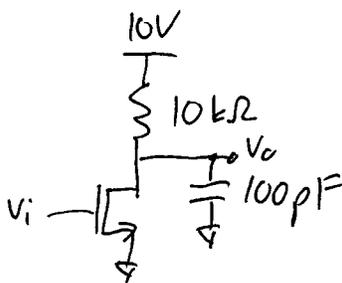


# EE140 HW3 Solutions

(1)



$$\mu C_{ox} = \frac{20 \mu A}{V^2}$$

$$\frac{W}{L} = 10^4$$

$$V_{th} = 1V$$

$$\lambda = 0.01 V^{-1}$$

$$a) I_D = \frac{V_{DD} - V_o}{R}$$

as  $V_o$  goes from 9V to 1V:

$$I_D = \frac{10 - 9}{10k} = 0.1 mA$$

0.1 mA to 0.9 mA

$$I_D = \frac{10 - 1}{10k} = 0.9 mA$$

$$b) I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} (V_{GS} - V_t)^2$$

$$@ I_D = 0.1 mA = \frac{20 \mu A}{2 V^2} 10^4 (V_{GS} - 1)^2 \quad V_{GS} = 1.032 mV$$

$$V_{ov} = 32 mV$$

$$@ I_D = 0.9 mA = \frac{20 \mu A}{2 V^2} 10^4 (V_{GS} - 1)^2 \quad V_{GS} = 1.095 V$$

$$V_{ov} = 95 mV$$

$$\Delta V_{ov} = 63 mV$$

$$c) g_m = \sqrt{2 \mu C_{ox} \frac{W}{L} (V_{DD} - V_o)}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{R}{\lambda (V_{DD} - V_o)}$$

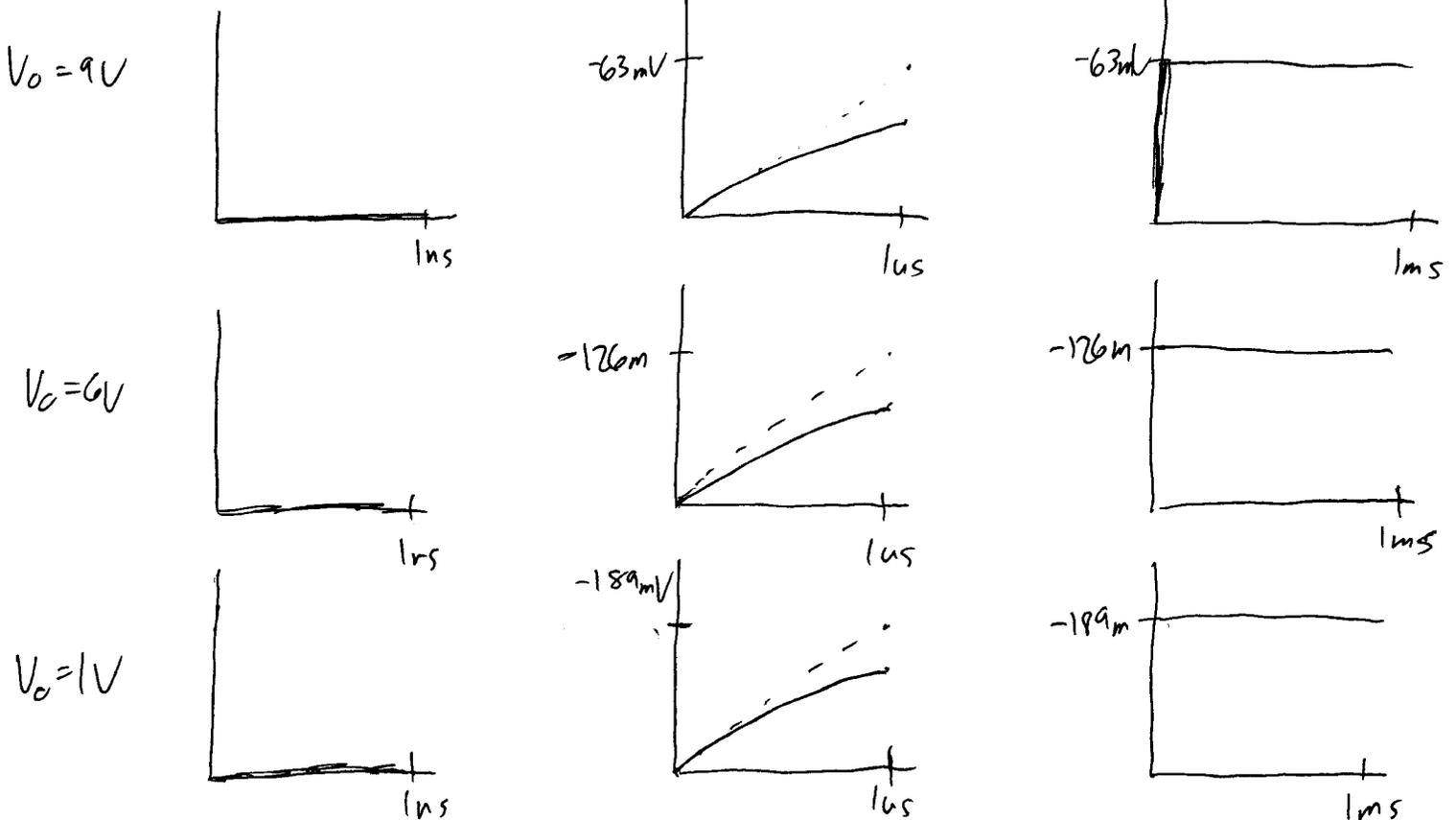
$$d) A_{vo} = -g_m r_o = \frac{2}{\lambda V_{ov}} \quad V_{ov} = \sqrt{\frac{2I_D}{\mu C_{ox} \frac{W}{L}}}$$

