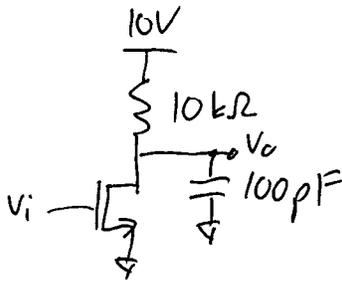


EE140 HW3 Solutions

In general, your answers don't need more than 1 sigfig

①



$$\mu C_{ox} = \frac{20 \mu A}{V^2}$$

$$\frac{W}{L} = 10^4$$

$$V_{th} = 1V$$

$$\lambda = 0.01 V^{-1}$$

$$a) I_D = \frac{V_{DD} - V_o}{R}$$

as V_o goes from 9V to 1V:

+1 for $I_D = f(V_o)$ equation

$$I_D = \frac{10 - 9}{10k} = 0.1 mA$$

0.1mA to 0.9mA

$$I_D = \frac{10 - 1}{10k} = 0.9 mA$$

+1 for I_D range

$$b) I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} (V_{GS} - V_t)^2$$

$$@ I_D = 0.1 mA = \frac{20 \mu A}{2 V^2} 10^4 (V_{GS} - 1)^2$$

$$V_{GS} = 1.032 mV$$

$$V_{ov} = 32 mV$$

+1 each for high and low V_{ov} and V_{GS}

$$@ I_D = 0.9 mA = \frac{20 \mu A}{2 V^2} 10^4 (V_{GS} - 1)^2$$

$$V_{GS} = 1.095 V$$

$$V_{ov} = 95 mV$$

$$\Delta V_{ov} = 63 mV$$

$$c) g_m = \sqrt{2 \mu C_{ox} \frac{W}{L} (V_{DD} - V_o)}$$

+1 for $g_m = f(V_o)$

$$r_o = \frac{1}{\lambda I_D} = \frac{R}{\lambda (V_{DD} - V_o)}$$

+1 for $r_o = f(V_o)$

$$d) A_{vo} = -g_m r_o = \frac{2}{\lambda V_{ov}} \quad V_{ov} = \sqrt{\frac{2I_D}{\mu C_{ox} \frac{W}{L}}}$$

$$A_{vo} = \frac{2}{\lambda \sqrt{\frac{2I_D}{\mu C_{ox} \frac{W}{L}}}} = \frac{2}{\lambda \sqrt{\frac{2(V_{DD}-V_o)}{\mu C_{ox} \frac{W}{L} R}}}$$

