

# Key

EECS140 Final  
Fall 2019

Name Good Luck, Have Fun!

SID top secret

Prob.	Score
1	15
2,3	/20
4	/16
5	/16
6	20 21
7	/8
8	/15
9	/14
10	/10
11	/15
12	/22
Total	/165

For q1: All points for correct procedure, no H's for this one...

[8] The I-V curves below are from an Indium Gallium Zinc Oxide transistor used in flat panel display TVs.

a. Estimate  $g_m$  when  $V_{GS}=10V$  and  $V_{DS}=10V$ . Write down what  $\Delta I$  and  $\Delta V$  you are using for your calculation.

$$g_m = \frac{\Delta I}{\Delta V} \quad \text{for } \Delta V = 2.5V$$

$$= \frac{0.01mA}{2.5V} \quad \Delta I \approx 0.06 - 0.01 \approx 0.05mA$$

b. Estimate  $g_o=1/r_o$  under the same conditions. Draw the line that you are using to estimate  $g_o$ . Say slope  $\approx \frac{0.01mA}{10V} \approx 0.001 = g_o$

c. Calculate the intrinsic gain from parts a and b

$$\frac{0.01mA}{2.5V} \cdot \frac{10}{0.01mA} = 40$$

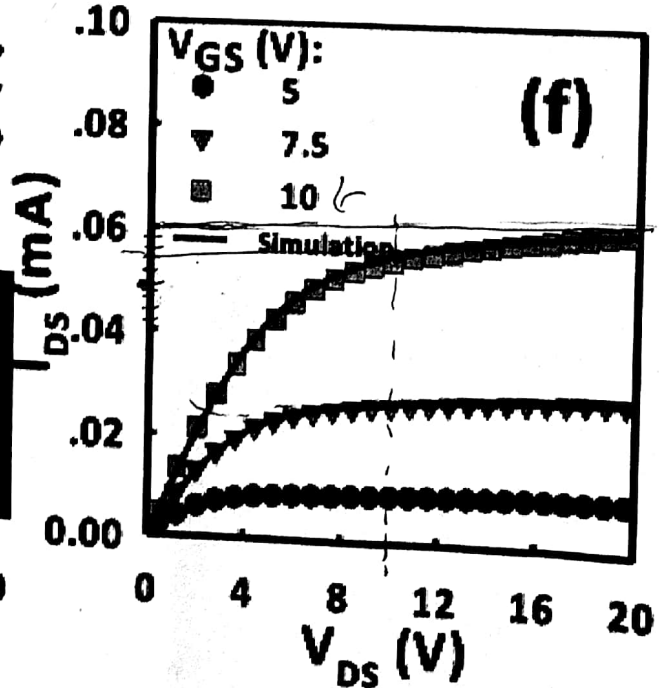
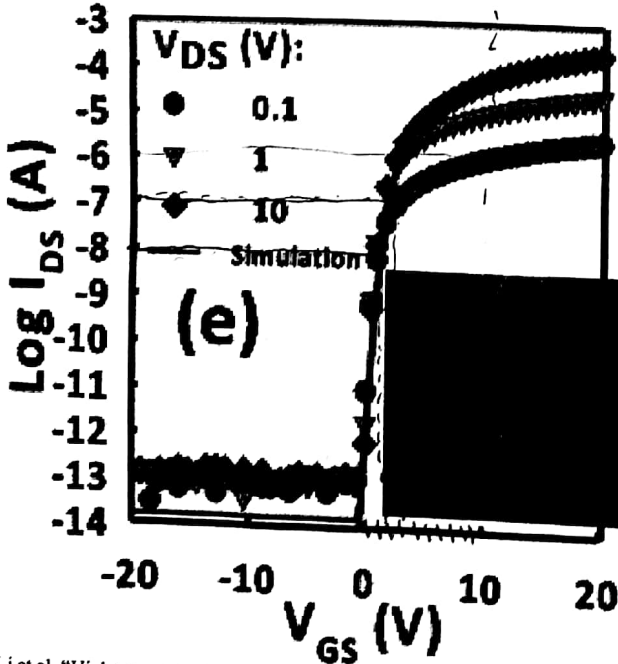
$$\frac{0.4}{2.5 \cdot 0.01} = \frac{40}{2.5} = 16$$

d. If  $I_{DS}=10nA$  when  $V_{GS}=2V$ , estimate the sub-threshold slope.

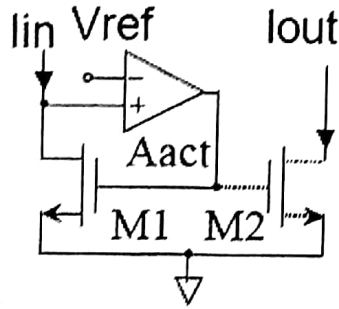
$10nA \rightarrow -8$  on a  $\log_{10}$  scale

$0V \approx -13$

so  $\sim 5 \text{ decades} / 2V \approx 2.5$

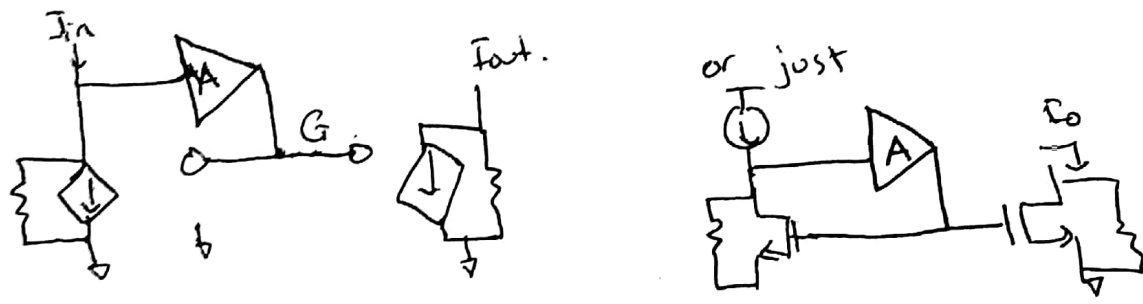


(Li et al, "High-speed dual-gate a-IGZO TFT-based circuits ...", 2014)



2. [10] For the current mirror shown here,

- 1 a. Draw the small signal model of the circuit, assuming that the amplifier is an ideal op-amp with a large positive gain  $A$ , and that  $V_{ref}$  is a constant voltage.

