1.

a. **1 pt. for effort**

1 pt. for an approximately right answer

$$\boldsymbol{C(x) = \epsilon_0 t L\left(\frac{1}{g_1 - x} + \frac{1}{g_2 + x}\right)}$$

b. **1 pt. for effort**

1 pt. for an approximately right answer

$$\boldsymbol{\frac{dC}{dx} = \epsilon_0 t L\left(\frac{1}{(g_1 - x)^2} - \frac{1}{(g_2 + x)^2}\right)}$$

c. **1 pt. for effort**

1 pt. for an approximately right answer

$$AD = \frac{\left.\frac{dC}{dx}\right|_{x=0}}{A_{combs}}$$

$$= \frac{\epsilon_0 t L\left(\frac{1}{g_1^2} - \frac{1}{g_2^2}\right)}{L(2a + g_1 + g_2)}$$

$$\boldsymbol{AD = \frac{\epsilon_0 t\left(\frac{1}{g_1^2} - \frac{1}{g_2^2}\right)}{(2a + g_1 + g_2)}}$$

d. **1 pt. for effort**

1 pt. for an approximately right answer

Take the derivative of the aerial density with respect to g_2, and solve for g_2 when the derivative is zero.

$$\frac{d(AD)}{dg_2} = \frac{\epsilon_0 t\left(\frac{2}{g_2^3}\right)}{2a + g_1 + g_2} - \frac{\epsilon_0 t\left(\frac{1}{g_1^2} - \frac{1}{g_2^2}\right)}{(2a + g_1 + g_2)^2} = 0$$

$$\frac{2}{g_2^3(2a + g_1 + g_2)} - \frac{\left(\frac{1}{g_1^2} - \frac{1}{g_2^2}\right)}{(2a + g_1 + g_2)^2} = 0$$

$$\frac{\left(\frac{1}{g_1^2} - \frac{1}{g_2^2}\right)}{(2a + g_1 + g_2)^2} = \frac{2}{g_2^3(2a + g_1 + g_2)}$$

$$g_2^3 - g_2 g_1^2 = 2g_1^2(2a + g_1 + g_2)$$

$$\boldsymbol{g_2^3 - (3g_1^2)g_2 - 2g_1^3 - 4ag_1^2 = 0}$$

Extra note (not critical for grading):

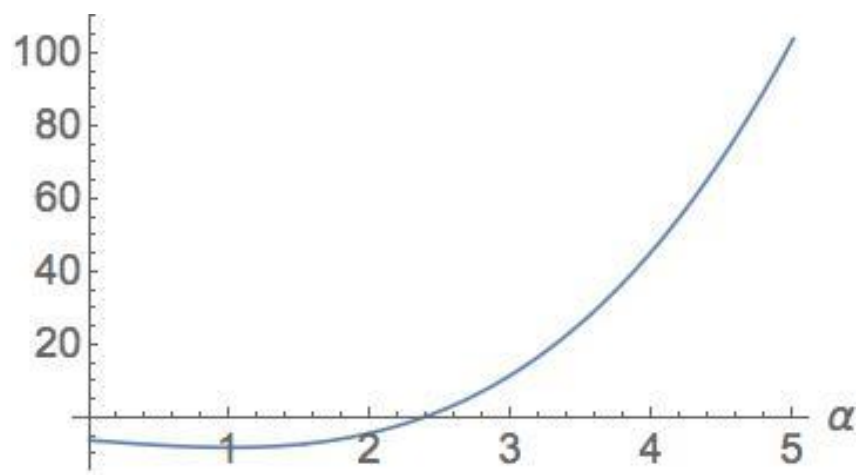
Usually when designing comb fingers for maximum aerial density, it is convenient to make g_1 the minimum feature size allowed by the design rules and then have $g_2 = \alpha g_1$. If we do this, we get a new cubic equation in terms of α

$$\alpha^3 - 3\alpha - 2 - \frac{4a}{g_1} = 0$$

In most cases, we also make the finger width equal to the finger spacing, $a = g_1$. This gives

$$\alpha^3 - 3\alpha - 6 = 0$$

We can plot this equation for ratios greater than 0



The only real root is $\alpha = 2.35$, which fits with the result in part e, below.

