EE 147/247A Prof. Pister
Fall 2021

Homework Assignment #2

Due on bcourses Wednesday 9/15/2021 (late 9 AM thursday)

Answers Must have units.
Within ~20% is correct due to rounding and simplification.
Ask on piazza if you think that you deserve credit for something.

1. 14 points: You have a solid cube of silicon of side length *S* sitting a distance *d* beneath a large flat conductive plate, creating a parallel plate gap between them. You apply a voltage *V* between the cube and the plate, which generates an upwards force on the cube. If *d=S/100*
   a. 3 points: Does the magnitude of the force depend upon S?

      1 point for effort
      1 point for parallel plate force equation
      1 point for "no"

      $F_{parallel\ plate} = \frac{1}{2}\epsilon_0 V^2 \frac{A}{d^2} = \frac{1}{2}\epsilon_0 V^2 \frac{A}{d^2}$, where A is the area perpendicular to the field lines and d is the spacing between the plates. However, d= d=$\frac{S}{100}$

      $F_{parallel\ plate} = \frac{1}{2}\epsilon_0 V^2 10^4$, so no F does not depend on S

   b. 6 points: Using the approximation that ½ $\varepsilon_0(15V)^2$=1nN, what is the force on the cube if V=1.5V, 15V, and 150V?

      1 point for effort on each of 3 parts
      1 point for answer on each of 3 parts
      Note: if you used your calculator and got more than 1 digit of precision, you did it wrong. Give yourself full credit, but that is *not* what I want you to do with electrostatics and voltages that are powers of 10 times 15V.

      d=$\frac{S}{100}$ so $=\frac{S^2}{d^2} = 10^4$

      $$F_{1.5V} = \frac{1}{2}\epsilon_0 V^2 \frac{S^2}{d^2} = 10^4 * 1nN * 10^{-2} = 100nN$$

      $$F_{15V} = \frac{1}{2}\epsilon_0 V^2 \frac{S^2}{d^2} = 10^4 * 1nN = 10uN$$

      $$F_{150V} = \frac{1}{2}\epsilon_0 V^2 \frac{S^2}{d^2} = 10^4 * 1nN * 100 = 1mN$$

      Notice that as the force increases by a factor of 100 as the voltage increases by 10x

   c. 5 points: For the three voltages above, calculate the range of values of *S* for which the mass of the cube is less than the electrostatic force lifting it up.

      2 points for relating mass and force: 1 for effort, 1 for correctness
      3 points total for answers
      On an exam: -1 point if used = instead of <, but give full credit on HW.

      Mass of cube $=\rho S^3 = \frac{2300kg}{m^3} S^3$

      Mass of cube*gravity $< F_{parallel\ plate}$

      $$F_{1.5V} = 100nN > \frac{2300kg}{m^3} * 10\frac{m}{s^2} S^3 \rightarrow S < 160um$$

      $$F_{15V} = 10uN > \frac{2300kg}{m^3} * 10\frac{m}{s^2} S^3 \rightarrow S < 750um$$

