

**Homework Assignment #5 v1.1 solution**

Due by online submission **Wednesday 3/2/2016** (Thursday 9am)

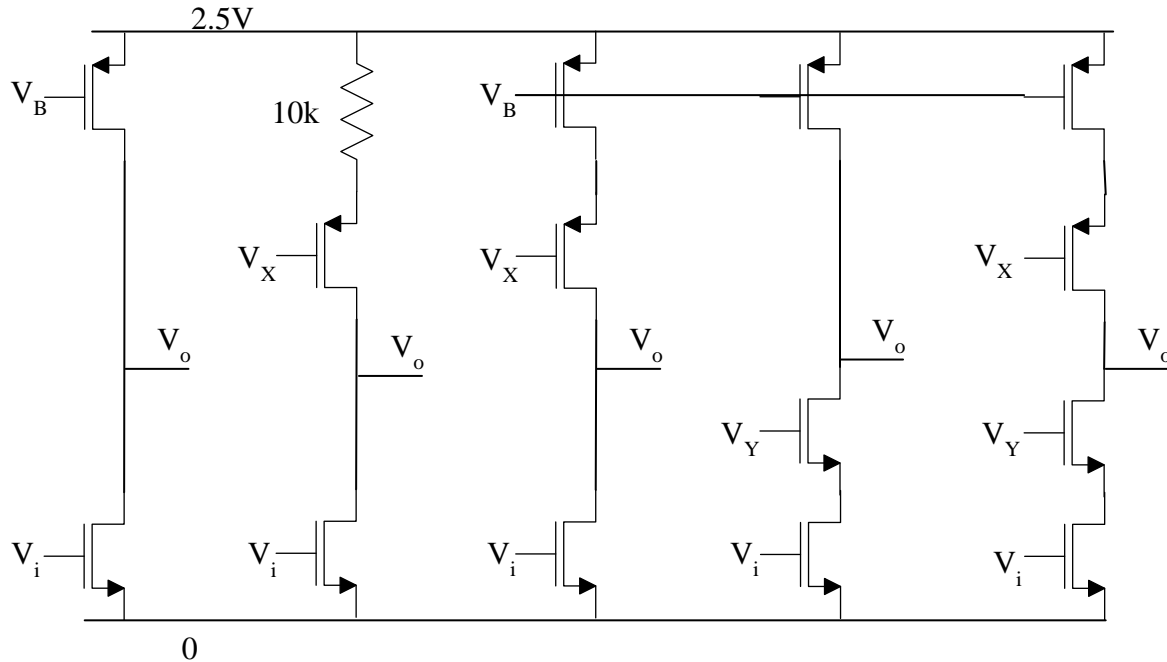
1. Check out the datasheet for the LM324 quad op-amp. <http://www.ti.com/lit/ds/symlink/lm324.pdf>  
TI has been selling this op-amp for more than 40 years! There's a schematic on page 10. Copy that schematic, and identify (circle and label)
  - a. input differential pair (Darlington)
  - b. current mirror active load
  - c. compensation capacitor
  - d. common collector level shifter (emitter follower)
  - e. common emitter amplifier (Darlington)
  - f. output stage

[1 pt each] Full credit as long as you included the correct transistors (you don't need to have circled the current sources for d and e, although technically that's the right answer). See scanned solution.
2. For the LM324, use the typical input bias current spec and the differential pair bias current from the schematic to estimate the beta of the input transistors.  

[4pts – 1 each for input bias, tail current, calculation, and effort] 6uA for the tail current means 3uA in each leg. The input bias current should be  $\beta^2$  times less (squared because of the Darlington configuration – each transistor multiplies base current by  $\beta$ ). Typical input bias current is 20nA, so  $\beta = \sqrt{3\mu\text{A}/20\text{nA}}$ , roughly 12.
3. Given a diode-connected NPN transistor Q1 which has a 1uA reference current flowing through it, design a bipolar circuit to generate all four of the current supplies shown in the LM324 schematic. Label your transistors as multiples of each other as appropriate, e.g. Q2 = 5 Q1.  

[8 pts. 2 for each current source. 1 for effort, 1 for getting the topology and device size right.] See scanned solution.

4. For each of the circuits below, assume that all transistors are biased in saturation with  $I_D=100\mu\text{A}$ ,  $V_{ov}=0.2\text{ V}$ , and  $\lambda=1/(5\text{ V})$ , and that for the NMOS devices,  $C_{gs}=10\text{fF}$ , and  $C_{gd}=2\text{fF}$ . Each amplifier is driving a  $200\text{fF}$  load. For each amplifier, calculate  $G_m$ ,  $R_o$ ,  $A_v$ ,  $\omega_p$ ,  $\omega_u$ ,  $C_{in}$ ,  $V_{X,max}$  and  $V_{Y,min}$  (if present), and the output swing  $V_{out,max}$  and  $V_{out,min}$ . Which amplifier has the highest gain? Lowest input capacitance? Widest output swing?



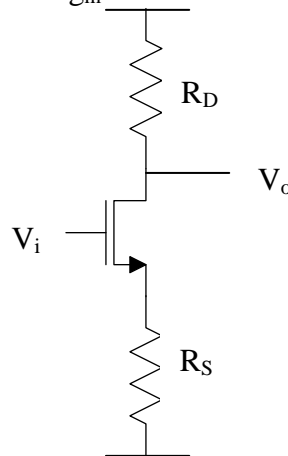
[25 pts - 5 pts for each column. Gotta get at least 5 right per column for full credit.]