+1 for an Av equation and
+1 if it is a function of Vo

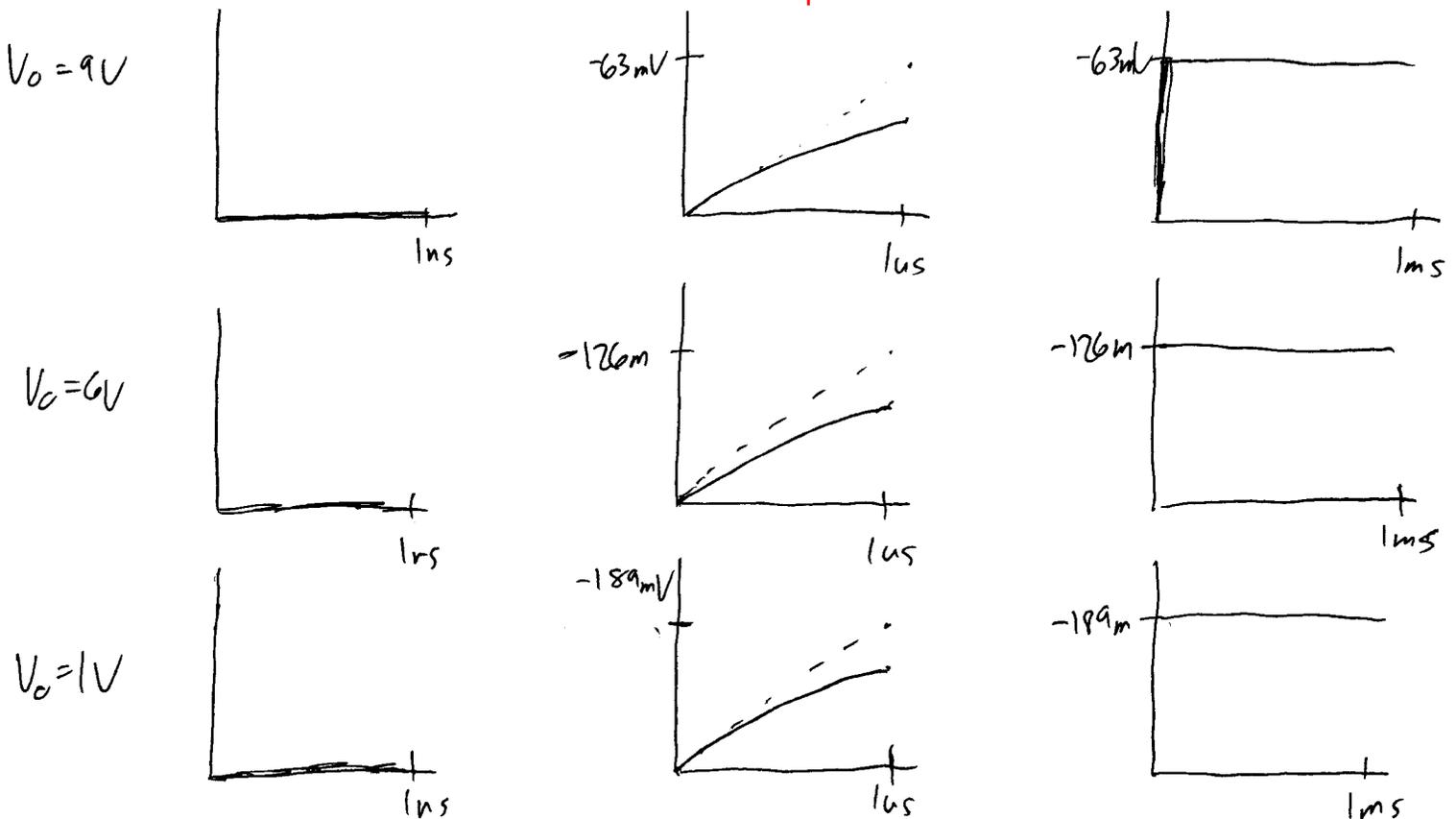
e)

	I_D	g_m	r_o	A_{vo}	ω_p	ω_u
$V_o = 9V$	100 μA	6.3 mS	1 M Ω	63 $\frac{V}{V}$	1 Mrad/s	63 Mrad/s
$V_o = 6V$	400 μA	12.6 mS	250 k Ω	126 $\frac{V}{V}$	1 Mrad/s	126 Mrad/s
$V_o = 1V$	900 μA	18.9 mS	111 k Ω	189 $\frac{V}{V}$	1 Mrad/s	189 Mrad/s

+0.5 per entry, within ~10-20% is fine, 1 sig fig is sufficient

f) $\tau = 1 \mu s$,

+1 per plot,
ns scale should be ~flat
us scale should look exponential and end at 0.63 of the final value
ms scale should look like a step



(2)



$$\lambda = 0.1 \text{ V}^{-1}$$

$$a) R_L = r_o = \frac{1}{\lambda I_D} = \frac{1}{\lambda (V_{DD} - V_c) / R_L}$$

+1 for equation
+1 for 11V as answer

$$\frac{10}{V_{DD} - 1} = 1 \quad \boxed{V_{DD} = 11 \text{ V}}$$

$$b) \text{ If } V_{DD} = 2 \text{ V and } V_{D,RL} = 1 \text{ V}$$

+1 for approx as $R_o = R_L$
+1 for some reasoning why
+2 for getting reasonably close to 10% error

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.1 \frac{(2-1)}{R_L}} = 10 R_L$$

So we should approximate that $R_o = R_L$.