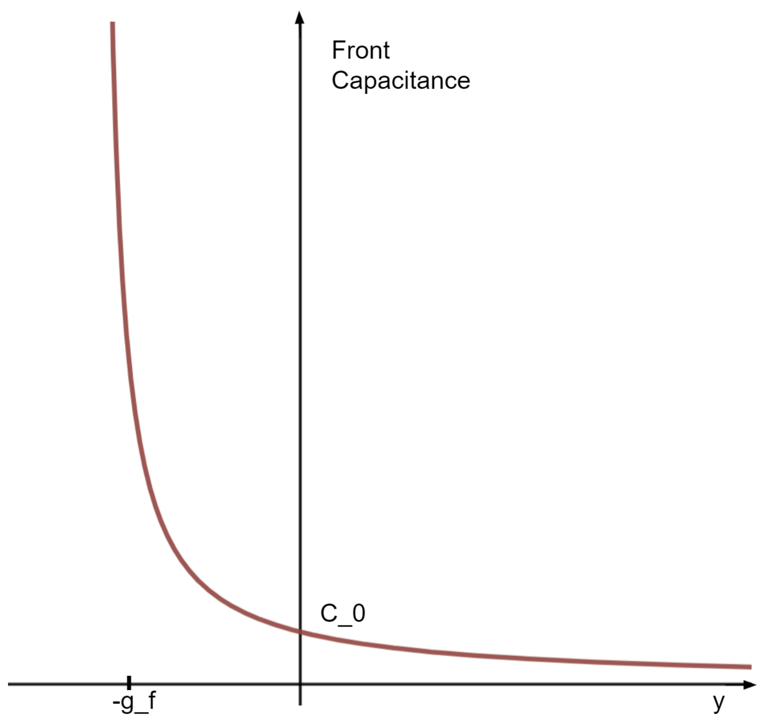
$$F_{150V} = 1mN > \frac{2300kg}{m^3} * 10\frac{m}{s^2}S^3 \rightarrow S < 3.5mm$$

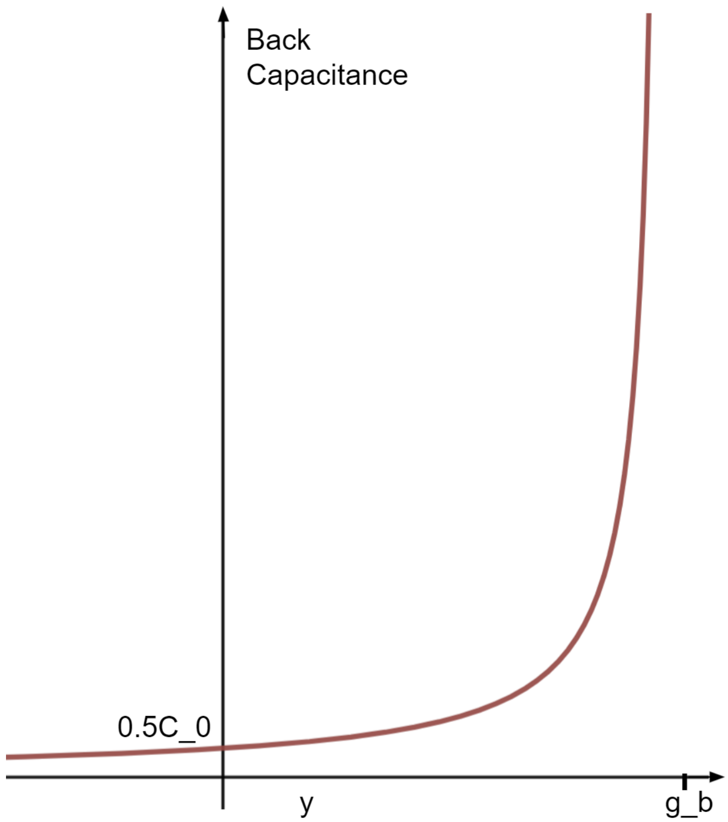
Notice that as the force increases by a factor of 100, S increases by ~5 because (100^3)≈ 5