$$A_{vo} = \frac{2}{\lambda \sqrt{\frac{2I_D}{\mu C_{ox} \frac{W}{L}}}} = \frac{2}{\lambda \sqrt{\frac{2(V_{DD} - V_o)}{\mu C_{ox} \frac{W}{L} R}}}$$

e)

	$I_D$	$g_m$	$r_o$	$A_{vo}$	$\omega_p$	$\omega_u$
$V_o = 9V$	100 $\mu A$	6.3 mS	1 M $\Omega$	63 $\frac{V}{V}$	1 Mrad/s	63 Mrad/s
$V_o = 6V$	400 $\mu A$	12.6 mS	250 k $\Omega$	126 $\frac{V}{V}$	1 Mrad/s	126 Mrad/s
$V_o = 1V$	900 $\mu A$	18.9 mS	111 k $\Omega$	189 $\frac{V}{V}$	1 Mrad/s	189 Mrad/s

f)  $\tau = 1 \mu s$ ,



(2)



$$\lambda = 0.1 \text{ V}^{-1}$$

$$a) R_L = r_o = \frac{1}{\lambda I_D} = \frac{1}{\lambda \frac{(V_{DD} - V_c)}{R_L}}$$

$$\frac{10}{V_{DD} - 1} = 1 \quad \boxed{V_{DD} = 11 \text{ V}}$$

b) If  $V_{DD} = 2 \text{ V}$  and  $V_{o,DC} = 1 \text{ V}$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.1 \frac{(2-1)}{R_L}} = 10 R_L$$

So we should approximate that  $R_o = R_L$ .

$$10 R_L // R_L = \frac{10 R_L}{11} \approx 0.9 R_L \quad \text{Approximation is off } \sim 10\%$$

$$c) A_v = -g_m R_L$$

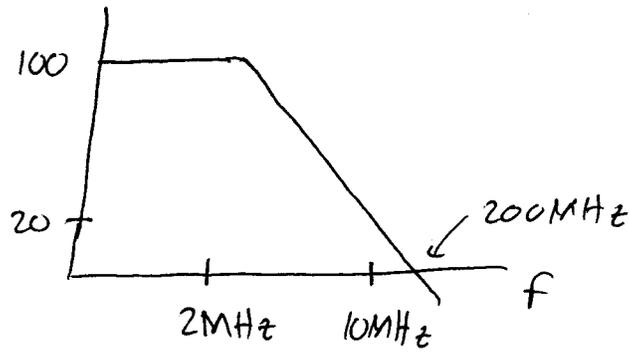
$$g_m = \frac{2 I_D}{V_{ov}}$$

$$I_D = \frac{2-1}{R_L} = \frac{1}{R_L}$$

$$A_v = \frac{-2}{V_{ov} R_L} \cdot R_L$$

$$\boxed{A_v = \frac{-2}{V_{ov}}}$$

(3)  $A_{vo} = 100$



$GBW = 10MHz \cdot 20 = 200MHz$

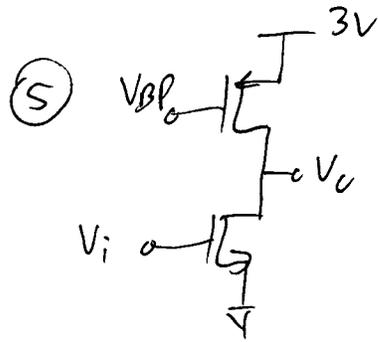
$f_p$  has gain of 100:  $\frac{200MHz}{100} = 2MHz$