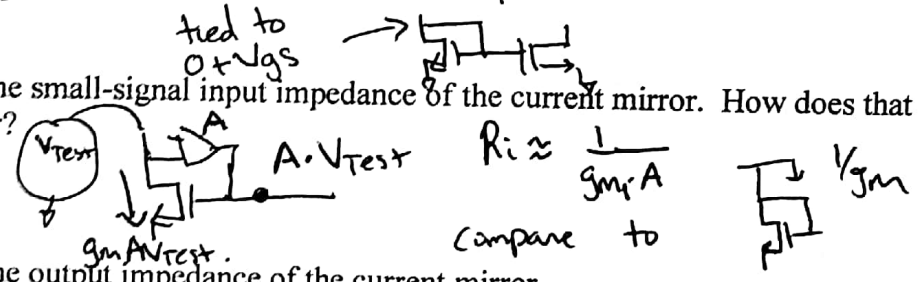


- 3 b. What is the minimum value of  $V_{ref}$  for which transistor M1 will remain in saturation? If we compare this circuit to a typical simple current mirror, which has a lower minimum input voltage? Why?

$V_{ref} > V_{ov1}$ , which is lower than the simple mirror.

- 4 c. Write an expression for the small-signal input impedance of the current mirror. How does that compare to a simple current mirror?

(actual answer is  $\frac{r_{o1}}{1+g_m r_{o1} A}$ )



- 2 d. Write an expression for the output impedance of the current mirror

$R_o = r_{o2}$  nothing new here

3. [10] You are designing a single-stage CMOS op-amp to be used in feedback to achieve a gain of 5. The gain must be accurate to 1% from DC to 1 Mrad/s (including any 3dB losses). The load is a 10pF capacitor. You are restricted to biasing your transistors with overdrives between 100mV and 1V, and they look reasonably quadratic in that range.

- 4 a. What is the minimum open-loop gain and dominant pole location of the op-amp?

$\frac{1}{A\beta} = 0.01$ ,  $A = \frac{1}{0.01} = 100$   
 $A = \frac{1}{[1\% \cdot f]} = 5 \cdot 100 = 500$

- 2 b. What is the minimum unity gain frequency of the op-amp?

$\omega_u = GBW = 500 \cdot \frac{1 \text{ Mrad/s}}{1 \text{ Mrad/s}} = 500 \text{ Mrad/s}$

- 2 c. What is the minimum transconductance of the input transistors?

$\omega_u = g_m / C_L \Rightarrow g_m = 500 \text{ Mrad/s} \cdot 10^{-11} \text{ F} = 5 \cdot 10^{-3} \text{ S}$

- 2 d. What is the minimum current in each of the input transistors?

$g_m = \frac{2I_D}{V_{ov}}$ ,  $V_{ov} = 100 \text{ mV}$ , then  $\frac{2I_D}{0.1} = 5 \cdot 10^{-3}$   
 $I_D = 5 \cdot 10^{-4} / 2 \text{ A} = 250 \text{ nA}$

[16] For the figure below you may assume that the switches and op-amp are ideal.  $\phi_1$  and  $\phi_2$  are non-overlapping clocks. Assume all of your capacitors are initially discharged.

a. After the first round of  $\phi_1$  and  $\phi_2$  (we'll call it cycle 0), what is the charge,  $Q_F[0]$ , on the right side of  $C_F$  in terms of that cycle's input,  $V_{IN}[0]$ ?

$$Q_S[0] = Q_X[0] = C_S(0 - V_{IN}[0]) = C_F(0 - V_{OUT}[0]) = -Q_F[0] \quad \boxed{Q_F[0] = C_S V_{IN}[0]}$$

b. What is the corresponding voltage,  $V_{OUT}[0]$ ?

$$\boxed{V_{OUT}[0] = \frac{C_S}{C_F} V_{IN}[0] = \frac{Q_F[0]}{C_F}}$$

c. Now the input voltage has changed to  $V_{IN}[1]$ . What is the charge  $Q_F[1]$  on the right side of  $C_F$  after the next cycle in terms of  $Q_F[0]$  and  $V_{IN}[1]$ ?

$$\begin{aligned} Q_S[1] &= C_S(-V_{IN}[1]) \\ Q_X[1] &= Q_X[0] + Q_S[1] \end{aligned} \quad \rightarrow \quad \begin{aligned} Q_F[1] &= Q_F[0] - Q_S[1] \\ \boxed{Q_F[1] &= Q_F[0] + C_S V_{IN}[1]} \end{aligned}$$

d. Write an expression for  $V_{OUT}[n]$ , the output voltage after n cycles.

$$V_{OUT}[n] = \frac{Q_F[n]}{C_F} = \frac{C_S}{C_F} \sum_{L=0}^n V_{IN}[L]$$

e. What do we call this kind of circuit?

integrator

Same circuit, different types of questions:

f. During  $\phi_2$  what is the feedback factor for the amplifier?

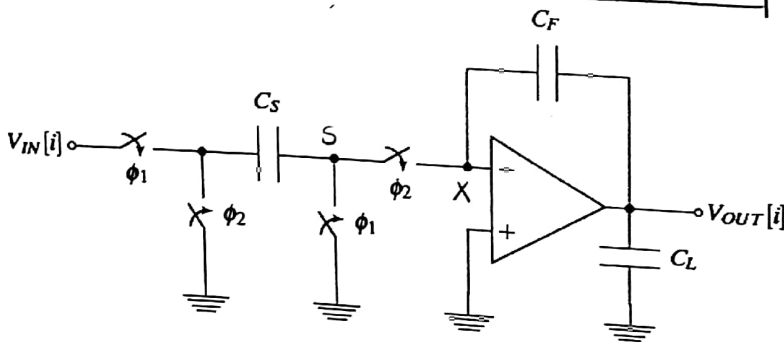
$$\boxed{f = \frac{C_F}{C_S + C_F}}$$

g. During  $\phi_2$  what is the total capacitance at the output of the amplifier?

$$\boxed{C_{total} = C_L + (1-f)C_F}$$

h. If there is parasitic capacitance  $C_p$  on node  $V_-$ , does that change the feedback factor during  $\phi_2$ , and if so what is the new feedback factor?

$$\boxed{f = \frac{C_F}{C_F + C_S + C_P}}$$



16

[15] Given the cascode amplifier below,  $g_m=1\text{ms}$  and  $r_o=10\text{k}$ .  $C_L=2\text{pF}$ ,  $C_{gs}=100\text{fF}$  and you may assume that all other capacitors are zero. The current source has an output impedance of  $1\text{M}$ .

a. On the next page, plot the magnitude of the output impedance,  $Z_{out}$ , vs. frequency. LABEL AXES

**CLEARLY**  $Z_o = g_m r_o^2 \parallel 1\text{M} = 10^{-3} \cdot 10^8 \parallel 1\text{M} \approx 10^5$

b. What is the output pole frequency?