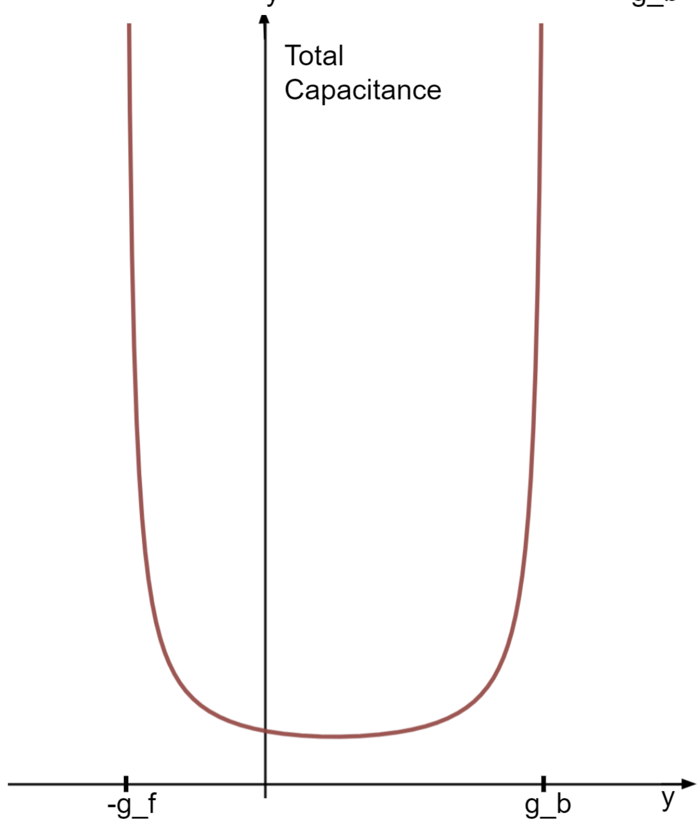
2. 18 points: In a single-mask SOI process in which your minimum lithography line and space is λ, you wish to make an accelerometer with a variable capacitor readout, like our simplest accelerometer (e.g. in lecture W2L2 at around 34 minutes). Two of the many capacitor finger pairs are shown below, showing the desired front gap g_f, and the undesired back gap g_b.
   a. For a displacement of the proof mass over the range $y = [-g_f, g_b]$, sketch the front gap capacitance, back gap capacitance, and total capacitance assuming $g_b = 2\ g_f$. You should have some asymptotes! Show clearly the relative front and back capacitance values when y=0.

1 point per plot shape

2 points total for relative front and back capacitance values when y=0







b. What is the impact of the back gap on dC/dy near y=0?

1 point for some equation for total capacitance

1 point for taking a derivative

1 point for answer

$$C(y) = \epsilon_0 t L\left(\frac{1}{g_b - y} + \frac{1}{g_f + y}\right)$$

$$C' = \frac{dC(y)}{dy} = \epsilon_0 t L\left(\frac{1}{(g_b - y)^2} - \frac{1}{(g_f + y)^2}\right)$$

$$\frac{dC'(0)}{dy} = \epsilon_0 tL(\frac{1}{(g_b)^2} - \frac{1}{(g_f)^2})$$

Since $g_b = 2g_f$

$$\frac{dC'(0)}{dy} = \epsilon_0 tL\left(\frac{1}{(2g_f)^2} - \frac{1}{(g_f)^2}\right)$$

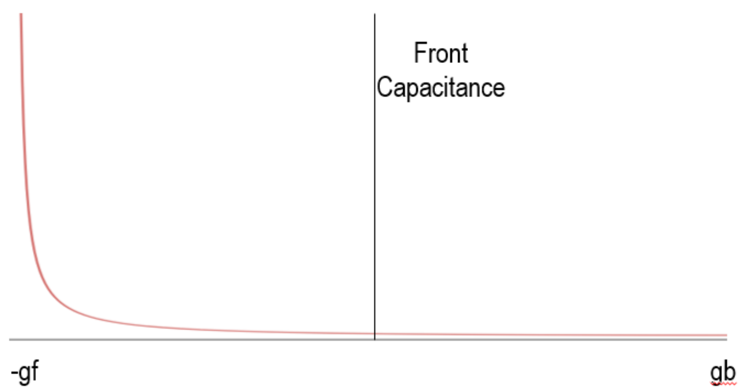
$$\mathit{This\ reduces\ the\ magnitede\ of}\ \frac{dC'(0)}{dy}\ \mathit{by}\ 25\%$$

