency divided by the Q?
    c. What is the difference in the phase at these two frequencies?
6. Using the results above, plot the magnitude and phase of the resonator displacement in response to a sine wave $f(t)=f_0\sin(\omega t)$ a few different ways. Note that you will be plotting the magnitude and phase of the complex number $H(jw) = 1/[\ \omega_n^2-\omega^2 + (\omega_n/Q)j\omega]$
    a. Use a linear axis for frequency, magnitude, and phase, and zoom in on the frequency range ω_n +/- ω_n/Q
    b. Use a linear axis for frequency, magnitude, and phase. The range of frequency should be 0 to 2 ω_n. The range for magnitude should include 0 and the peak magnitude. If you do this accurately, your plots will look like a couple of straight lines – that's ok.
    c. Use a log/log axis for magnitude vs. frequency plot, and a linear phase, log frequency axis for the phase plot. This is called a Bode plot
7. A piezoresistive sensor is used with a digital voltmeter with +/-2V range and 0.1 mV accuracy. The gauge factor is 20, and the peak allowed strain is 0.5%
    a. The piezoresistor is used in a half bridge (voltage divider) with 2V excitation. What is the voltage output of the bridge as a function of strain?
    b. What is the minimum and maximum value of the output voltage, given the allowed strain?
    c. What is the resolvable strain?
    d. If there is 2 mV of noise on the power supply (excitation voltage), what is the noise-equivalent strain?

    The sensor is now put into a Wheatstone bridge with the same 2V excitation, and is the only active resistor. An amplifier with a gain of 10 is used between the bridge output and the voltmeter.

    e. What is the minimum and maximum input voltages to the voltmeter?
    f. What is the resolvable strain?
    g. With the same supply noise as above, what is the noise-equivalent strain at ε=1ppm, and at ε =0.5%?
    h. What is the gain of the amplifier that matches the allowable strain range to the input voltage range of the voltmeter?
8. In the SOIMUMPs process with 25 um thick SOI, you fabricate a beam of length L=1 mm with a gap-closing electrostatic actuator on the end. Assume that E=170GPa, the beam is 2um wide and the gap is 2 um wide and 100um long. You may ignore fringing fields, and treat this as a 1D problem (ignore rotation of the tip of the beam, and the corresponding messy electrostatics).
    a. what is the pull-in voltage?
    b. If there are gap-stops at 0.5 microns, what is the pull-out voltage?

c. Sketch the DC displacement vs. applied voltage, showing the hysteresis loop and clearly labeling "voltage increasing" and "voltage decreasing" parts of the curves.

9. If you were to apply a sinusoidal voltage to this structure, could you get it to pull in at a lower voltage? Why/how? What frequency would you use if you were applying a pure sine wave with no DC bias?
10. In Spencer et al. (reference [1]), an electrostatic gap-closing relay is made with a gate/body overlap area A_{ov}, and initial gap g_o, a displacement g_d (the amount that it moves before contact), and a spring constant K (see table 1 for specific values of the "current devices" reported in the paper).
    a. Checkout the pictures of the people on the last page – you may recognize some of them
    b. calculate the expected pull-in voltage based on the specs for "current devices"
    c. calculate the expected value for the pull-out voltage based on the same specs
    d. Compare your results from parts b and c to the measured results in Figure 11. Are their results close to what the theory says that should be? What do you think might explain any differences between theory and experiment?
    e. How fast is the mechanical response of these devices today, and how fast do they hope to make them in the "scaled model" future? (see Table 1)

    [1] Spencer, et al., "Demonstration of Integrated Micro-Electro-Mechanical Relay Circuits for VLSI Applications", IEEE JSSCV46N1, 2011.

11. [247] As the beam gets shorter in the problem above, the parallel plate approximation for the capacitor becomes an increasingly poor model for the actuator, and the linear spring becomes a poor model for the beam, because both ignore rotation and torque. How would you go about calculating the actual pull-in voltage for this structure? Would the pull-out voltage be easier to calculate (assuming gap stops on both sides