1pt  $P_L = \frac{1}{10^5 \cdot 10^{-12} \cdot 2} = \frac{1}{5 \cdot 10^{-7}} = 5\text{M rad/s}$

c. What is the low frequency impedance seen looking into the source of M2?

2pts  $\frac{1}{g_{m2} \left(1 + \frac{R_d}{r_o}\right)} = 10^3 (1+100) \approx 10^5 \Omega$

d. What is the low frequency gain from  $V_{in}$  to the drain of M1?

1pt  $|g_{m1} \cdot (r_{o1} \parallel Z_{S2})| \approx 10^4 \cdot 10^3 = -10\text{V/V}$

e. What is the low frequency input capacitance if  $C_{gd}=20\text{fF}$ ?

2pts Miller Cap!  $C_{gd}(1+10) = 220\text{fF}$ . Add  $C_{gs}$  for  $C_{in}$

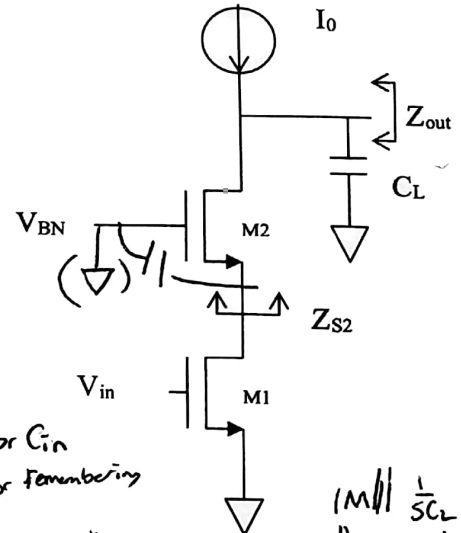
$C_{in} = 220\text{fF} + 100\text{fF} = 320\text{fF}$

f. Plot the magnitude of the impedance looking into the source of M2,  $Z_{S2}$ , vs. frequency. LABEL AXES CLEARLY

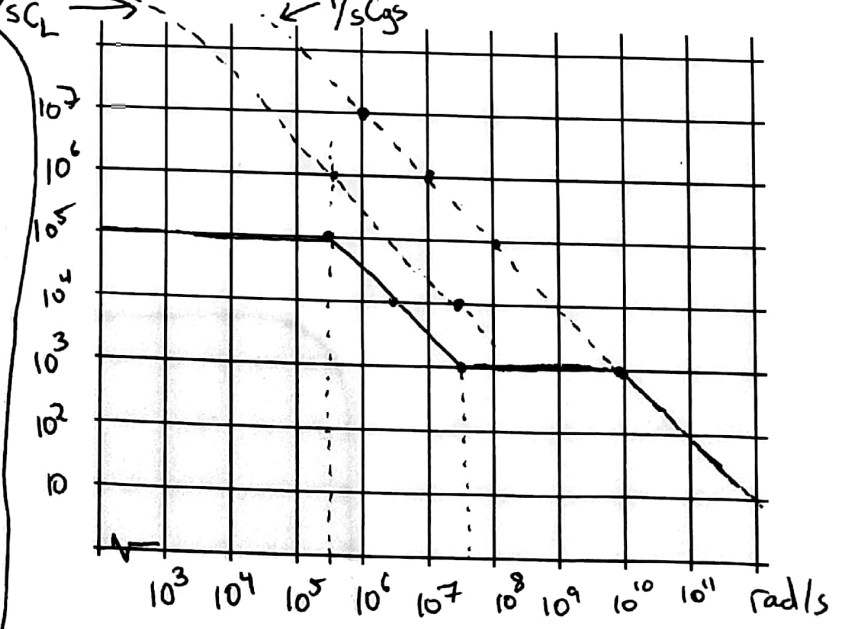
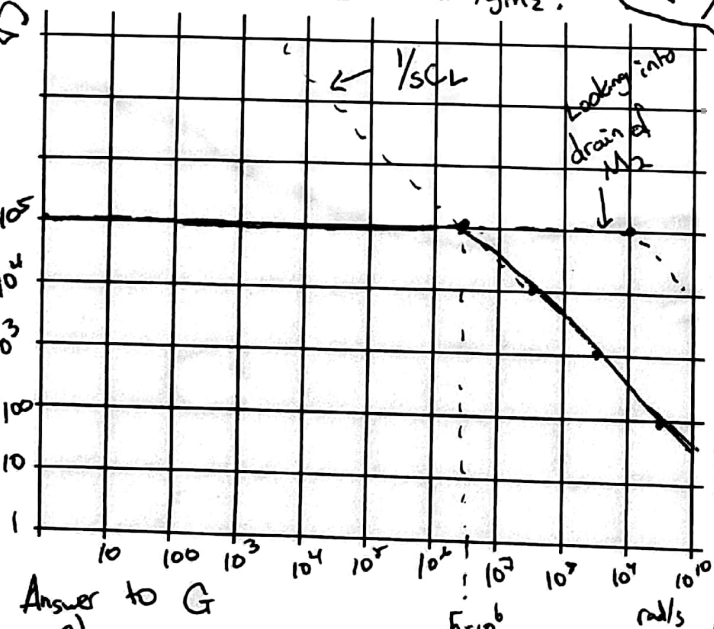
g. What is the second pole frequency?

Note  $Z_d$  will roll-off when  $\frac{1}{sC_L} = 1\text{M}\Omega$ ,  $\omega = 5 \times 10^5 \text{ rad/s}$ . At this frequency  $\frac{1}{sC_{gs2}}$  is still very large, & so is inconsequential.

But wait, there's more! There is a zero! recall  $Z_{S2} = \frac{1}{g_{m2} \left(1 + \frac{R_d(s)}{r_o}\right)}$ , Magnitude of  $Z_{out}$  so when  $Z_d(s)/r_o \ll 1$ ,  $Z_{S2}$  is still  $1/g_{m2}$ !