$f_u$  has gain of 1:  $\frac{200MHz}{1} = 200MHz$

$f_p = 2MHz$   
 $f_u = 200MHz$

(4)

$A_{vo}$ [V/V]	$\omega_p$ [rad/s]	$\omega_u$ [rad/s]	$g_m$ [A/V]	$r_o$ [ $\Omega$ ]	$C_L$ [F]
100	1M	100M	$10^{-4}$	$10^6$	1p
200	10M	2G	$2 \cdot 10^{-3}$	100k	1p
100	10M	1G	$10^{-4}$	1M	100f
$10^6$	10	10M	$10^{-4}$	$10^{10}$	10p



$$\mu_{Cox} \frac{W}{L} = 1 \text{ mA/V}^2 \quad |V_T| = 1 \text{ V} \quad \lambda = 0.1 \text{ V}^{-1}$$

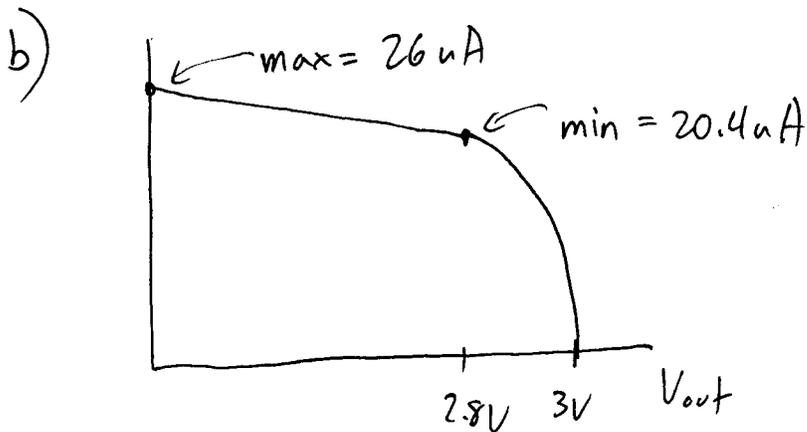
a)  $V_{BP} = 1.8 \text{ V}$        $V_{dsatp} = V_{GS} - V_T = 3 - 1.8 - 1 = 200 \text{ mV}$

$$V_{Dp} = 2.8 \text{ V}$$

$$I_{Dp} = \frac{\mu_{Cox}}{2} \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

$$I_{Dp} = \frac{1}{2} 1 \text{ mA/V}^2 (0.2 \text{ V})^2 \left(1 + \frac{0.1}{\text{V}} 0.2 \text{ V}\right) = \frac{1}{2} 0.04 \text{ m} (1 + 0.02)$$

$$I_{Dp} = 20.4 \text{ } \mu\text{A}$$

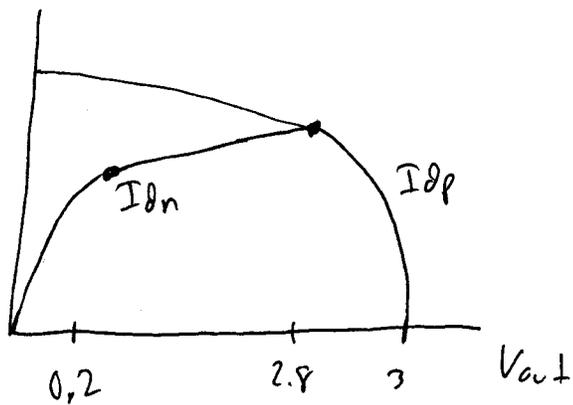


$$I_{Dp, \text{max}} = \frac{1}{2} \text{ mA/V}^2 (0.2)^2 \left(1 + \frac{0.1}{\text{V}} \times 3\right) = 26 \text{ } \mu\text{A}$$

c)  $I_D = 20.4 \text{ } \mu\text{A}$        $V_o = 2.8 \text{ V}$

$$I_D = \frac{1}{2} \frac{\text{mA}}{\text{V}^2} (V_i - V_T)^2 (1 + 0.1 (2.8)) \quad \underline{V_i = 1.18 \text{ V}}$$

c) cont)