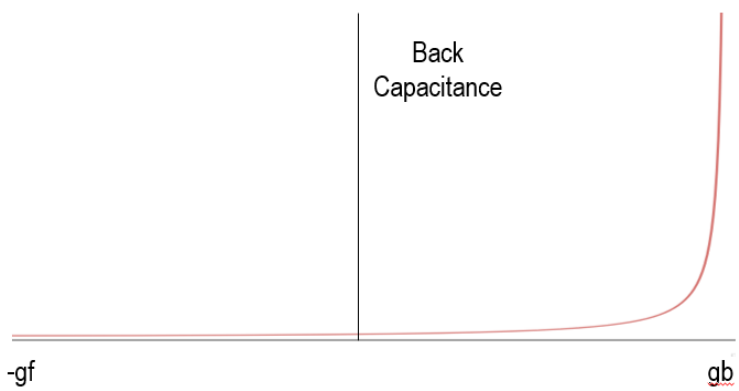
c. Repeat parts a and b, assuming $g_b = g_f$.

1 point per plot for correct shape

1 point stating that capacitance is now the same at y=0

1 point for stating that dC/dy=0





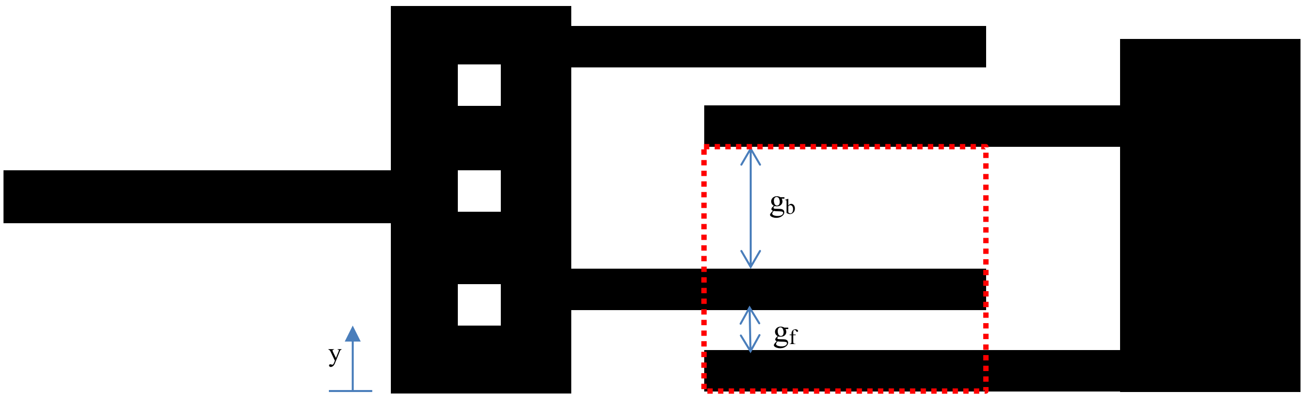
$$C' = \frac{dC(y)}{dy} = \epsilon_0 tL(\frac{1}{(g_b - y)^2} - \frac{1}{(g_f + y)^2})$$

$$\mathit{if}\ g_b = g_f\ \mathit{and}\ y = 0, \mathit{then}\ C'(0) = \frac{dC(0)}{dy} = 0$$

That means there is no net electrostatic force *and* no capacitive signal to detect.

Making gb too big means we don't get very many fingers, so we don't have much capacitance per layout area, and there is an upper limit on how much area we can use. Making gb too small means that we can put in a lot of fingers, but the change in capacitance per finger gets smaller. Somewhere there is an optimum.

d. Assuming that the width of the capacitor fingers and the front gap is λ, what is the value of the back gap that maximizes dC/dy at y=0 per unit layout area? I've drawn a representative tile-able layout area in red below.