6pts 2 pole pole/zero balance  
 Multiple things here!!!  
 $Z_{S2} = \frac{1}{sC_{gs2}} \parallel \left[ \frac{1}{g_{m2} \left(1 + \frac{C_{gd} \parallel \frac{1}{sC_L}}{r_{o2}}\right)} \right]$   
 all pts for remembering Miller cap!  
 $1\text{M} \parallel \frac{1}{sC_L}$  is the impedance  $Z_d$  seen e drain of M2



9) 10^10 based on the plot (intersection of 1/sCgs loadline)

∴ Zero @  $1 = \frac{R_d(s)}{r_o}$  or when  $Z_d(s) = r_o$   
 This is when  $\frac{1}{sC_L} = 10^4$  @  $s = 5 \cdot 10^7 \text{ rad/s}$

[20] For the figure below,  $V_{tn} = -V_{tp} = 1V$ . The overdrive voltage on M5 and M7 is 100 mV. You may ignore body effect, and assume that the devices all look quadratic.

- Assuming  $\lambda = 0$ , estimate the currents in on the transistors in the table below
- Assuming  $\lambda = 0$ , estimate the bias voltages on the gates of the transistors in the table below

The square root of 20 is approximately 4.5

	M1	M3	M5	M7	M9	M11	M16	M17
$ I_D $	100 $\mu A$	200 $\mu A$	200 $\mu A$	200 $\mu A$	200 $\mu A$	100 $\mu A$	100 $\mu A$	200 $\mu A$
$V_G$	3V	3.9V	3.9V	1.1V	1.1V	1.45V	3.9V	3.55V

- Estimate the input common mode range (give a voltage for each)

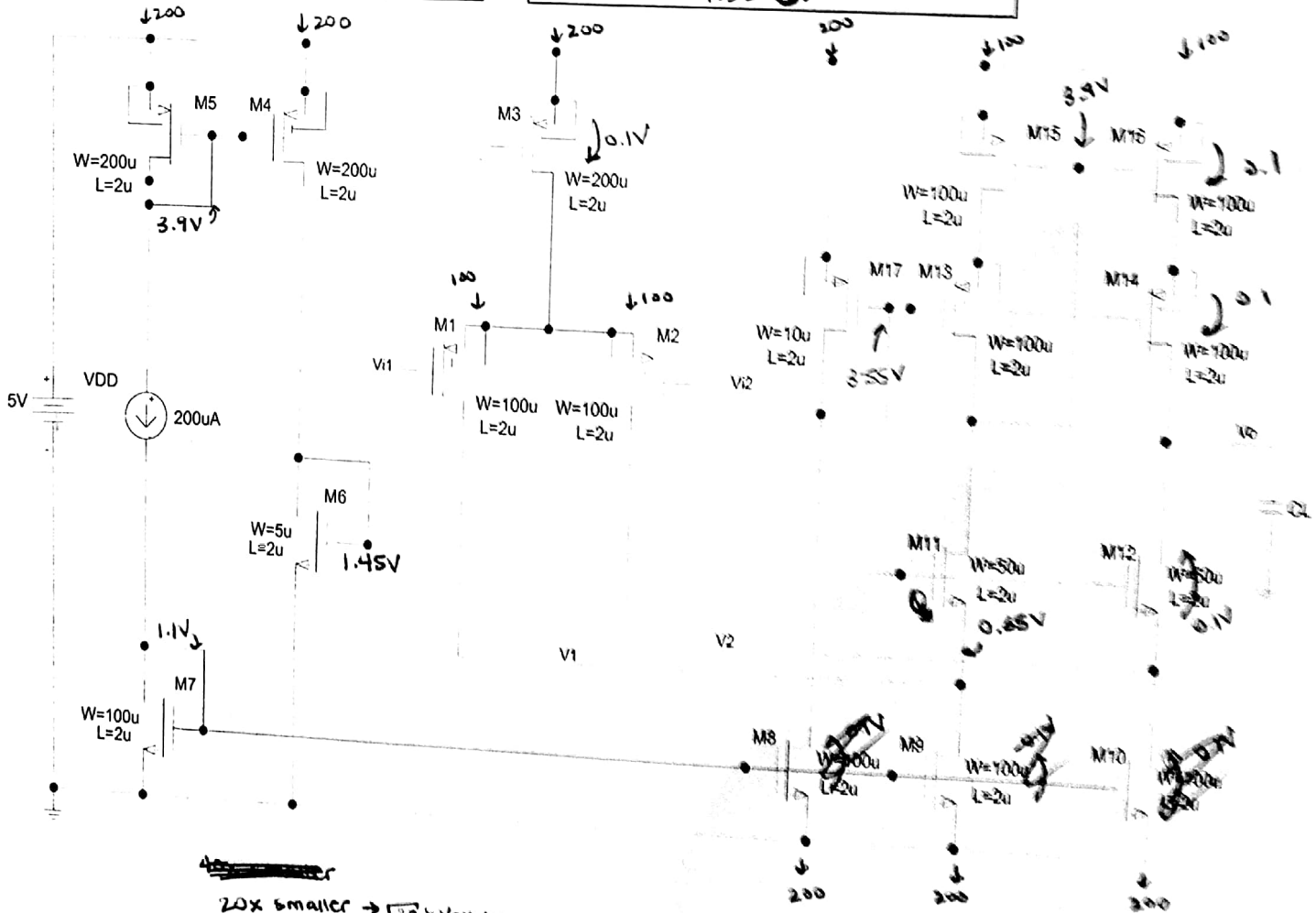
$V_{icm, min} = 0.65V$

$V_{icm, max} = 3.8V$

- Estimate the output swing for which all transistors will remain in saturation

$V_{o, min} = 0.45V$

$V_{o, max} = 4.55V$



For the following problems related to the project, you will be working in a CMOS process with  $\mu_n C_{ox} = 200 \mu A/V^2$ ,  $\mu_p C_{ox} = 100 \mu A/V^2$ ,  $V_{tn} = -V_{tp} = 0.5V$  and  $\lambda = 1/(10V)$  ( $1 \mu m/L$ ) with  $W_{min} = L_{min} = 1 \mu m$ . Note that this is NOT the same process as you used for your project this semester.

