EE 147/247A Prof. Pister
Fall 2021

Homework Assignment #3 Rubric

Due on bcourses Thursday 9/23/2021 (late 9 AM Friday)

40 points

1. 4 points: A polysilicon beam with a 2x2 um2 cross-section and a fracture strain of 1% is bent to a radius of 200 um. What is the surface strain on the concave surface and the convex surface of the beam?

   1 point for effort, 1 point for equation, 1 point for same magnitude, 1 point for correct sign

   $$\epsilon = \frac{z}{p}$$

   z= 1um for the convex surface and -1um for the concave surface so

   $$z_{convex} = \frac{1um}{200um} = 0.5\%$$

   $$z_{concave} = \frac{-1um}{200um} = -0.5\%$$

   The magnitude of the strain is what determines if it will break and the sign tells you if it's compressive (negative) or tensile (positive).

2. 4 points: A 1 mm long straight rod with a square cross-section is bent into the shape of a circle.
   a. What end loading condition will produce this result? Be specific, and give your answer in terms of EI

      1 point for effort, 1 point for loading condition, 1 point for equation

      1mm long beam bent into a circle means that the circumference of the circle is 1mm.

      $$1\text{mm}=2\pi r = 2\pi\rho \rightarrow \rho = 160um$$

      Loading condition needed: pure end moment M_0

      $$\frac{1}{\rho} = \frac{M}{EI} = 1/160um = \frac{M}{EI} \rightarrow M = \frac{EI}{160um}$$

   b. What is the maximum allowable beam width, in terms of the fracture strain?

      1 point for effort, 1 point for equation

      $$\epsilon_{frac} = \frac{z}{p} \rightarrow z = \epsilon_{frac}\rho$$

      A square cross section means w=t=2z

      $$w_{max} = 2z = 2\epsilon_{frac}\rho = 2\,\epsilon_{frac} * 160um$$

3. 12 points: Derive the formula for the shape of a cantilever deflecting under its own weight. Assume a beam of length L, width b, and thickness a, bending in the a direction.
   a. At a point x along the beam, what is the weight (m*g) of the cantilever past that point?

      1 point for effort, 1 point for equation

      ρ_{beam} *is the density of the beam*

      $F(x)=mg=(L-x)ab\rho_{beam}*g$

   b. At a point x along the beam, what is the moment M(x) due to the weight past that point?

      1 point for effort, 1 point for equation

      $$M(x) = \frac{L-x}{2}F = (L-x)^2 ab\rho_{beam}*g/2$$

   c. Integrate that moment subject to the boundary conditions y(0)=0, y'(0)=0 to get the shape of the beam.

      2 points for effort, 1 point for integrating, 1 point for answer

$$\frac{1}{\rho}=\frac{M(x)}{EI} \rightarrow y''(x) = M(x) = \frac{1}{2EI}abg\rho_{beam}\,(x^2-2Lx+L^2)$$

$$y'(x)=\frac{1}{2EI}abg\rho_{beam}\,(\frac{x^3}{3}-Lx^2+L^2x)+C$$

$$y'(0)=0\rightarrow C=0$$

$$y(x)=\frac{1}{2EI}abg\rho_{beam}\,(\frac{x^4}{12}-L\frac{x^3}{3}+L^2\frac{x^2}{2})+C'$$

$$y(0)=0\rightarrow C'=0$$

$$y(x)=\frac{1}{2EI}abg\rho_{beam}\,(\frac{x^4}{12}-L\frac{x^3}{3}+L^2\frac{x^2}{2})$$

d. What is the tip deflection?

1 point for effort, 1 point for equation

$$y(L)=\frac{1}{2EI}abg\rho_{beam}\frac{3L^4}{12}=\frac{12}{2Ea^3b}abg\rho_{beam}\frac{L^4}{12}=\frac{3}{2Ea^2}g\rho_{beam}L^4$$

e. How does the tip deflection compare to what we would have gotten if we'd just calculated the spring constant of the beam, and divided that into m*g?

1 point for effort, 1 point for ratio

$$y=\frac{4F_g}{Eba^3}L^3=\frac{4(abLg\rho_{beam})}{Eba^3}L^3=\frac{4}{Ea^2}g\rho_{beam}L^4$$

The analysis using distributed load is (3/2)/4=3/8 of the deflection calculated by the simple "all of the mass on the end" approximation.