	Simple CS	Source Resistor	Source Cascode	Cascode in	Cascode in/out
$G_m$	$g_{mn}=1\text{mS}$	$g_{mn}=1\text{mS}$	$g_{mn}=1\text{mS}$	$g_{mn}=1\text{mS}$	$g_{mn}=1\text{mS}$
$R_o$	$25\text{k}\Omega$	$r_{on}=50\text{k}\Omega$	$r_{on}=50\text{k}\Omega$	$r_{op}=50\text{k}\Omega$	$1.25\text{M}\Omega$
$A_v$	-25	-50	-50	-50	-1,250
$\omega_p$	200 Mrad/s	100 Mrad/s	100 Mrad/s	100 Mrad/s	4 Mrad/s
$\omega_u$	5 Grad/s	5 Grad/s	5 Grad/s	5 Grad/s	5 Grad/s
$C_{in}$	60 fF	110 fF	110 fF	14 fF	60 fF
$V_{X,max}$		$1.3- V_{tp} $	$2.1- V_{tp} $		$2.1- V_{tp} $
$V_{Y,min}$				$0.4+V_{tn}$	$0.4+V_{tn}$
$V_{out,max}$	2.3 V	1.3 V	2.1 V	2.3 V	2.1 V
$V_{out,min}$	0.2 V	0.2 V	0.2 V	0.4 V	0.4 V

“Cascode in/out” is highest gain of course, but relatively high input capacitance. “Cascode in” has the lowest input capacitance. “Simple CS” has the widest output swing.

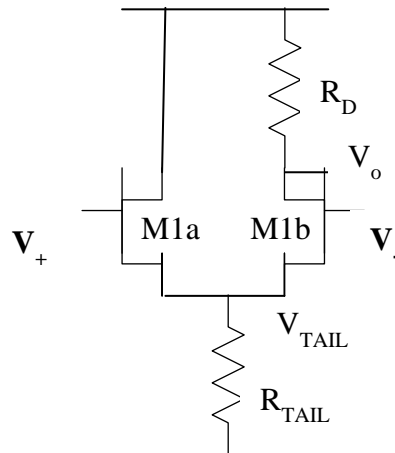
5. While keeping the same unity gain frequency, change the length of the transistors in the leftmost figure to get the same gain as the rightmost figure. What is the new input capacitance, compared to the input capacitance in the rightmost figure?  
 [8pts - 2 for getting L right, 2 each for  $C_{gs}$  and  $C_{gd}$ , and 2 for input capacitance. Half credit for honest attempt on each] To get the same gain as the cascode, we need to make L 50 times longer. To keep the same unity gain frequency, we need the same  $g_m$ . The simplest way to do that is to make W 50 times wider as well. So  $C_{gd}$  goes up by 50x (proportional to W) to  $100\text{fF}$ , and  $C_{gs}$  goes up by  $50^2$ , to  $25\text{pF}$ , since it goes as WL. The input capacitance is  $25\text{pF}+1,251*100\text{fF}=150\text{pF}$ , more than a thousand times more than the cascode!
6. For the amplifier in the figure below
- Draw the small signal model labeling the small signal variables  $v_i$ ,  $v_o$ ,  $i_o$ ,  $v_s$
  - Write an expression for  $G_m$  as the ratio of two small signal parameters while a third is held equal to zero.

- Write  $v_s$  in terms of  $i_o$
- Write KCL @  $v_s$  and solve for  $G_m$ .
- Find the approximate value for  $G_m$  and  $A_v$  for each of three different values of  $R_S$ : much less than  $1/g_m$ , equal to  $1/g_m$ , and much greater than  $1/g_m$ ?



[10 pts total. 2 for each part] See scanned solution

- For the differential amplifier in the figure below, estimate the change in  $V_{tail}$ ,  $I_{tail}$ ,  $I_{d1a}$ ,  $I_{d1b}$ , and  $V_o$  due to
  - An increase of  $\Delta V$  in both  $V_+$  and  $V_-$
  - An increase of  $\Delta V$  in just  $V_+$
  - An increase of  $\Delta V$  in just  $V_-$
  - What is the common mode rejection ratio of this amplifier?



[17 pts - 15 for the table (1 pt each) and 2 for part d] If you got the signs wrong just take off one point for the first one, not the rest. All results assume  $R_D \ll r_o$

	$V_{tail}$	$I_{tail}$	$I_{d1a}$	$I_{d1b}$	$V_o$
both	$\Delta V$	$\Delta V/R_{tail}$	$\Delta V/(2R_{tail})$	$\Delta V/(2R_{tail})$	$-\Delta V R_D/(2R_{tail})$
just $V_+$	$\Delta V/2$	$\Delta V/(2R_{tail})$	$+g_{m1a}\Delta V/2$	$-g_{m1b}\Delta V/2$	$+\Delta V g_{m1b}R_D/2$
just $V_-$	$\Delta V/2$	$\Delta V/(2R_{tail})$	$-g_{m1b}\Delta V/2$	$+g_{m1b}\Delta V/2$	$-\Delta V g_{m1b}R_D/2$

d) common mode gain is  $R_D/(2R_{tail})$ . Differential gain is  $g_{m1b}R_D/2$ . Common mode rejection ratio is differential over common, or  $g_{m1b}R_{tail}$