If g_1 is much larger than a, then the limiting value of α is sqrt(3). If g_1 is much smaller than a, then the value of α approaches the cube root of $4a/g_1$. So making the

bigger gap 2.35 times bigger than the smaller gap is going to be pretty close to optimal over a wide range of line vs. space values for the fingers and primary gap.

e. **1 pt. for optimal g_2**

   1 pt. for maximum aerial density

   Numerically solving the above equation, with $g_1 = a = 2\mu\text{m}$, we get $\boldsymbol{g_2 = 4.7\mu\text{m}}$ as the only real solution. The maximum aerial density is $\mathbf{3.4F/m^3}$.

2. **1 pt. for finding the desired capacitance at zero displacement (total length of required fingers)**

   1 pt. for finding the dimensions of the mass and total mass based on the finger length and number of fingers

   1 pt. for finding the necessary spring constant and spring dimensions from the mass and max displacement requirements

   1 pt. for sketch with all dimensions labeled

   1 pt. for dC/dx at $x = 0$

   At $x = 0$, we want a capacitance of 1pF. We must find the number of finger units, N, required for the accelerometer. Using the dimensions from hw1 and the optimal gap spacing found above

$$C(0) = 10^{-12}\text{F} = N\epsilon_0 t L_{fingers}\left(\frac{1}{g_1} + \frac{1}{g_2}\right)$$

$$N = \frac{10^{-12}\text{F}}{\epsilon_0 t L_{fingers}\left(\frac{1}{g_1} + \frac{1}{g_2}\right)}$$

$$N = \frac{10^{-12}\text{F}}{(8.85 \times 10^{-12}\text{F/m})(20 \times 10^{-6}\text{m})(20 \times 10^{-6}\text{m})\left(\frac{1}{2 \times 10^{-6}\text{m}} + \frac{1}{4.7 \times 10^{-6}\text{m}}\right)}$$

$$= \frac{10^{-12}F}{(8.85)(20)(20)\left(\frac{1}{2} + \frac{1}{4.7}\right) \times (10^{-12}\text{F/m})(10^{-6}\text{m})(10^{-6}\text{m})\left(\frac{1}{10^{-6}\text{m}}\right)}$$

$$\boldsymbol{N \cong 396}$$

   We will assume the mass is still 20μm in the finger-less dimension, W. With 396 finger units, the total length of the mass will be

$$L = 396(2a + g_1 + g_2)$$

$$= 396(2(2\mu\text{m}) + 2\mu\text{m} + 4.7\mu\text{m})$$

$$L = 4200\mu\text{m}$$

That gives us a really long and thin shape, which is fine for your solutions but not what anyone would do in practice. If the array is about 20um in one dimension, and 4,000 in the other, we can increase the length of the fingers by 10x to 200um,

and reduce the number of fingers by 10x to 40. Now we have a shape that is closer to square – a little over 200um by a little over 400um.

Neglecting the fingers (probably not a good approximation!), the mass will be

$$mplate = \rho V_{mass}$$

$$= 2300\ \mathrm{kg/m^3}(423 \times 10^{-6}\mathrm{m})(20 \times 10^{-6}\mathrm{m})(20 \times 10^{-6}\mathrm{m})$$

$$\boldsymbol{m = 4 \times 10^{-10}\mathrm{kg}}$$

The mass of the fingers is

$m_{fingers}$ = (2300 kg/m3)(40)(2e-4m)(2e-6m)(2e-5m)

= (2.3)(4)(2)(2)(2)x $10^{3+1-4-6-5}$ kg

= 7.4 x 10^{-10} kg

So the total weight is dominated by the fingers, and is about 10^{-9} kg.

At $2g$, we want an approximate deflection of the finger gap, 2μm. Equating the force equations for spring deflection and acceleration, we get

$$kx = ma = m(2g)$$

$$k = \frac{m(2g)}{x}$$

$$k = \frac{(10^{-9}\mathrm{kg})\big(2(10\mathrm{m/s^2})\big)}{2 \times 10^{-6}\mathrm{m}}$$

