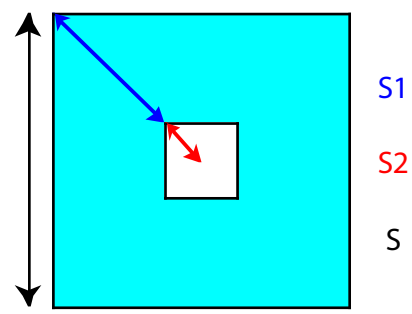
Problem 1

F2013 Midterm (EE147 F15 HW#7 Solutions)

1. **1 pt. for effort, 1 pt. for the right answer**
The etch will come in 10.1μm on any side of a silicon square. A 20.2μm will be the largest square to be completely undercut. The **22μm square** will the be smallest to survive

2. **1 pt. for effort, 1 pt. for the right answer**
The diagonal dimension, from the corner of the hole to the corner of the square, will be the longest undercut distance. The dimensions are labeled in the figure. The hole is a 2μm x 2μm hole, so from the figure $S1 = \sqrt{2}\mu\text{m} \cong 1.4\mu\text{m}$.

For S1, from the outer corner, the etch will come in 10.1μm on any side of the square, as in problem 1. This diagonal distance will be $\sqrt{2}(10.1\mu\text{m}) = 1.4(10.1\mu\text{m}) = 14.14\mu\text{m}$



From the inner corner, the etch will come in radially from the corner, so this distance will just be 10.1μm.

The full diagonal of the square will be $2(14.14 + 10.1 + 1.4)\mu\text{m} = 51.3\mu\text{m}$ $S = \frac{51.3\mu\text{m}}{\sqrt{2}} = \frac{51.3\mu\text{m}}{1.4} = 36.6\mu\text{m}$. The **38μm square** will the smallest to survive.

4. **1 pt. for effort, 1 pt. for the right answer**

$$\epsilon = \frac{\sigma}{E} = \frac{F}{AE}$$

$$F_{max} = \epsilon_{max}AE = 0.04(10^{-6}\text{m})^2(150\times10^9\ \text{N/m}^2)$$

$$\boldsymbol{F_{max} = 6\text{mN}}$$

5. **1 pt. for effort, 1 pt. for the right answer**

$$\dot{Q} = \frac{\Delta T}{R_{th}}$$

$$R_{th} = \frac{L}{kA} = \frac{10^{-4}\text{m}}{\left(100\frac{\text{W}}{\text{mK}}\right)(10^{-1}\text{m})^2} = 10^{-4}\frac{\text{K}}{\text{W}}$$

$$\Delta T = T_{front} - T_{back} = \dot{Q}R_{th}$$

$$T_{front} = T_{back} + \dot{Q}R_{th}$$

$$T_{front} = 40\text{C} + (0.1\text{W})\left(10^{-4}\frac{\text{K}}{\text{W}}\right)\left(\frac{1\text{C}}{1\text{K}}\right)$$

$$\boldsymbol{T_{front} = 40.00001\text{C}}$$

6.

c. **1 pt. for the right answer for each relation (2 pts. total)**

$$\boldsymbol{k \propto \frac{1}{L^3} \rightarrow Decrease\ by \frac{1}{8}}$$

$$\boldsymbol{\omega_n \propto \sqrt{k} \rightarrow Decrease\ by \frac{1}{\sqrt{8}}}$$

d. **1 pt. for effort, 1 pt. for the right answer for each frequency (2pts.)**
For a sinusoidal AC voltage with a DC voltage bias, the force will be

$$F = \frac{1}{2}C_0\left[V_{DC}^2 - \frac{V_{AC}^2}{2} + 2V_{DC}V_{AC}\sin(\omega t) - \frac{V_{AC}^2}{2}\cos(2\omega t)\right]$$

F_0 occurs at ω (we are driving the structure at 1Hz and looking at the force response at 1Hz). So $F_0 \propto 2V_{DC}V_{AC}$. There will be a force component at DC and another at 2ω.

@DC

$$\frac{F_{DC}}{F_0} = \frac{\left(V_{DC}^2 - \frac{V_{AC}^2}{2}\right)}{2V_{DC}V_{AC}} \cong \frac{V_{DC}^2}{2V_{DC}V_{AC}}$$

$$\boldsymbol{F_{DC} = \frac{V_{DC}}{2V_{AC}}F_0 = \frac{150\text{V}}{2(1.5\text{V})}F_0 = 50F_0}$$

@2Hz

$$\frac{F_{2Hz}}{F_0} = \frac{\frac{V_{AC}^2}{2}}{2V_{DC}V_{AC}}$$

$$F_{2Hz} = \frac{1}{4}\left(\frac{V_{AC}}{V_{DC}}\right)F_0 = \frac{1}{4}\left(\frac{1.5\text{V}}{150\text{V}}\right)F_0$$

$$\boldsymbol{F_{2Hz} = 0.0025F_0}$$

7. **0.5 pts. for each correct scaling relation**

$$k \propto t \rightarrow Increase\ by\ 2$$

$$b = \frac{\mu A}{h} \rightarrow Stays\ the\ same$$

$$m \propto V \propto t \rightarrow Increase\ by\ 2$$

$$f_n \propto \sqrt{\frac{k}{m}} \rightarrow Stays\ the\ same$$

$$Q = \frac{k}{b(2\pi f_n)} \rightarrow Increase\ by\ 2$$

$$x_n = \frac{F_0}{b\omega_n} \rightarrow x_n \propto F_0 \propto t \rightarrow Increase\ by\ 2$$

8. **1 pt. for effort, 1 pt. for the right answer for each part (3 pts.)**

a. The output of a Wheatstone bridge is

$$V_{out} = \frac{V_x}{4}\epsilon_R$$

Where $\epsilon_R = \frac{\Delta R}{R}$

$$\boldsymbol{V_{out} = \frac{10\text{V}}{4}(0.01) = 0.025\text{V}}$$

b.

$$\boldsymbol{\frac{\Delta R}{R} = G\epsilon = \frac{G\Delta L}{L} = (-20)(0.001) = -0.02}$$

c.

$$\boldsymbol{\frac{\Delta R}{R} = \alpha_{TCR}\Delta T = (0.001\text{C}^{-1})(100\text{C}) = 0.1}$$

Problem 2
Solutions on course webpage

Problem 3

A. **1 pt. for effort, 1 pt. for finding the snap-back voltage**
For a 3μm gap, the pull-in displacement will be at 1μm. The spring force at pull-in is given by

$$F_{spring\_PI} = kx = k(1\mu\text{m})$$

At this point the spring force and the electrostatic force are equal

$$F_{es\_PI} = F_{spring\_PI}$$

The gap will close to the gap stop after the pull-in voltage. The capacitor gap size will go from 2μm to 1μm. Since the force in inversely proportional to the square of the gap, the force will become 4x stronger

$$F_{es\_2} = 4F_{es\_PI} = 4F_{spring\_PI}$$

The spring will now be displaced by 2μm so it will be 2x stronger than at just before full pull-in voltage

$$F_{spring\_2} = 2F_{spring\_PI}$$

We want to lower the voltage until the electrostatic force is just lower than the spring force. We can see that $F_{es\_2} = 2F_{spring\_2}$. We want the electrostatic force lowered by a factor of 2. Since the force is proportional to the square of the voltage, the voltage should be lowered by a **factor of $\sqrt{2}$**. Since we found a pull-in voltage of 1.5V, the **snap-back voltage will be 1.5V/$\sqrt{2}$ = 1.06V**

B. **1 pt. for the plot, 1 pt. for pointing out the bistable region**