$$10 R_L // R_L = \frac{10 R_L}{11} \approx 0.9 R_L$$

Approximation is off $\sim 10\%$

$$c) A_v = -g_m R_L$$

$$g_m = \frac{2 I_D}{V_{ov}}$$

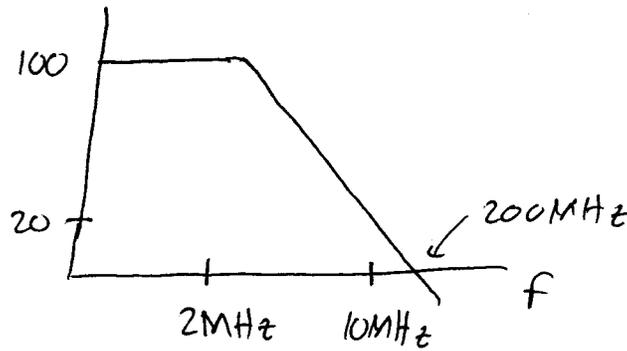
$$I_D = \frac{2-1}{R_L} = \frac{1}{R_L}$$

$$A_v = \frac{-2}{V_{ov} R_L} \cdot R_L$$

$$\boxed{A_v = \frac{-2}{V_{ov}}}$$

+1 for a gain equation
+1 for correct answer

(3) $A_{vo} = 100$



+2 for setting up equations for f_p and f_u
 +1 for correct f_p
 +1 for correct f_u

$GBW = 10MHz \cdot 20 = 200MHz$

f_p has gain of 100: $\frac{200MHz}{100} = 2MHz$

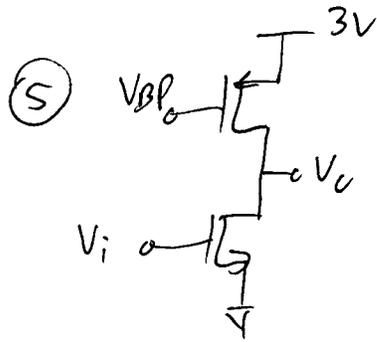
f_u has gain of 1: $\frac{200MHz}{1} = 200MHz$

$f_p = 2MHz$
 $f_u = 200MHz$

(4)

A_{vo} [V/V]	ω_p [rad/s]	ω_u [rad/s]	g_m [A/V]	r_o [Ω]	C_L [F]
100	1M	100M	10^{-4}	10^6	1p
200	10M	2G	$2 \cdot 10^{-3}$	100k	1p
100	10M	1G	10^{-4}	1M	100f
10^6	10	10M	10^{-4}	10^{10}	10p

+1 per blank (12 points total)
 1 sig fig, but answers should be exact



$$\mu_{Cox} \frac{W}{L} = 1 \text{ mA/V}^2 \quad |V_t| = 1 \text{ V} \quad \lambda = 0.1 \text{ V}^{-1}$$

a) $V_{BP} = 1.8 \text{ V}$ $V_{dsatp} = V_{GS} - V_t = 3 - 1.8 - 1 = 200 \text{ mV}$

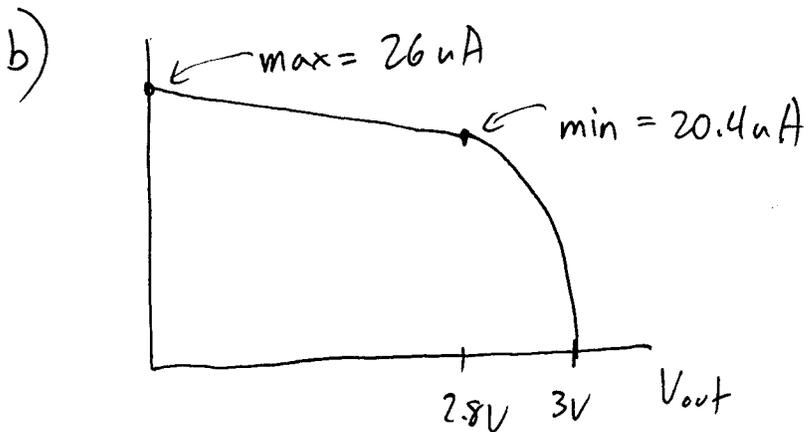
$$V_{Dp} = 2.8 \text{ V}$$

- +1 for Vdp equation
- +1 for Idp equation setup
- +1 for correct Vdp
- +1 for correct Id (don't need 0.1 uA accuracy)