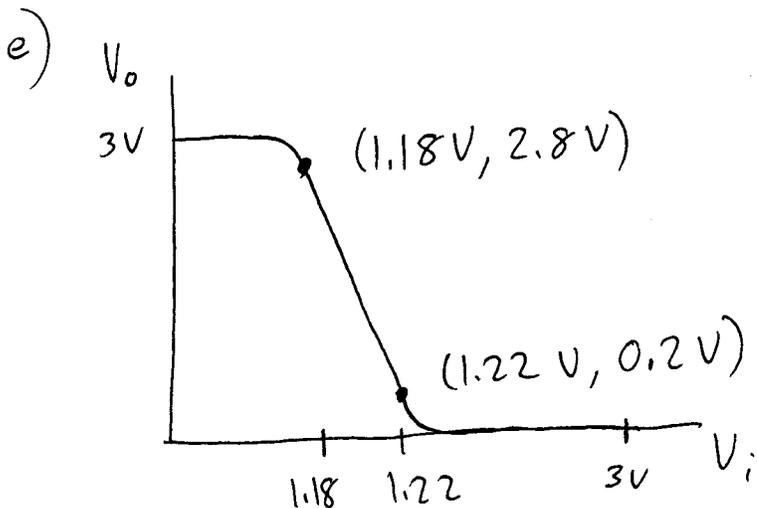
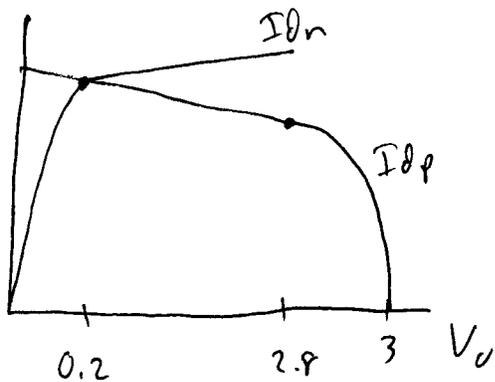


d) NMOS leaves saturation at 0.2V Vout

$I_{Dp}$  @  $V_{out} = 0.2V$  is:

$$\frac{1}{2} \frac{mA}{V^2} (0.2V)^2 (1 + 0.1(2.8)) = 25.6 \mu A$$

$$25.6 \mu A = \frac{1}{2} \frac{mA}{V^2} (V_i - V_t)^2 (1 + 0.1(0.2)) \quad \underline{V_i = 1.22 V}$$



$$f) A_v = \frac{\Delta V_o}{\Delta V_{in}} = \underline{2.6V} \quad \boxed{A_v = -65 \text{ V/V}}$$

output range is 0.2V to 2.8V (2.6V swing)

input range is 1.18V to 1.22V (40mV)

$$g) \text{ At } V_{o,DC} = 2.8V$$

$$g_m = \frac{2I_D}{V_{ov}} = \frac{2(20.4\mu A)}{1.18-1} = 230 \mu S$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.1(20.4\mu A)} = 245 k\Omega$$

$$\text{Or } A_v = -g_m r_o = \frac{-1}{\lambda V_{ov}} = \frac{-1}{(0.1)(0.18)} = -55.6 \text{ V/V}$$

$$\text{At } V_{o,DC} = 0.2V, I_D = 25.6\mu A$$

$$A_v = \frac{-1}{\lambda V_{ov}} = \frac{-1}{0.1(0.22)} = -45.5 \text{ V/V}$$

$$\text{At } V_{o,DC} = 1.5V, I_D = \frac{1}{2} \text{ mA/V}^2 (0.2V)^2 (1 + 0.1(1.5)) = 23\mu A$$

$$A_v = \frac{-1}{\lambda V_{ov}} = \frac{-1}{(0.1)(0.2)} = -50 \text{ V/V} \quad (\text{NMOS } V_{ov} \text{ must be } 0.2V)$$

In summary :

$V_{o,DC} = 2.8V$	$A_v = -55.6 \text{ V/V}$
$V_{o,DC} = 1.5V$	$A_v = -50 \text{ V/V}$
$V_{o,DC} = 0.2V$	$A_v = -45.5 \text{ V/V}$

### HW3 grading rubric

1) 28 pts total

1a) 2

1b) 4

1c) 2

1d) 2

1e) 9 (0.5 for each entry in the table)

1f) 9, 1 for each plot

2) 8 pts total

2a) 2 pts

2b) 4 pts; 1 for right answer, 1 for some reasoning, 2 for error calc

2c) 2 pts.

3) 4 pts, 2 for each frequency

4) 12 pts, 1 per blank

5) 33 pts total

5a) 4 pts

5b) 6 pts: 2 for plot, 2 each for min/max  $I_{dp}$

5c) 4 pts: 2 for  $V_i$ ; 2 for plot

5d) 4 pts: 2 for  $V_i$ ; 2 for plot

5e) 7 pts: 1 for "nmos off" region, 1 for pmos triode, 1 each for the X and Y location of the beginning of high gain, 1 each for the X and Y location of the end of high gain, 1 for nmos triode. The triode regions should be vaguely quadratic, and the high gain region should be a straight line.

5f) 2 pts

5g) 6 pts