The supply is 2 alkaline cells, so  $V_{bat} = 1.6..3.2V$

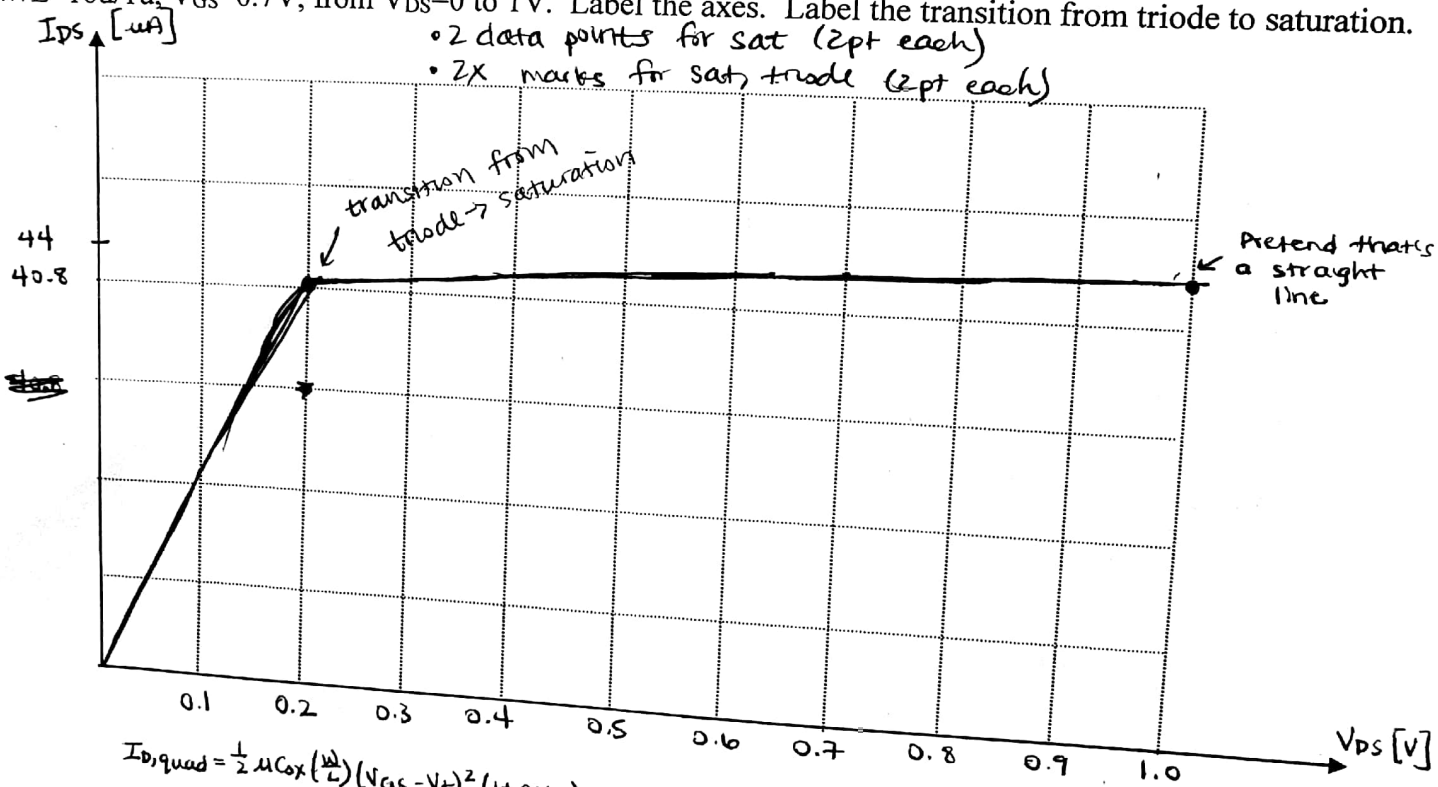
You will design parts of the following analog blocks

- A low voltage bandgap reference
- an analog regulator with an output of 1.25 V
- an ADC reference voltage of 1V
- a digital regulator with an output of 1.5 V
- a 2-input analog multiplexor with a single control bit
- a programmable gain amplifier with one gain select bit, and a gain of either 1 or 5
- a 10 bit switched capacitor ADC. **The ADC has an input range from 0-1V.** This is less than  $V_{DDA}$ .

op-amp topologies for parts "b" below: N or P input; 5 transistor, current mirror, 2 stage, folded cascode

7.

[8] To get started, carefully sketch the drain current vs. drain voltage for an NMOS transistor with  $W/L = 10 \mu/1 \mu$ ,  $V_{GS} = 0.7V$ , from  $V_{DS} = 0$  to 1V. Label the axes. Label the transition from triode to saturation.



$$I_{D,quad} = \frac{1}{2} \mu C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

$$V_{GS} = 0.7V \Rightarrow V_{GS} - V_t = 0.2V$$

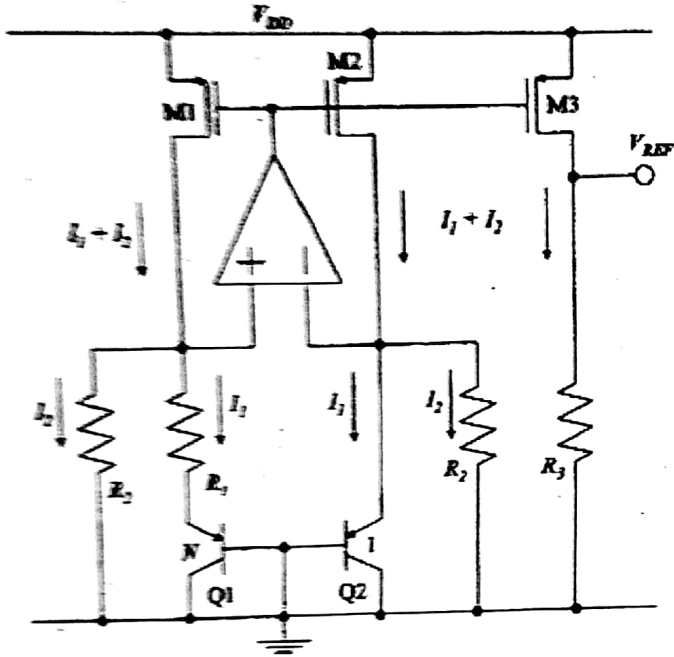
$$\text{@ } V_{DS} = 0.2V: \frac{1}{2} (200 \frac{\mu A}{V^2}) (10) (0.2)^2 (1 + 0.02)$$

$$= 100 (0.04) (10) (1.02) = 40 (1.02)$$

$$\text{@ } V_{DS} = 1.0V: \frac{1}{2} (200) (10) (0.04) (1.1) = 40 (1.1) \rightarrow 44 \mu A$$

$$= 40.8 (10) = 40.8 \mu A$$

Low-voltage Bandgap



For the above circuit, assume that the op-amp is high gain and the transistors are identical with high output impedance

- 2 a) Explain why (prove) the three currents labeled " $I_1+I_2$ " must be equal, why the two currents labeled  $I_2$  must be equal, and why the two currents labeled  $I_1$  must be equal.