2 points for writing an equation for dC/dy per layout area (1 point if you wrote an equation for capacitance per area instead)
1 point for taking a derivative with respect to the back gap gb
1 point for getting the right answer
1 point for effort

As in part b: $\frac{dC(y)}{dy} = \epsilon_0 tL\left(\frac{1}{(g_b-y)^2} - \frac{1}{(g_f+y)^2}\right)$

$$\frac{dC(0)}{dy} = \epsilon_0 tL\left(\frac{1}{(g_b)^2} - \frac{1}{(g_f)^2}\right)$$

The Area denoted by the dotted red line is A=L$(g_b + g_f + 2w)$; $where\ w\ is\ the\ finger\ width$

$$\text{Area Density AD} = \frac{\frac{dC(0)}{dy}}{A}$$

$$\text{AD} = \frac{\epsilon_0 tL\left(\frac{1}{(g_b-y)^2} - \frac{1}{(g_f+y)^2}\right)}{\text{L}(g_b+g_f+2w)}$$

But $g_f = w = \lambda$ so $\text{AD} = \frac{\epsilon_0 tL\left(\frac{1}{(g_b)^2} - \frac{1}{\lambda^2}\right)}{\text{L}(g_b+3\lambda)}$

And we can cancel and L before deriving

$$\frac{dAD}{dg_b} = \frac{\epsilon_0 t({g_b}^3 - 3g_b\lambda^2 - 6\lambda^3)}{{g_b}^3\lambda^3(g_b+3)^2} = 0$$

$$\epsilon_0 t({g_b}^3 - 3g_b\lambda^2 - 6\lambda^3) = 0$$

The only real root is $g_b = 2.35\,\lambda$

As an example, if you have 2 um line and space lithography and you are making a gap-closing actuator or capacitive sensor, you maximize your force or capacitance change per unit area by making the front gap and fingers 2 um wide, and the back gap 4.7 um wide. Note that if the fingers and the front gap are not the same size (sometimes we make the fingers wider to avoid electrostatic pull-in) then the optimal back gap will vary somewhat.

3. 4 points: For this problem, assume that the fracture strain of silicon is 1%. You may use 50 kg as your mass if you'd rather not disclose personal information.
   a. What is the minimum cross-sectional area of a silicon wire that will hold your weight without breaking?

2 points if you get the right answer.
1 point for some explanation and the wrong answer

$$m = 50kg$$
$$F = mg = kx = \frac{EA}{L}x = EA\epsilon \rightarrow A = \frac{F}{E\epsilon} = \frac{mg}{E\epsilon}$$
$$A = \frac{50kg * 10\frac{m}{s^2}}{150GPa * 0.01} = \frac{10}{3 * 10^{9-2}} = 3 * 10^{-7}\ m^2$$

   b. If we etched a long straight piece of the device layer on a 40/2/550 SOI wafer and supported you by it, how wide would it need to be?

2 points if you get the right answer.
1 point for some explanation and the wrong answer

