1. POLY2 is \( h = 1.5 \mu m \) thick and the minimum feature size is \( w = 2 \mu m \)
   a. **1 pt. for each correct spring constant (4 pts. total)**
      Assuming every loading case is a force at the tip and the beam is
      oriented with its length along the x-axis, width along the y-axis, and
      height along the z-axis

      \[
      K_{xx} = \frac{EA}{L} = \frac{Ewh}{L}
      \]
      \[
      = \frac{(150 \times 10^9 \text{ N/m}^2)(2 \mu m)(1.5 \mu m)}{100 \mu m}
      \]
      \[
      K_{xx} = (1.5)(2)(1.5) \times 10^{11-6-6+4} \text{N/m}
      \]
      \[
      K_{xx} = 4.5 \times 10^3 \text{N/m}
      \]

      \[
      K_{yy} = \frac{Ehw^3}{4L^3}
      \]
      \[
      = \frac{(150 \times 10^9 \text{ N/m}^2)(1.5 \mu m)(2 \mu m)^3}{4(100 \mu m)^3}
      \]
      \[
      = \frac{(1.5)(1.5)(8)}{4} \times 10^{11-6-18+12} \text{N/m}
      \]
      \[
      K_{yy} = 0.45 \text{N/m}
      \]

      \[
      K_{zz} = \frac{Ewh^3}{4L^3}
      \]
      \[
      = \frac{(150 \times 10^9 \text{ N/m}^2)(2 \mu m)(1.5 \mu m)^3}{4(100 \mu m)^3}
      \]
      \[
      = \frac{(1.5)(2)(3.4)}{4} \times 10^{11-6-18+12} \text{N/m}
      \]
      \[
      K_{zz} \approx 0.3 \text{N/m}
      \]

      \[
      K_{\theta z} = \frac{2EI_z}{L^2} = \frac{Ewh^3}{6L^2}
      \]
\[ \begin{align*}
= \frac{(150 \times 10^9 \text{ N/m}^2)(2\mu\text{m})(1.5\mu\text{m})^3}{6(100\mu\text{m})^2} \\
= \frac{(1.5)(2)(3.4)}{6} \times 10^{11-6+18+8}\text{N}
\end{align*} \]

\[ K_{\theta z} = 2 \times 10^{-5}\text{N} \]

Note that the units of \( K_{\theta z} \) are in Newtons/radian

b. 1 pt. for the force
   1 pt. for the moment
   The force at the tip of the beam is simply \( F = mg \), where \( m \) is the mass of the plate

\[ F = mg = \rho Vg = \left( 2300 \frac{\text{kg}}{\text{m}^3} \right) (100\mu\text{m})^2 (1.5\mu\text{m})(10\text{m/s}^2) \]

\[ F = 3.5 \times 10^{-10}\text{N} \]

The moment is caused by the offset of the plate mass from the beam tip. The center of mass of the plate is \( L_{cm} = 50\mu\text{m} \) from the tip assuming the mass is uniform, so \( M = mgL_{cm} = 2 \times 10^{-14}\text{Nm} \)

c. 1 pt. for beam tip linear deflection
   1 pt. for beam tip angular deflection
   1 pt. for the deflection of the end of the plate

\[ z = \frac{1}{EI} \left[ \frac{ML^2}{2} + \frac{FL^3}{3} \right] \]

\[ = \frac{12}{(150 \times 10^9 \text{ N/m}^2)(2\mu\text{m})(1.5\mu\text{m})^3} \left[ \frac{(2 \times 10^{14}\text{Nm})(100\mu\text{m})^2}{2} + \frac{(3.5 \times 10^{-10}\text{N})(100\mu\text{m})^3}{3} \right] \]

\[ z = 2\mu\text{m} \]

\[ z' = \theta_z = \frac{1}{EI} \left( ML + \frac{FL^2}{2} \right) \]

\[ \theta_z = 4 \times 10^{-5} \text{ (radians)} \]
Since the angular deflection at the beam tip is small, the deflection of the end of the plate is given by simple trig approximations

\[ dz_{\text{plate-tip}} = \theta_z L_{\text{plate}} = 4\text{nm} \]

d. 1 pt. for the acceleration

Assuming the fracture strain of silicon is about 2%  

\[ F = ma = kx = \frac{EA}{L} x \]

\[ a = \frac{EA x}{m L} = \frac{EA}{m} \epsilon \]

\[ a = \left( \frac{150 \times 10^9 \text{ N/m}^2}{3.5 \times 10^{-11} \text{ kg}} \right) (1.5 \mu\text{m})(2 \mu\text{m}) (0.02) \]

\[ a = 3 \times 10^8 \frac{\text{m}}{\text{s}^2} \]

e. 1 pt. for the acceleration  
1 pt. for noting where the beam will break

The maximum strain in a bent cantilever is always at the base of the anchored end, furthest from the neutral axis. This is where the beam will break. At the top surface of the beam

\[ \epsilon_{\text{max}} = \frac{z}{\rho(x)} = \frac{h}{2\rho(x)} \rightarrow \rho(x) = \frac{h}{2\epsilon_{\text{max}}} \]