$V^+ = V^-$ , &  $I_2 = V^+/R_2$  for both sides.

$I_1+I_2$  is equal both sides due to mirror M1 & M2

$I_1$  equal both sides due to above (algebra)

- 2 b) What is the voltage across  $R_1$ ?

$\Delta V_{be} = \ln(N) \cdot V_{thermal}$

- 2 c) Write expressions for  $I_1$  and  $I_2$  in terms of  $V_{be2}$  (the right diode voltage),  $V_{th}$  (thermal voltage),  $R_1$ ,  $N$ , and  $R_2$ .

$I_1 = \frac{\Delta V_{be}}{R_1}$ , PTAT

$I_2 = \frac{V_{be2}}{R_2}$

- 2 d) For  $I_1$  and  $I_2$ , which one is PTAT, which one has a negative temperature coefficient?

$I_1$  PTAT       $I_2$  CTAT

- 2 e) Put it all together: Find the equation for  $V_{ref}$ !

$V_{ref} = (I_1 + I_2)R_3 = \frac{R_3}{R_2} \left( V_{be2} + \left( \frac{R_2}{R_1} \right) \ln N \cdot V_{th} \right)$

- 2 f) Roughly what should the ratio between  $R_1$  and  $R_2$  be to get zero temperature coefficient on  $V_{ref}$ ? Write one as an integer times the other. if  $V_{be} \approx -2mV/K$ ,  $V_{th} \approx 0.085mV/K$

$\frac{R_2}{R_1} = \frac{2mV/K}{0.085mV/K \cdot \ln(N)}$

- 2 g) Roughly what should the ratio between  $R_2$  and  $R_3$  be to get  $V_{ref} = 0.6V$ ? Write one as an integer times the other. Bandgap  $\approx 1.2V$ .

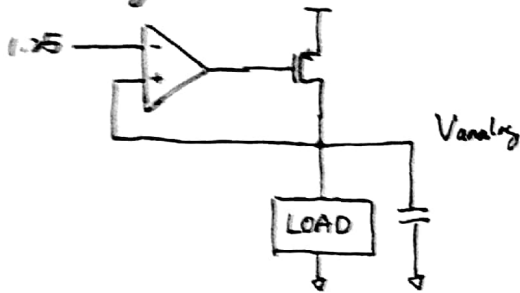
so  $R_3 = \frac{1}{2}R_2$  or  $\frac{R_3}{R_2} = \frac{1}{2}$

- 2 h) Give the minimum required supply voltage for this circuit if the mirror transistors have a  $V_{ov}$  of 200mV and  $V_{be2} = 0.65V$ .

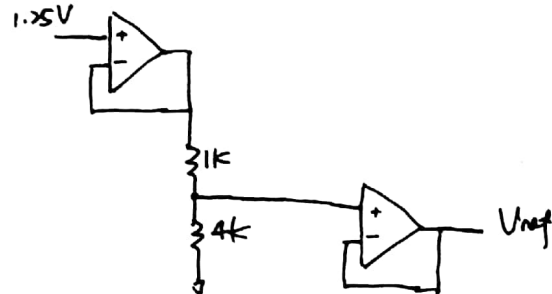
$0.85V$

Regulators. You decide not to use the fancy low-voltage bandgap, and go back to a traditional one.  
 a. [9] draw the schematic for the analog regulator (1.25V), digital regulator (1.5V), and ADC reference regulator (1V) using op-amp symbols, assuming that the bandgap is producing  $V_{BG}=1.25V$ . If you use resistors, clearly indicate what their ratios should be.

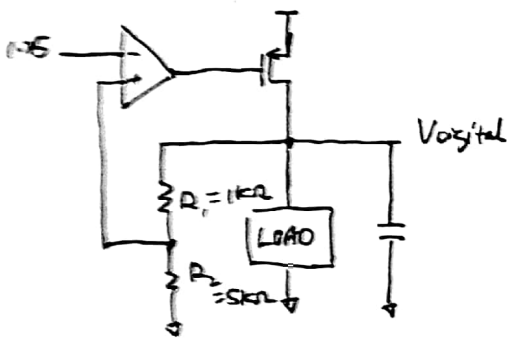
Analog



ADC reference:



Digital



$$V_{digital} \cdot \frac{R_2}{R_1 + R_2} = 1.25$$

(1.5)

$$R_1 = R_2 = 1:5$$

$$R_1 = 1k\Omega, R_2 = 5k\Omega$$

b. [5] for the digital regulator op-amp:

i. what is the common mode input?

1.25V

ii. what is the output swing needed?

$$V_{DD} - V_{th} - V_{ovp} \approx V_{DD} - V_{th}$$

iii. what is the voltage supply?

1.6V - 3.2V

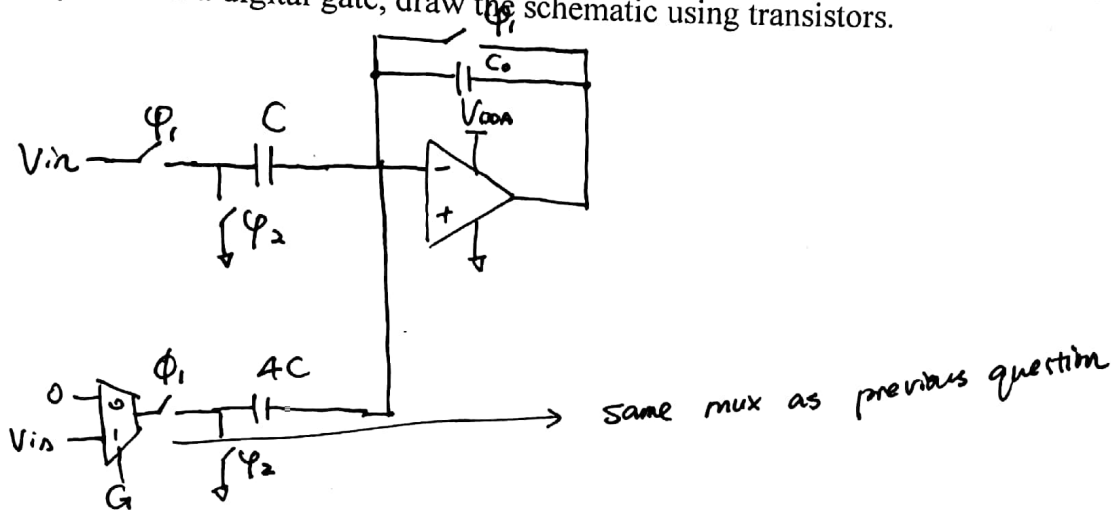
iv. which topologies will NOT work, and why?