A = t*w

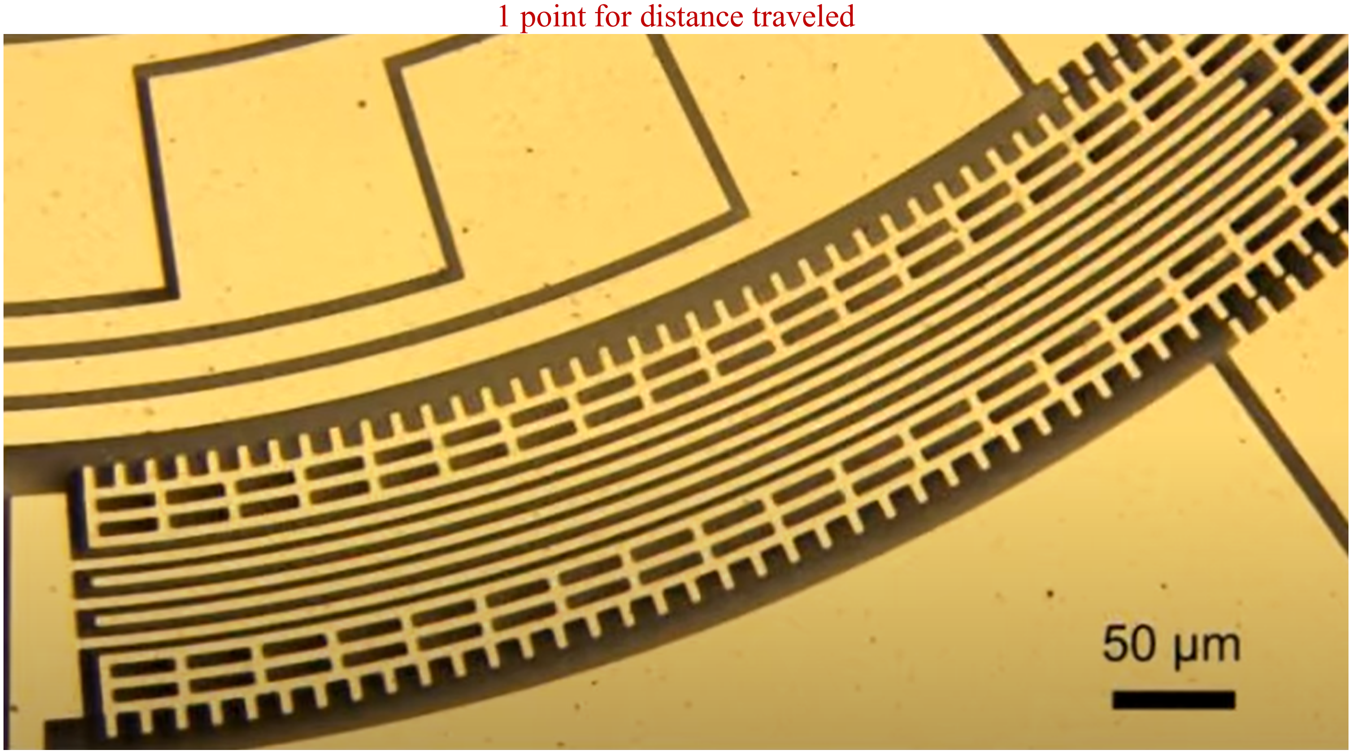
$$w = \frac{A}{t} = \frac{3 * 10^{-7}\ m^2}{40 * 10^{-6}\ m} = \frac{3\ m^2}{400\ m} = 7.5\text{mm}$$

4. 11 points: Look at the video on the link "single-mask SOI motor" on the class webpage.
   a. There's a scale bar at 0:05. How big is the overall structure, from bond wire to bond wire? (the bond wires are in 400x400 um2 holes in the SOI layer, and connect to the substrate, most likely to ground it)

1 point for reasonable answer with units
~3000um aka 3mm

   b. Estimate the change in sense capacitance over the range of travel (shown 0:20 to 0:35), and the distance traveled. With a sensor able to resolve 4 aF, what position resolution would this give?

1 point for stating assumptions and trying something
1 point for C_max
1 point for C_min
1 point for distance traveled



6 capacitors (3 fixed curved beams that have a moving curved beam on each side.

5um gap
Assume 50um thick
Max length overlap L_{ov_max}= 5000um=5mm
Min length overlap L_{ov_min}= 75um
~4925um traveled

$$C_{max} = \frac{\epsilon_0 L_{ov\_max} t}{g} = \frac{9 * 10^{-12}\frac{F}{m} 5mm * 50um}{5um} = 45 * 10^{-14}F = 450fF$$

$$C_{min} = \frac{\epsilon_0 L_{ov\_min} t}{g} = \frac{9 * 10^{-12}\frac{F}{m} 75um * 50um}{5um} = 675 * 10^{-17}F = 6.75fF$$

443fF change in capacitance

$$\frac{443fF}{4925um} = \frac{4aF}{x_{min}} \rightarrow x_{min} = 45nm\ position\ resolution$$

c. Estimate the change in capacitance for one of the three poles, over a single step (0:44 to 0:57). You'll need to do some counting of crenelations.