$$I_{Dp} = \frac{\mu_{Cox}}{2} \frac{W}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

$$I_{Dp} = \frac{1}{2} 1 \text{ mA/V}^2 (0.2 \text{ V})^2 \left(1 + \frac{0.1}{\text{V}} 0.2 \text{ V}\right) = \frac{1}{2} 0.04 \text{ m} (1 + 0.02)$$

$$I_{Dp} = 20.4 \text{ } \mu\text{A}$$



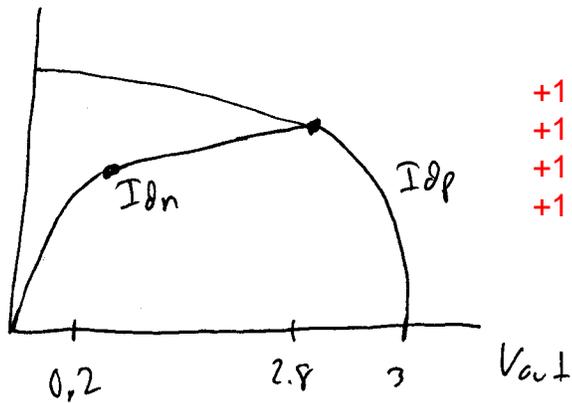
- +2 for correct plot shape, axis
- +2 for max current
- +2 for min current

$$I_{Dp, \text{max}} = \frac{1}{2} \text{ mA/V}^2 (0.2)^2 \left(1 + \frac{0.1}{\text{V}} \times 3\right) = 26 \text{ } \mu\text{A}$$

c) $I_d = 20.4 \text{ } \mu\text{A}$ $V_o = 2.8 \text{ V}$

$$I_d = \frac{1}{2} \frac{\text{mA}}{\text{V}^2} (V_i - V_t)^2 (1 + 0.1 (2.8)) \quad \underline{V_i = 1.18 \text{ V}}$$

c cont)



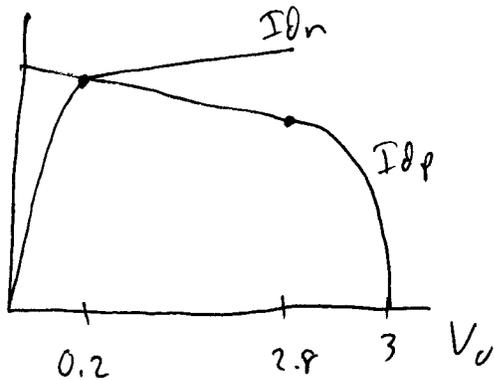
- +1 for equation to find V_i
- +1 for V_i within 0.1 V
- +1 for I_{dn} curve
- +1 for I_{dp} curve

d) NMOS leaves saturation at 0.2V V_{out}

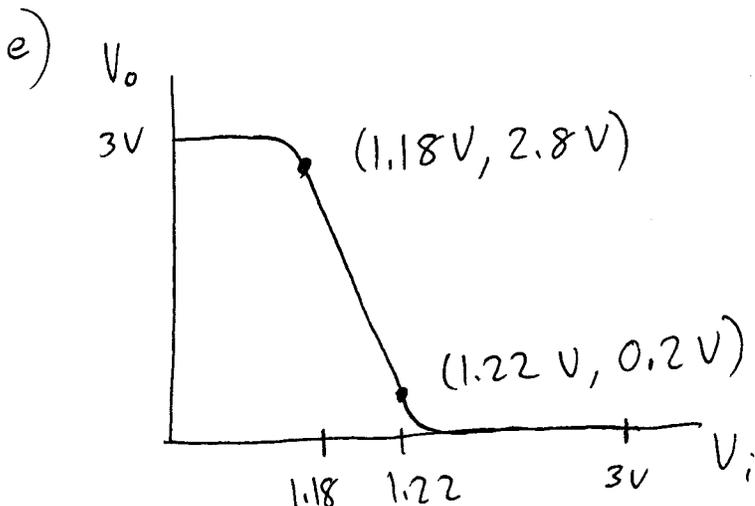
I_{dp} @ $V_{out} = 0.2V$ is:

$$\frac{1}{2} \frac{mA}{V^2} (0.2V)^2 (1 + 0.1(2.8)) = 25.6 \mu A$$

$$25.6 \mu A = \frac{1}{2} \frac{mA}{V^2} (V_i - V_t)^2 (1 + 0.1(0.2)) \quad \underline{V_i = 1.22 V}$$