- Phases input will not work



Gain of 1 or 5 PGA

- a. [10] draw the schematic using an op-amp symbol, transistors, and capacitors. Assume that the gain control bit  $G$  and two non-overlapping clock phases  $\phi_1$  and  $\phi_2$  are in the analog power domain ( $0, V_{DDA}$ ). If you need a digital gate, draw the schematic using transistors.

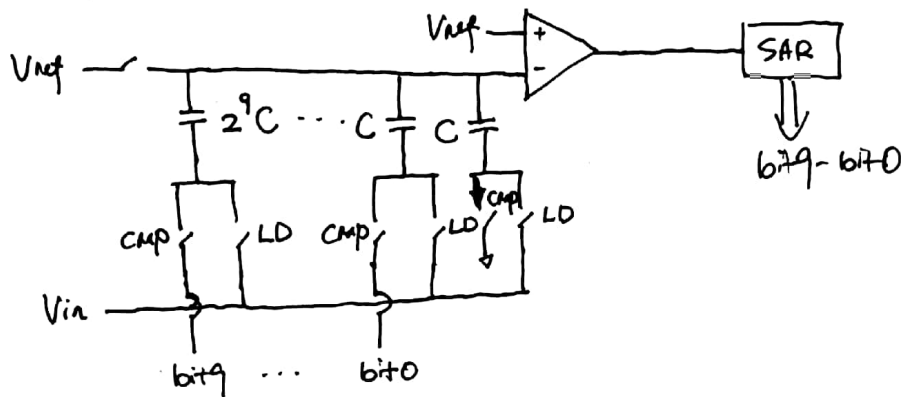


- b. [5] for the op-amp:

- i. what is the common mode input?  
0V
- ii. what is the output swing needed?  
0 -  $V_{DDA}$  or 0 -  $V_{DDref}$
- iii. what is the voltage supply  
 $V_{DDA}$
- iv. which topologies will NOT work, and why?
  - 1) NMOS input not work
  - 2) 5T
  - 3) without a output stages
  - ⋮

ADC.  $V_{in}=0$  to  $1V$ . The reference voltage is  $1V$ . The analog supply is  $1.25V$ .

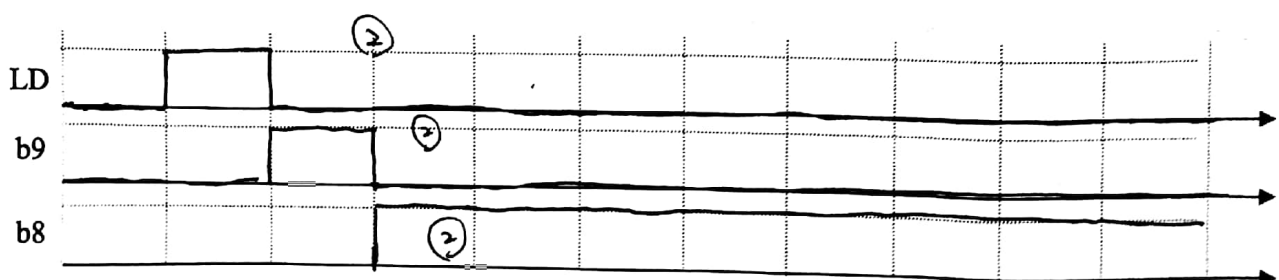
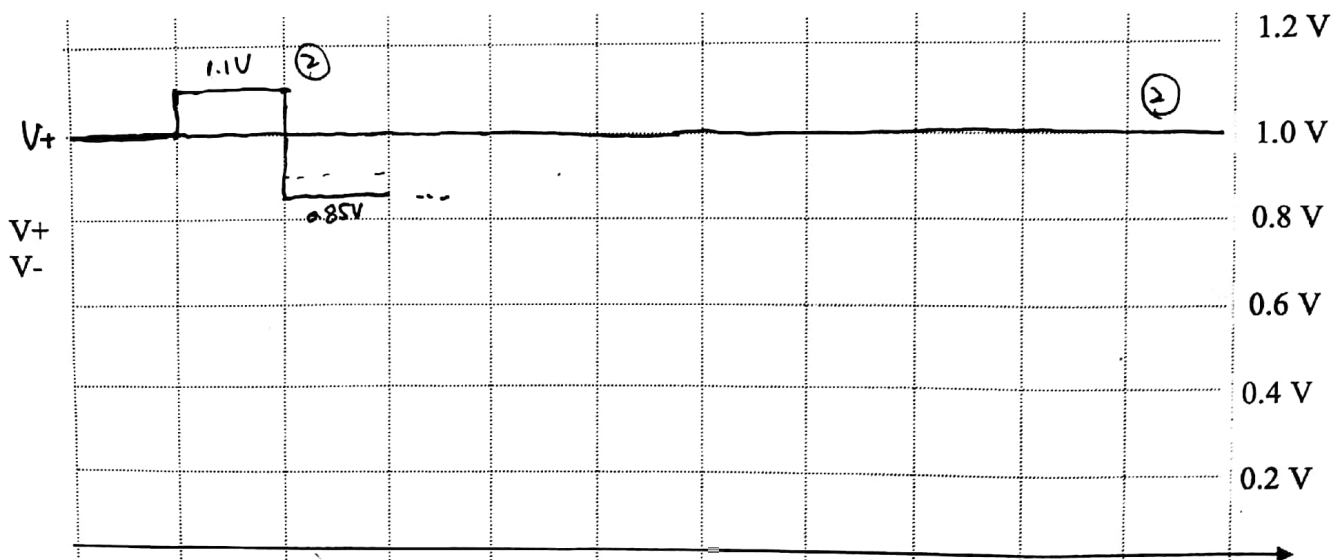
- a. [10] draw a schematic of the ADC using an op-amp symbol for the comparator, and transistors and capacitors. Don't draw anything for bits 1 through 8, but do draw bits 0 and 9. Assume that there is a 10 bit digital SAR controller supplied to you.



