LTI ODE

steady state

transient

zeros

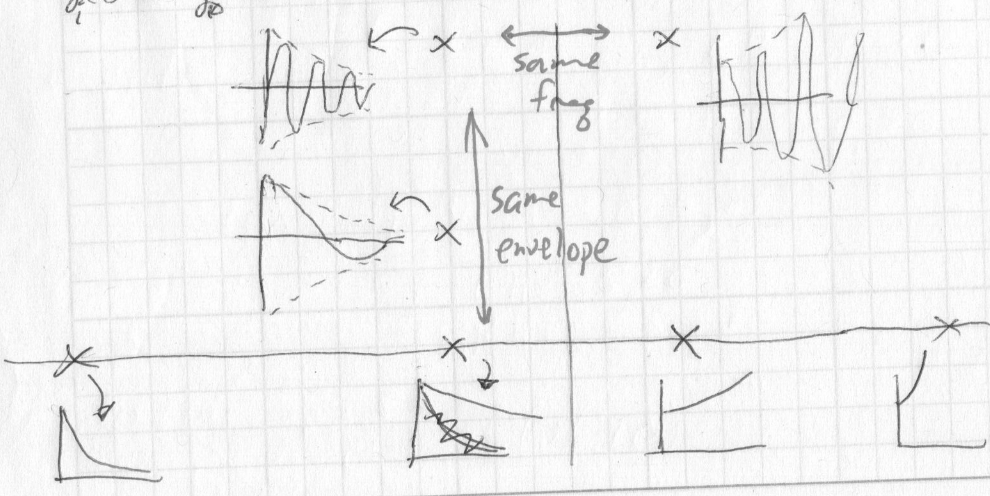
1, 2, 3 poles in feedback

externally compensated op-amps

poles are the natural response $p_i = \sigma + j0$

$p_{i,i+1} = \sigma \pm j\omega$

$y_i(t) \propto y_0 e^{p_i t}$



Linear Time Invariant Ordinary Differential Equ.
(circuits, physics, biology, economics, ...)

$$u \rightarrow \boxed{\text{LTI ODE} \quad \sum a_i \frac{d^{n-i} y}{dt^{n-i}} = \sum b_i \frac{d^{m-i} u}{dt^{m-i}}} \rightarrow y$$

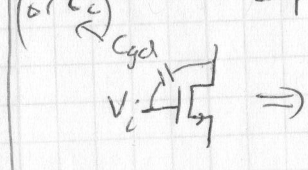
$\Downarrow$ Heaviside

$$u(t) = \sum A_i \sin(\omega_i t) \rightarrow \boxed{H(s)} \rightarrow y(t) = \sum A_i |H(j\omega_i)| \sin(\omega_i t + \angle H(j\omega_i))$$

steady state response

zeros: - at least 1 derivative of input in ODE

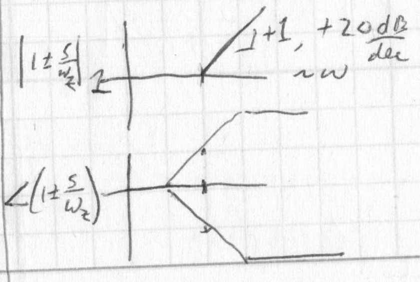
- 2 paths ~~that~~ from input to output

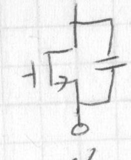


$G_m = g_m - sC_c$

$= g_m \left(1 - \frac{s}{\omega_z}\right)$

RHP zero vs.





$G_m = -g_m - sC_c$

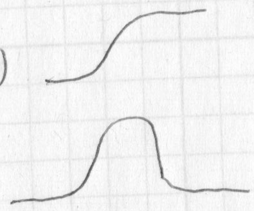
$= -g_m \left(1 + \frac{s}{\omega_z}\right)$

LHP

$$\sum_{i} a_i \frac{d^i y}{dt^{n-1}} = u + a\frac{du}{dt}$$

(b)

$u(t)$

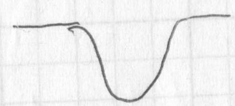


$\frac{du}{dt}$

tends to speed up response

what about RHP?

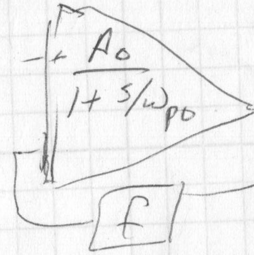
$-\frac{du}{dt}$

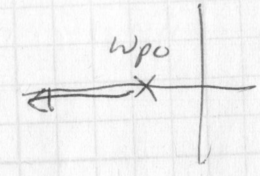


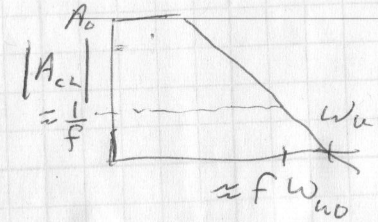
can start response in opposite direction!

