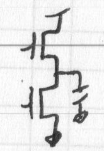
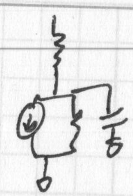
Linear Systems

Heavisido

poles & zeros



Non-linear system



linearized system

$\Rightarrow v_i = v_o + RC\frac{dv_o}{dt} \Rightarrow H(s)$, poles & zeros

ODE (KCL + algebra)

Laplace (Heavisido)

Every capacitor (or inductor) adds another $\frac{d}{dt}$ of v_o

and sometimes a $\frac{d}{dt}$ of v_i

n caps $\Rightarrow$ $\underbrace{\sum_{i=0}^{n-1} a_i \frac{d^i}{dt^i} v_o}_{\text{poles}} = \underbrace{\sum_{i=0}^{?} b_i \frac{d^i}{dt^i} v_i}_{\text{zeros}}$

Heavisido

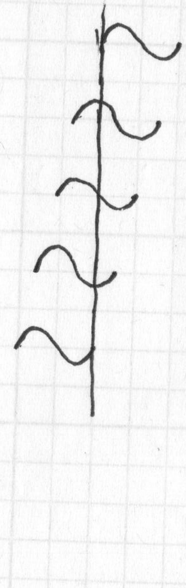
assume $v_o = V_o \sin(\omega t + \phi)$

$\frac{dv_o}{dt} = \omega V_o \cos(\omega t + \phi)$

$\frac{d^2 v_o}{dt^2} = -\omega^2 V_o \sin(\omega t + \phi)$

$\frac{d^3 v_o}{dt^3} = -\omega^3 V_o \cos(\omega t + \phi)$

$\frac{d^4 v_o}{dt^4} = \omega^4 V_o \sin(\omega t + \phi)$

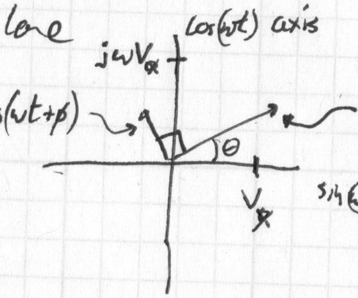


derivative = scales by ω

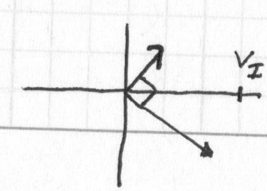
- 90° phase shift

represent a sine wave as a point in the complex plane

magnitude & phase



$\frac{V}{I}\sin(\omega t) = RC\omega V_o \cos(\omega t + \phi) + V_o \sin(\omega t + \phi)$



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$V_I \sin(\omega t) = RC \frac{d}{dt}\left(V_o \sin(\omega t+\phi)\right) + V_o \sin(\omega t+\phi)$

$V_I = RC\, j\omega\, V_o + V_o$

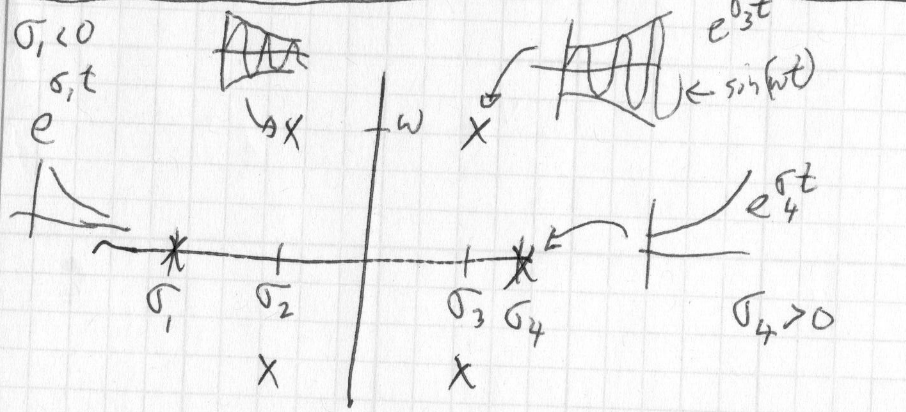
$\frac{1}{1+RCj\omega} = \frac{V_O}{V_I}$

$H(j\omega) = \frac{1}{1+RCj\omega}$

let $s = j\omega$ one fewer character

$H(s) = \frac{1}{1+sRL}$

__only__ care about $s = j\omega$



poles of $H(s)$

__but__ $H(s)$ function has some interesting structure

$H(s) = A\frac{\Sigma(s+z_i)}{\Sigma(s+p_i)}$

$\Sigma a_i \sin(\omega_i t+\phi_i) \rightarrow \boxed{H(s)} \rightarrow \Sigma |H(j\omega)|\, a_i \sin(\omega_i t+\phi_i + \angle H(j\omega_i))$