1 point for stating assumptions and trying something
1 point for answer

$\#_{cren}$ = ~70 crenelations per pole (control voltage pad)
$w_{cren} = 5um$

Assume capacitance when crenelations are lined up >> then when they are misaligned so the max (aligned) capacitance is the change in capacitance

$$C_{pole} = \#_{cren}\frac{\epsilon_0 w_{cren} t}{g} = 70\frac{9 * 10^{-12}\frac{F}{m} 5um * 50um}{5um} = 70 * 45 * 10^{-17}F = 32fF$$

d. At 1:06 it gives a peak angular velocity. Estimate the corresponding velocity of the pointer tip in m/s.

1 point for effort
1 point for answer

~25um between degree tick marks

$$1\frac{\circ}{ms} \rightarrow 25\frac{um}{ms} = 0.025\frac{m}{s}$$

e. What is the approximate frequency of the electrical excitation signal at the peak angular velocity?

1 point for effort
1 point for answer

Crenelations spaced ~10um, so excitation voltage turns on every 10um movement. At 25 $\frac{um}{ms}$ the period is 0.4ms. $f = \frac{1}{T}$

$$f = 2.5kHz$$

5. 14 points: The pictures below are of the ADXL202, which is now obsolete but looks a lot like the newer parts that replaced it. Die photo (CMOS+MEMS), SEM of the MEMS device, detail SEM, and simulation of response to a vertical acceleration.

a. Using the scale bar, estimate the mass of the proof mass, assuming a 3um thick polysilicon film.

1 point for effort
1 point for answer

mass=2300$\frac{kg}{m^3} * 3um * (4 * 75um * 75um + 200um * 200um)$ =450p kg

b. Using the sensor resonant frequency quoted on the datasheet, estimate the combined spring constant of all of the support springs.

1 point for effort

1 point for answer

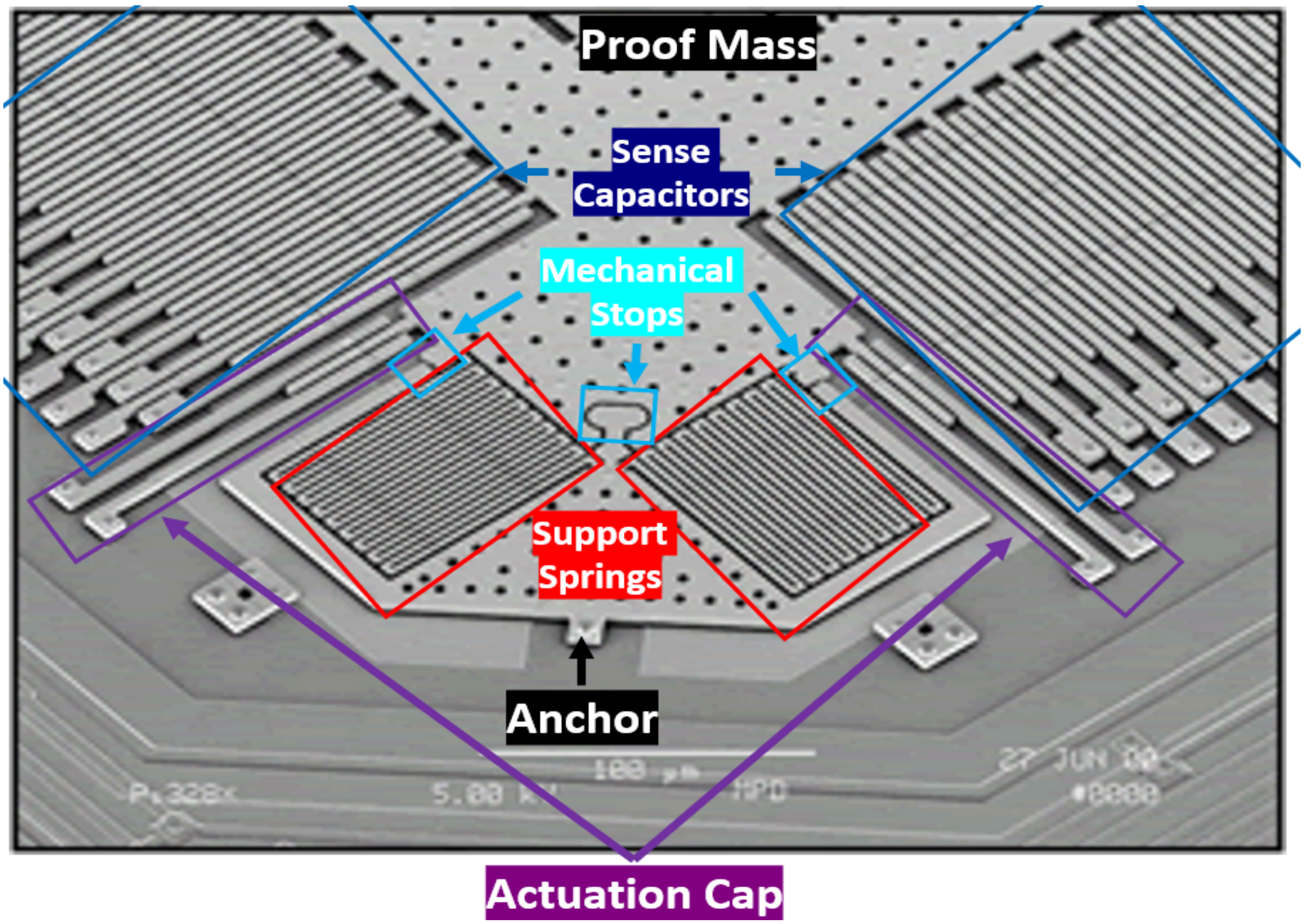
$k = m * \omega_n^2 = 450\text{p kg} * (14\text{kHz})^2 = 0.09\frac{N}{m}$