- b. [2] if the unit capacitance is  $2\text{ fF}$ , what is the total input capacitance of the ADC?

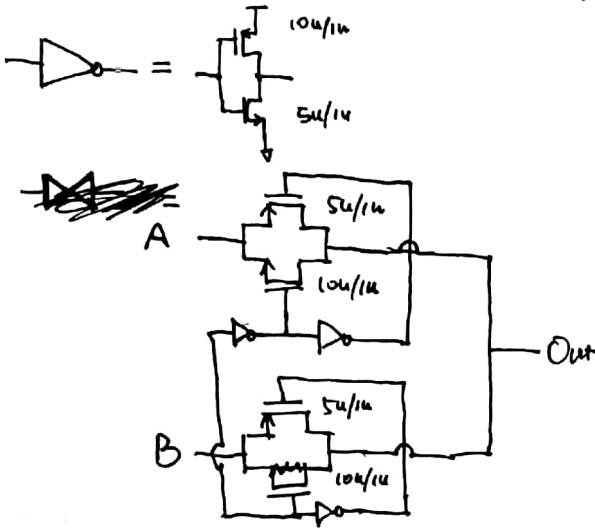
$$C = 2^{10} \cdot 2 = 2^{11} = 2048\text{ fF}$$

- c. [10] sketch the waveforms for the first two bit decisions ( $b_9$  and  $b_8$ ) for an input of  $0.4V$ . Specifically, sketch LD,  $b_9$  and  $b_8$  on the lower axes, and both  $V_+$  and  $V_-$  of the comparator on the upper axes. Clock edges correspond to the vertical dotted lines. LD goes high on the second rising edge.



2-input MUX. Inputs will be between 0 and 1V (not 0 and  $V_{DDA}$ )

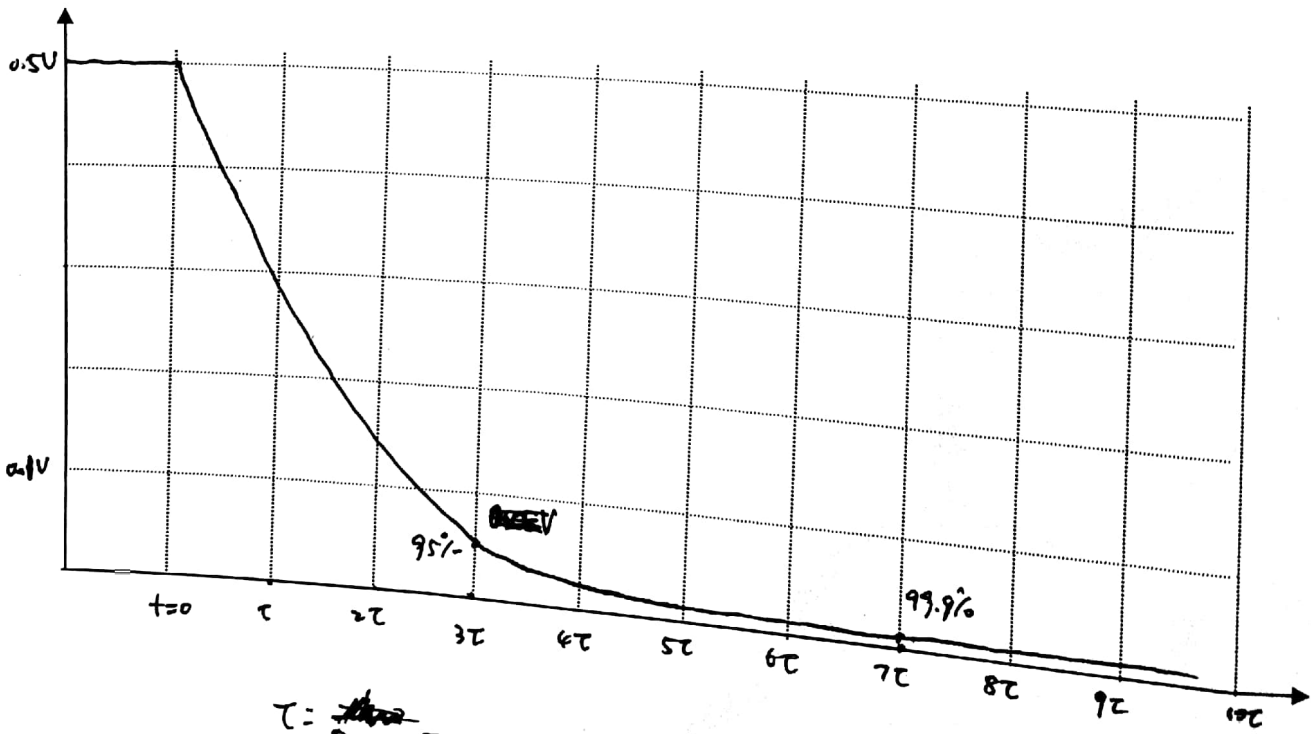
- a. [4] draw the schematic of the transistor implementation. Label W/L values. Assume that the control bit is in the analog power domain (0,  $V_{DDA}$ ). If you need a digital logic cell, draw the transistors schematic.



- b. [2] estimate the MUX "on" resistance for an input near 0V (I want an answer in Ohms, not a formula)

$$R_{on} = \frac{1}{\mu C_{ox} \frac{W}{L} (V_{DDA} - V_{TH})} = \frac{1}{200 \mu \times 5 \times 0.75V} = \frac{1}{750} K \approx 1.3 K\Omega$$

- c. [4] The MUX, with one input at 0V and one input at 0.5V, is driving a 1pF capacitor. Carefully sketch the voltage waveform when the input to the MUX select switches from the 0.5V input to the 0V input. Label the axes. Label the times when the settling accuracy is at 95%, and 99.9%



$$\tau = R_{on} C = 1.3K \cdot 1pF = 1.3ns$$