Homework Assignment #6
Due by online submission Saturday 3/14/2020 (late Sunday 9am)

1. Check out the datasheet for the K2-W tube op-amp. This op-amp, released in 1952, was the first production op-amp. It runs from a +/-300V supply, and has a bandwidth of 300kHz (or k-cycles/s, as they said back then – the unit Hertz not having been established yet). There’s a schematic on page 2. (You may want to draw the circuit with NMOS FETs instead of tubes to help you figure it out.) Pins 1, 2, and 6 on the bottom of the figure are V+, V-, and Vout. VR1 and VR2 are neon bulbs that provide a low impedance level shift of roughly 100V to center the output between the rails. Identify (circle and label)
   a. input differential pair
   b. diff-pair load resistor
   c. tail current resistor.
   d. Estimate the common mode gain and write it near the tail resistor.
   e. Common-cathode gain stage (like CS or CE)
   f. Cathode-follower output stage (like source-follower or emitter follower, CD, CC)
   g. Miller-multiplied compensation capacitor from the output back to the input of the gain stage.
   h. Bonus points if you can identify positive feedback in this amplifier, designed to increase the low-frequency gain (which ended up at about 20,000).

2. You have an opamp with a low-frequency gain of 1,000 and a single pole at 1 M rad/s. Plot the location of the pole as a function of the feedback factor f from f=0 to 1. Now with f=0.1
   a. Sketch the Bode plot of the closed-loop amplifier
   b. What is the fractional gain error at low frequency?
   c. What is the time constant of the step response? How does it compare to the open-loop time constant?
   d. What is the unity gain frequency? How does it compare to the open-loop unity gain frequency?

3. You now have an opamp with a low-frequency gain of 1,000 and three poles at 1 M rad/s.
   a. Plot the location of the three poles as a function of the feedback factor f.
   b. At the point where the poles cross the jw axis, annotate the plot with the value of f that gives this pole location.
   c. Using this value for f, draw a Bode plot of the loop gain Af

4. A two-stage CMOS op-amp running at a particular bias point has the following parameters:
   Gm1=1mS, Rm1=1MΩ, C1=0.1pF, Cc=0pF, Gm2=1mS, Rm2=100kΩ, C2=10pF.
   a. Plot the magnitude and phase of the overall gain of this uncompensated amplifier.
   b. Where are the poles of the uncompensated amplifier? Is it unity-gain stable?

5. For the same amplifier as above, we now add Cc=1pF. For this problem, you may ignore the RHP zero that this introduces. On the figures provided below,
   a. Plot the magnitude of the second stage gain vs. frequency
   b. Plot the magnitude of the input capacitance of the second stage (including Cc) vs. frequency
   c. Plot the magnitude of the input impedance of the second stage vs. frequency. Add a line for the output impedance of the first stage.
   d. Now plot the magnitude of the gain of the first stage on the top plot, and the magnitude of the overall gain of the amplifier
   e. What are the compensated poles of the amplifier? If Cc were 0, where would the poles of the amplifier be?

6. For our standard 2 stage NMOS-input CMOS op-amp (e.g. lecture notes W6L1P4LL) once the device sizes are picked the resistor sets the overdrive voltage in all of the transistors. If
   µnCox=200µA/V², µpCox=100µA/V², λ= 1/(10V), -Vip=Vim=0.5V, VDD=2V, (W/L)1=100, (W/L)2=200, (W/L)3=400, and (W/L)3,5,6=200
a. What is the reference current $I_{ref}$ necessary for each of the following bias conditions? You may assume $\lambda = 0$ to make these calculations easier
   i. $v_{ov} = 100\text{mV}$
   ii. $v_{ov} = 500\text{mV}$
   iii. $v_{ov} = -80\text{mV}$ (assuming that subthreshold and inversion currents are equal when $v_{ov} = 10\text{mV}$, and $n=1.5$)

b. What are the bias resistor values needed to produce the bias conditions above?

c. Looking back at HW3 problem 3, on a single plot sketch the gain and bandwidth of this amplifier vs. bias resistor value

7. You have three op-amp topologies: single stage active load (our standard 5 transistor opamp), the two stage version of that, and the current mirror op-amp. Each topology can either have NMOS or PMOS inputs, for six different op-amps. Sketch the output swing vs. common mode input range for each of the versions.