Poles move in feedback

1 pole







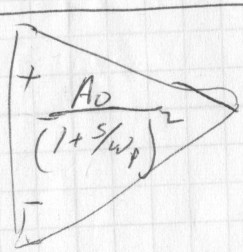
closed loop $A_{CL} = \frac{A_0}{1+A_0 f}$

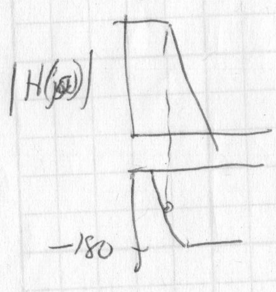
$$\omega_{pCL} = (1+A_0 f)\,\omega_{p0}$$

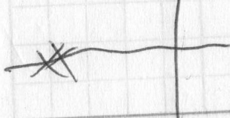
$$\omega_{u,CL} = A_{CL}\,\omega_{pCL} = A_0\omega_{p0} = \omega_{u0}$$

faster, lower gain

2 poles co-located



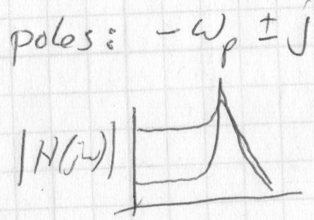


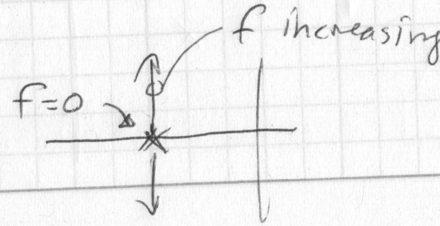


closed loop:

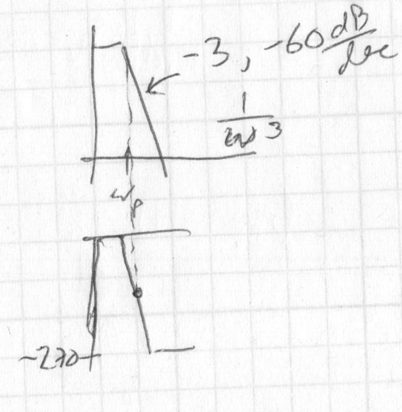
$$\frac{A_0}{1+A_0 f} = \frac{A_0}{\left(1+\frac{s}{\omega_p}\right)^2 + A_0 f}$$

poles: $-\omega_p \pm j\omega_p\sqrt{A_0 f}$





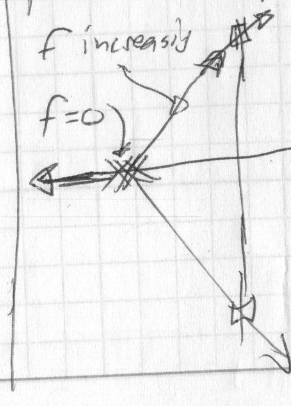
3 poles co-located $\frac{A_0}{(1+s/\omega_p)^3}$



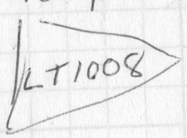
closed loop

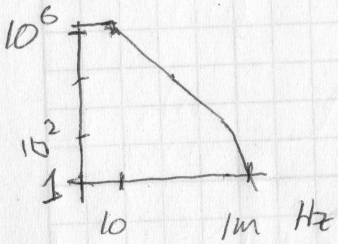
$$\frac{A_0}{1+A_0 f} = \frac{A_0}{\left(1+\frac{s}{\omega_p}\right)^3 + A_0 f}$$

poles: $s = -\omega_p + \sqrt[3]{A_0 f}\,\omega_p\, e^{j(\pm 60, -180)}$

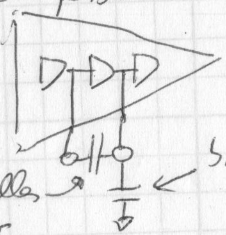


Uncompensated





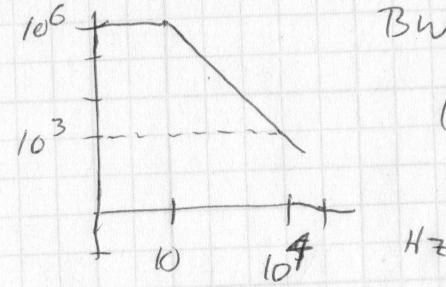
Compensation

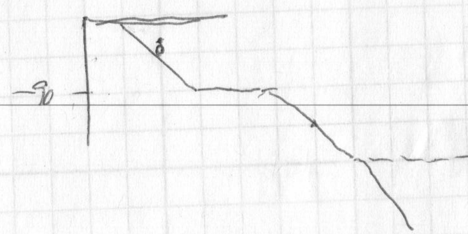


Uncompensated, $f = 10^{-3} \Rightarrow A_{VCL} \approx \frac{1}{f} = 10^{3}$

$BW \approx 10^{4}$ Hz

$(Gain)(BW) = 10^{3}10^{4} = 10^{7}$





Compensated $C_f = 30pf$ $f = 1$

$A_{VCL} \cong \frac{1}{f} = 1$

$BW = 1 MHz$

$(Gain)(BW) = (1)(10^6) = 10^6 MHz$