4. 5 points: For a beam of length ***L***, width ***a*** and thickness ***b***, where K_a is the spring constant with a pure force applied at the tip of the beam in the 'a' direction.

a. calculate K_x/K_a, the ratio of the spring constants in the axial and 'a' transverse directions.

1 point for effort, 1 point for equation

$$K_x=\frac{abE}{L}$$

$$K_a=\frac{Eba^3}{4L^3}$$

$$\frac{K_x}{K_a}=\frac{4L^2}{a^2}$$

b. calculate K_b/K_a, the ratio of the spring constants in the 'a' and 'b' transverse directions.

1 point for effort, 1 point for equation

$$K_b=\frac{Eab^3}{4L^3}$$

$$K_a=\frac{Eba^3}{4L^3}$$

$$\frac{K_b}{K_a}=\frac{b^2}{a^2}$$

c. If the first ratio is 10,000, and the second is 100, what can you say about the shape of the beam?

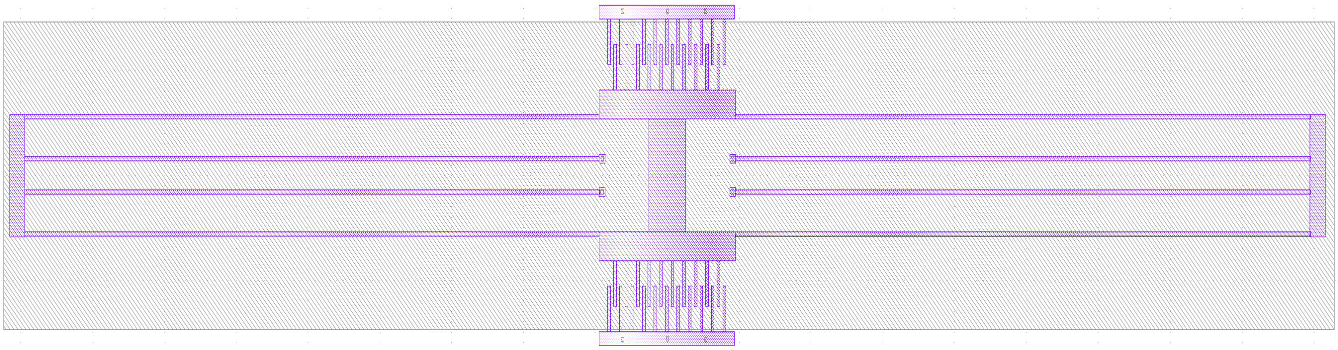
1 point for effort

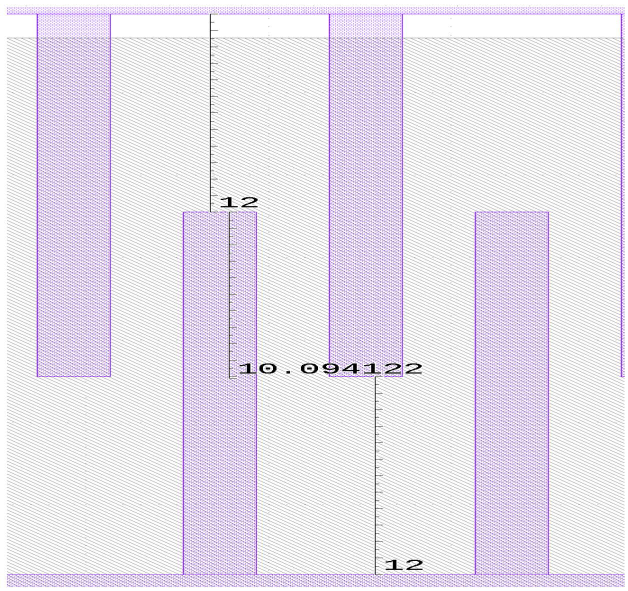
b=10a and L=50a

5. 16 points: Using a 3 mask polysilicon process with PLY0 (patterned ground plane), CONT (contact through a 2um oxide), and PLY1 (a 2 um thick polysilicon layer with 2 um line and space), design a folded-flexure comb-drive resonator with 10 moving comb fingers on each side, and a maximum displacement of +/-10um. Make the resonant frequency 10kHz. Draw the structure carefully, showing dimensions.

Different numbers and structure dimensions is ok.