http://www.analog.com/media/en/technical-documentation/obsolete-data-sheets/ADXL202_210.pdf

c. 7 points: In the detailed SEM, identify the proof mass, support springs, proof mass anchor, over-acceleration mechanical stops, differential sense capacitors (big), actuation capacitors (small).

1 point for effort

1 point for each labeled correctly (6 total)



d. Assuming a 3um thick film with 1um gaps, estimate the voltage needed across the actuator gap to support the weight of the proof mass (at 1 gravity).

1 point for effort

1 point for parallel plate force equation

1 point for answer

$F = mg = 4 * F_{cap}$ (There are four drive capacitors for each in plane direction)

$$F_{cap} = 0.5\epsilon_0 V^2 \frac{A}{g^2} = 0.5\left(8.854 * 10^{-12}\frac{F}{m}\right)V^2\frac{3um * 40um}{(1um)^2} = 450\text{p kg} * \frac{10\text{N}}{\text{m}^2}$$

$$V = \sqrt{\frac{450\text{p kg} * \frac{10\text{N}}{\text{m}^2} * (1um)^2}{0.5\left(8.854 * 10^{-12}\frac{F}{m}\right) * 3um * 40um}} = 3V$$

6. 5 points: [EE247A] In a three mask process (POLY0, C1S, POLY1) with only one structural polysilicon (POLY0 for wiring, and POLY1 for structures), design a 3 axis capacitive accelerometer. Can you do it with a single proof mass? Will the response be linear in all directions?

1 point for effort

1 each for an attempt at x, y, z sensors
1 point for some discussion of linearity of each sensor

To have a linear response you need a pair of capacitors: when one's capacitance increases, the other's decreases. Since we only have 2 conductive layers, we cannot make a pair of electrodes for the z axis. Thus the x and y response is linear, but z's is not.

Purple boxes around x sense gaps and red boxes around y sense gaps. They're labels, not in the design.

Blue lines is Poly0
Dark blue is C1S
Green is Poly1