8. Figure 6.15 in Razavi is a model of a two-stage amplifier. [For ee247A students: Fig 9.18 in GHLM, and equations 9.27 and 9.33 for parts b and c]
   a. re-draw it using our terminology from class: $G_{m1}, G_{m2}, R_{o1}, R_{o2}, C_1, C_2, C_c$.
   b. Equation 6.30 is the transfer function of the amplifier. Re-write that with our terminology.
   c. Equation 6.39 is the simplified expression for the 2nd pole location, assuming the first pole is given by Miller-multiplied $C_c$.
      i. Re-write that with our terminology
      ii. Assuming that the 2nd stage gain is much larger than 1, the Miller capacitance is all that matters in the compensated first stage pole $\omega_{p1,c}$, write the expression for the compensated second stage pole $\omega_{p2,c}$ in terms of only capacitors and the transconductance of the second stage.
      iii. With those same assumptions, and ignoring any other poles and zeros, what is the constraint on transconductance and capacitance that insures a unity gain phase margin of at least 45?

9. [not graded] Estimate the output resistance of a CMOS differential amplifier with current mirror load without making the virtual ground assumption. You may assume that $g_m R_o >> 1$ for all combinations of $g_m$ and $R_o$. The following steps may help.
   a. Estimate the impedance seen looking into the source of M1A
   b. Estimate the impedance seen looking down from the source of M1B
   c. Estimate the impedance seen looking into the drain of M1B
   d. For the $R_o$ calculation, estimate $i_{d1B}$ as a function of $v_o$.
   e. The current in $i_{d2B}$ is due to both the output resistance and the mirrored current. Estimate both parts.
   f. Estimate the total output current $i_o = i_{d1B} + i_{d2B}$
   g. Show that $R_o$ is equal to $\left( R_{o1B} || R_{o2B} \right)$. Magic!

10. [Not graded] A single-stage op-amp has a low frequency gain of 200 and a dominant pole at 10Mrad/sec.
   a. Draw the s-plane with the real axis from $-10^7$ to 0, and the imaginary axis from 0 to $10^7$. Mark the pole location with an x, and draw a dot at $10^7j$.
   b. Draw the vector from the pole to $10^7j$. Calculate the magnitude and phase of this vector.
   c. Draw a dot at $10^6j$. Draw the vector from the pole to $10^6j$. Calculate the magnitude and phase of this vector.
   d. Repeat parts a. and b., but with the imaginary axis from 0 to $10^8$ and the dot at $10^8j$.
   e. Draw a Bode plot of the gain of your amplifier, with frequency running from $10^5$ to $10^9\text{rad/s}$. Use the straight-line approximations for the Bode plot, and then add dots showing the results of parts b, c, and d.

11. [not graded] For a standard 5 transistor CMOS differential amplifier show that the gain from a differential input to the (so called virtual ground!) tail voltage is 1/4. You can assume that $g_m R_o >> 1$ for all combinations of $g_m$ and $R_o$. You can win bets with experienced IC designers with this knowledge!
Second stage gain – $|A_{v2}|$, and first stage and overall gains

| $|A_{v2}|$ | $|A_{v2}|$ |
|-----------|-----------|
| 100       | 1         |
| 1         |           |

magnitude of second stage input (Miller) capacitance

| $|Z_{in2}|$ | $|Z_{in2}|$ |
|-----------|-----------|
| 10pF      | 1pF       |
|           | 1         |
| 1         | 1k        | 1M | 1G | rad/s |

second stage input impedance, and $R_{o1}$

| $|Z_{in2}|$ | $|Z_{in2}|$ |
|-----------|-----------|
| 1M        | 100k      |
| 10k       | 1k        |
|           |           | 1 | 1k | 1M | 1G | rad/s |