1 point for general shape, 1 point for 10um or > spacing between end of fingers, 1 point for >=10um from dogbone to contacts, 1 point for correct number of fingers, 1 point for ground plane under moving structure, 1 point for anchors





a. What is the total mass of the dogbone, comb, and comb support structures?

1 point for effort, 1 point for equation, 1 point for mass and spring to give 10kHz resonance

$$m_{Dog\ bone} = \rho_{PSi} t * L * w = \frac{2300kg}{m^3} 2um * 50um * 25um = 5.75 * 10^{-12} kg$$

$$m_{comb} = 20\rho_{PSi} t * L * w = 20 \frac{2300kg}{m^3} 2um * 22um * 2um = 4 * 10^{-12} kg$$

$$m_{comb\_support} = 2\rho_{PSi} t * L * w = 2 \frac{2300kg}{m^3} 2um * 15um * 100um = 14 * 10^{-12} kg$$

$$m_{total} = 24 * 10^{-12} \text{kg}$$

With this mass we need out overall spring constant (aka spring constant of one half beam) to be

$$2\pi f_n = \sqrt{\frac{k}{m}} \rightarrow k = 3600M \frac{rad}{sec} * 24 * 10^{-12} kg = 85m \frac{N}{m}$$

$$k_{\frac{1}{2}} = \frac{Etw^3}{4L^3} \rightarrow \sqrt[3]{\frac{Etw^3}{4k_{\frac{1}{2}}}} = \sqrt[3]{\frac{170GPa(2um)(2um)^3}{4 * 72m\frac{N}{m}}} = 200um\ so\ the\ length\ of\ one\ beam\ is\ 400um$$

b. What is the mass of the suspension?

1 point for effort, 1 point for equation

$$m_{springs} = 8\rho_{PSi} t * L * w = 8 * \frac{2300kg}{m^3} 2um * 400um * 2um = 29 * 10^{-12} kg$$

$$m_{trusses} = 2\rho_{PSi} t * L * w = 2 * \frac{2300kg}{m^3} 2um * 50um * 10um = 4.6 * 10^{-12} kg$$

$$m_{suspension} = 33.6 * 10^{-12} kg$$

c. What is the displacement with an acceleration of 1g in the direction of the comb fingers?

1 point for effort, 1 point for F equation

$$F_{1g} = 10 * m_{total} = 240 * 10^{-12}\mathrm{N}$$

$$\Delta y = \frac{F}{k} = \frac{240 * 10^{-12}\mathrm{N}}{85m\frac{N}{m}} = 3nm$$

d. What voltage do you need to counteract the acceleration of 1g and bring the structure back to its rest position?

1 point for effort, 1 point for equation, 1 point for answer

$$F_{1g} = F_{comb}$$

$$240 * 10^{-12}N = N_{fingers}\epsilon_0 V^2\frac{t+g}{g} = 10*8.8 * 10^{-12}\frac{F}{m}V^2\frac{2um+2um}{2um} \rightarrow V = \sqrt{\frac{240*10^{-12}N}{88*10^{-12}\frac{F}{m}*2}} = 1.2V$$

6. [247A] A hollow square cross-section beam has sides that are 2um long and 0.1um thick. If this beam is bent to a radius of curvature rho,

Solve for a 2x2um solid beam and then subtract the 1.8x1.8um hole as if it was another beam. This works because k is directly proportional to I, which depends on the shape of the beam.

a. what is the strain as a function of position in the cross-section?
b. What is the internal moment due to this strain?
c. What is the spring constant relative to the spring constant of a filled square cross-section 2um on a side?
d. What is the spring constant relative to a square cross-section with the same weight per length (so same material area) as the hollow cross-section?
e. What is the mass of the beam relative to the mass of a full square cross-section beam?
f. What does this tell you about the speed with which a hollow structure can move relative to a filled structure?

The hollow structure can move much faster than the filled structure. The step response of a lightly-damped second-order system like this looks sinusoidal with a frequency of $\omega_n/2\pi$, so a rise time roughly proportional to $1/\omega_n$. Increase resonant frequency, speed up the response.

If you feel like it, check out the trapjaw ant, which has the fastest reaction time of any known animal, around 100us. It's jaws are hollow. It hunts the springtail thrip, which has the second-fastest reaction time of any animal. Nature is marvelous.

https://www.berkeley.edu/news/media/releases/2006/08/21_ant.shtml