At the base of the beam

\[ M(0) = \frac{E I_y}{\rho(0)} = M_o + F_o L = m\alpha_y(L_{cm} + L) \]

\[ a_y = \frac{E I_y}{\rho(0)(m(L_{cm} + L))} \]

Combining these equations, we get

\[ a_y = \frac{2\epsilon E I_y}{h(m(L_{cm} + L))} = 8 \times 10^5 \frac{\text{m}}{\text{s}^2} \]
2. 1 pt. for each scaling relation (11 pts. total)

a.

In part 1a.

\[ K_{xx} \propto \frac{A}{L} \propto \frac{S^2}{S} \rightarrow S \]

\[ K_{yy} \propto \frac{h w^3}{L^3} \propto S \]

\[ K_{zz} \propto \frac{w h^3}{L^3} \propto S \]

\[ K_{\theta z} \propto \frac{w h^3}{L^2} \propto S^2 \]

In part 1b.

\[ F \propto m \propto V \propto S^3 \]

\[ M \propto mL_0 \propto S^4 \]

In part 1c.

\[ z \propto \frac{F L^3}{I} \propto \frac{S^6}{S^4} \rightarrow S^2 \]

\[ \theta_z \propto \frac{F L^2}{I} \propto S \]

\[ dz_{plate} \propto \theta_z L \propto S^2 \]

In part 1d.

\[ a_{max} \propto \frac{A}{m} \propto \frac{S^2}{S^3} \rightarrow S^{-1} \]

In part 1e.

\[ a_{max} \propto \frac{I}{h m L} \propto \frac{S^4}{S^5} \rightarrow S^{-1} \]
b. 1 pt. for each scaling relation (2 pts. total)

The acceleration scales as $S^{-1}$

In x

$$S^{-1} = \frac{a_{max_2}}{a_{max_1}}$$

$$S = \frac{a_{max_1}}{a_{max_2}} = \frac{4 \times 10^7 g}{g}$$

$$S = 4 \times 10^7$$

In y

$$S = \frac{a_{max_1}}{a_{max_2}} = 4 \times 10^4$$
3. 1 pt. for each cross sectional area (4 pts.)

1 pt. for commenting on safety

\[ F = kx = \frac{EA}{L}x = EA\epsilon \]

\[ A = \frac{F}{E\epsilon} \]

Assuming \( \epsilon = 0.02 \)

\[ A = \frac{mg}{E\epsilon} = \frac{(50\text{kg}) \left(10\text{m} \right)}{(150\times10^9\text{N/m}^2)(0.02)} \]

\[ A_{Si} = 2\times10^{-7}\text{m}^2 \]

The ultimate tensile strength for aluminum is about 300MPa. For glass it is 33MPa. For Steel, it varies a lot, but stainless steel has a max tensile strength of about 800MPa. All values from https://en.wikipedia.org/wiki/Ultimate_tensile_strength

\[ A_{Al} = \frac{F}{\sigma_{max}} = \frac{mg}{\sigma_{max}} = 2\times10^{-6}\text{m}^2 \]

\[ A_{glass} = 2\times10^{-5}\text{m}^2 \]

\[ A_{steel} = 6\times10^{-7}\text{m}^2 \]

You would probably not feel safe hanging from the brittle materials (silicon, glass) since they are prone to snapping.
4. 1 pt. for an approximately correct final cross section
   1 pt. for the dimple under poly1
   1 pt. for putting metal on poly2
   1 pt. for having poly2 contact poly1 and poly0
   1 pt. for no metal on poly1 (it gets taken away during the oxide etch)
5. 1 pt. for each part correct (5 pts. total)

-1 pt. total if you forgot the nitride

a. Substrate, nitride, poly 0, 1\textsuperscript{st} oxide, poly 1, 2\textsuperscript{nd} oxide, poly 2, metal
b. Substrate, nitride, poly 0, poly 1, poly 2
c. Substrate, nitride, poly 2, metal
d. Substrate, nitride, 1\textsuperscript{st} oxide, 2\textsuperscript{nd} oxide, metal
e. Substrate, nitride, 1\textsuperscript{st} oxide, 2\textsuperscript{nd} oxide

6. 5 pts. distributed as you see fit