$$\boldsymbol{k = 0.01\ \mathrm{N/m}}$$

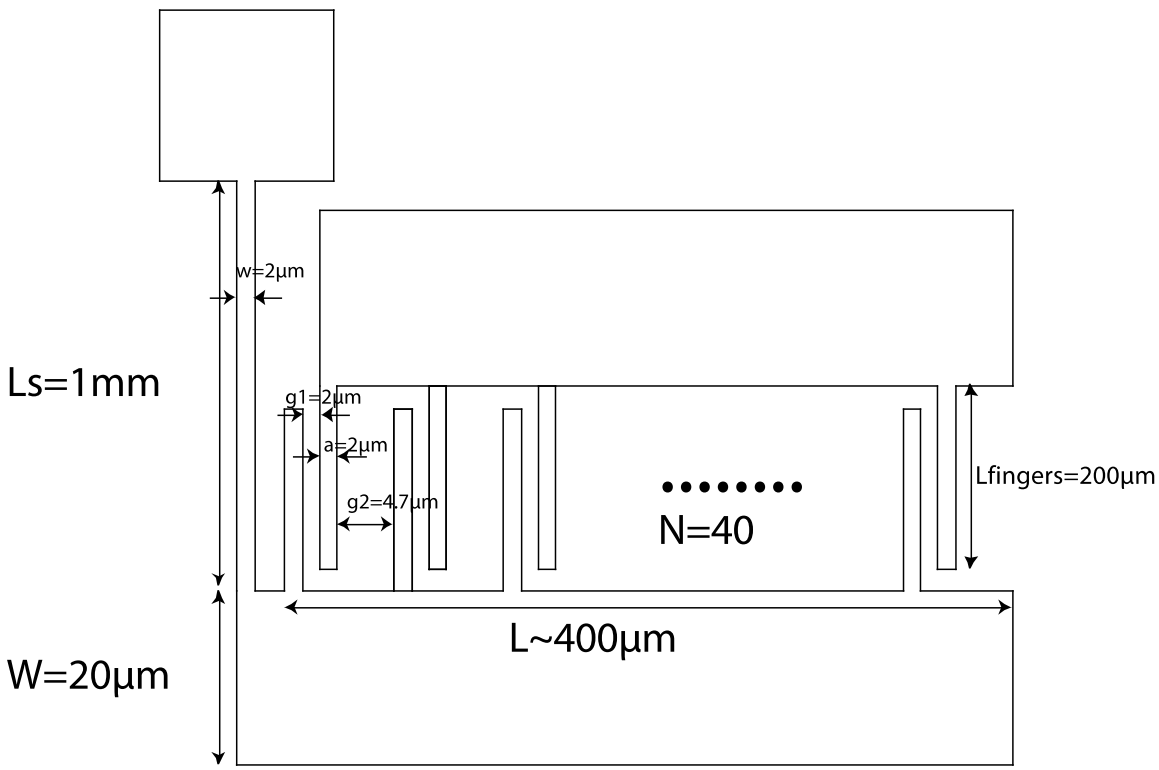
We must find the beam length necessary for this given spring constant. For a spring that is 2μm wide and 20μm high, we get

$$k = \frac{Ehw^3}{4L_s^3}$$

$$L_s = \sqrt[3]{\frac{Ehw^3}{4k}}$$

$$L_s = \sqrt[3]{\frac{150 \times 10^9\mathrm{N/m^2}(20 \times 10^{-6}\mathrm{m})(2 \times 10^{-6}\mathrm{m})^3}{4(0.01\mathrm{N/m})}}$$

$$\boldsymbol{L_s = 1\mathrm{mm}}$$



Note that this diagram and the mass calculation do not account for etch holes to release the mass.

At $x = 0$,

$$\frac{dC}{dx} = N\epsilon_0 tL\left(\frac{1}{g_1^2} - \frac{1}{g_2^2}\right)$$

$$= (396)\left(8.85 \times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}}\right)(20)(20)(10^{-12}\mathrm{m})\left(\frac{1}{(2 \times 10^{-6}\mathrm{m})^2} - \frac{1}{(4.7 \times 10^{-6}\mathrm{m})^2}\right)$$

$$= \mathbf{3 \times 10^{-7}F/m}$$

3. **For each cross section:**

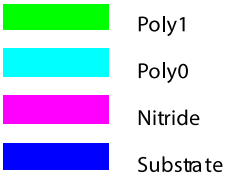
   1 pt. for an attempt

   1 pt. for including the correct layers in the correct order. From bottom to top, substrate, nitride, poly0, poly1 (the problem did not explicitly state that the oxide on top of poly0 has been etched, so if you included the oxide, you are fine)

   1 pt. for getting the cross sections correct (don't worry about getting the conformal depositions technically correct)

   Top cross section

   Bottom cross section



5. **5 pts. total.** Again, no straightforward solution. Reward yourself for appropriate effort.