- +1 for equation to find V_i
- +1 for V_i within 0.1 V
- +1 for I_{dn} curve (including finding the current at $V_{out} = 0.2V$)
- +1 for I_{dp} curve



- 7 pts total
- +1 nmos off and pmos off regions
- +1 nmos triode (roughly quadratic)
- +1 pmos triode (roughly quadratic)
- +2 for the high gain start (x,y) points
- +2 for the high gain end (x,y) points

$$f) A_v = \frac{\Delta V_o}{\Delta V_{in}} = \underline{2.6V} \quad \boxed{A_v = -65 \text{ V/V}}$$

output range is 0.2V to 2.8V (2.6V swing)

input range is 1.18V to 1.22V (40mV)

$$g) \text{ At } V_{o,dc} = 2.8V$$

+1 for gain estimate within 20% of -65 V/V
+1 for range (to within 0.1 V is fine)

$$g_m = \frac{2I_D}{V_{ov}} = \frac{2(20.4\mu A)}{1.18-1} = 230 \mu S$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.1(20.4\mu A)} = 245 k\Omega$$

$$\text{Or } A_v = -g_m r_o = \frac{-1}{\lambda V_{ov}} = \frac{-1}{(0.1)(0.18)} = -55.6 \text{ V/V}$$

$$\text{At } V_{o,dc} = 0.2V, I_D = 25.6\mu A$$

$$A_v = \frac{-1}{\lambda V_{ov}} = \frac{-1}{0.1(0.22)} = -45.5 \text{ V/V}$$

+1 for g_m, r_o at high V_o
+1 for gain at high V_o
+1 for g_m, r_o at mid V_o
+1 for gain at mid V_o
+1 for g_m, r_o at low V_o
+1 for gain at low V_o
Fewer sig figs OK

$$\text{At } V_{o,dc} = 1.5V, I_D = \frac{1}{2} \text{ mA/V}^2 (0.2V)^2 (1 + 0.1(1.5)) = 23 \mu A$$

$$A_v = \frac{-1}{\lambda V_{ov}} = \frac{-1}{(0.1)(0.2)} = -50 \text{ V/V} \quad (\text{NMOS } V_{ov} \text{ must be } 0.2V)$$

In summary:

$V_{o,dc} = 2.8V$	$A_v = -55.6 \text{ V/V}$
$V_{o,dc} = 1.5V$	$A_v = -50 \text{ V/V}$
$V_{o,dc} = 0.2V$	$A_v = -45.5 \text{ V/V}$

HW3 grading rubric

1) 28 pts total

1a) 2

1b) 4

1c) 2

1d) 2

1e) 9 (0.5 for each entry in the table)

1f) 9, 1 for each plot

2) 8 pts total

2a) 2 pts

2b) 4 pts; 1 for right answer, 1 for some reasoning, 2 for error calc

2c) 2 pts.

3) 4 pts, 2 for each frequency

4) 12 pts, 1 per blank

5) 33 pts total

5a) 4 pts

5b) 6 pts: 2 for plot, 2 each for min/max I_{dp}

5c) 4 pts: 2 for V_i ; 2 for plot

5d) 4 pts: 2 for V_i ; 2 for plot

5e) 7 pts: 1 for "nmos off" region, 1 for pmos triode, 1 each for the X and Y location of the beginning of high gain, 1 each for the X and Y location of the end of high gain, 1 for nmos triode. The triode regions should be vaguely quadratic, and the high gain region should be a straight line.

5f) 2 pts

5g) 6 pts