Grasping and Fixturing as Submodular Coverage Problems

John Schulman, Ken Goldberg, Pieter Abbeel

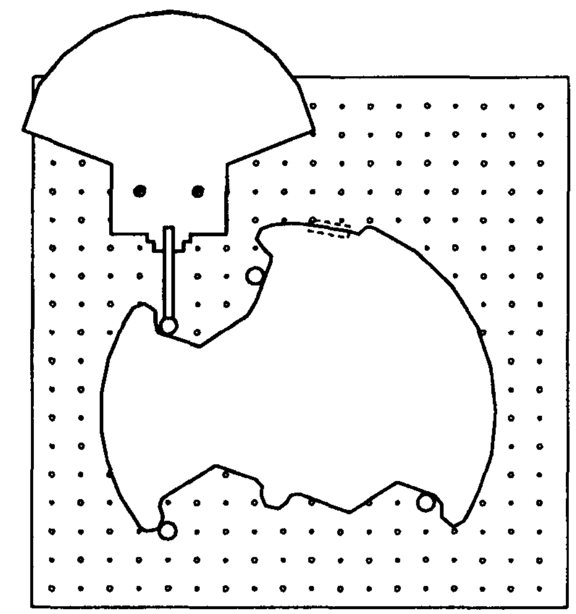
UC Berkeley

Optimal contact point selection

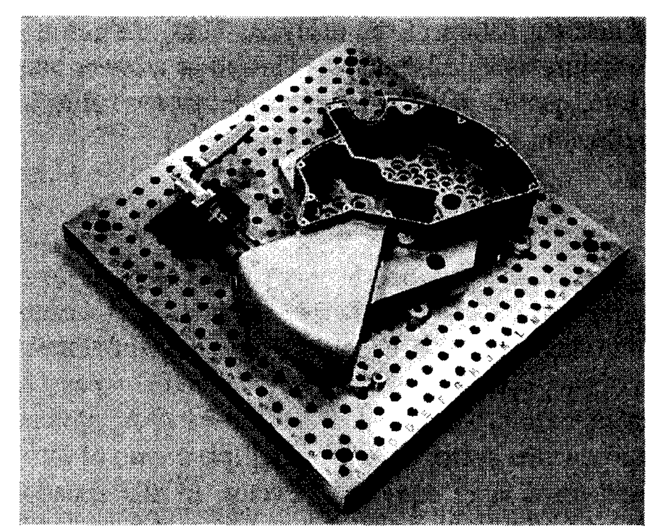
Grasping

http://robotics.naist.jp

Fixturing



(b)



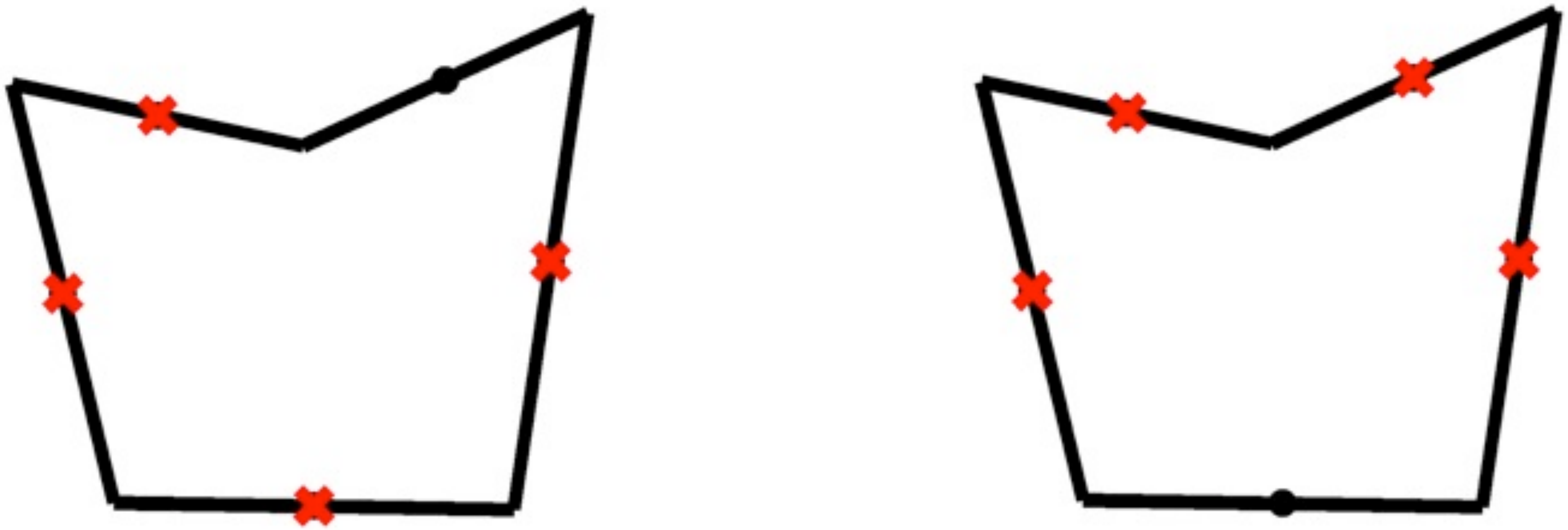
(c)

Brost & Goldberg, 1994

Outline

- Clarify how to quantify if a grasp is good
- Efficient algorithm to find good grasps
- Examples from grasping, fixturing and towing

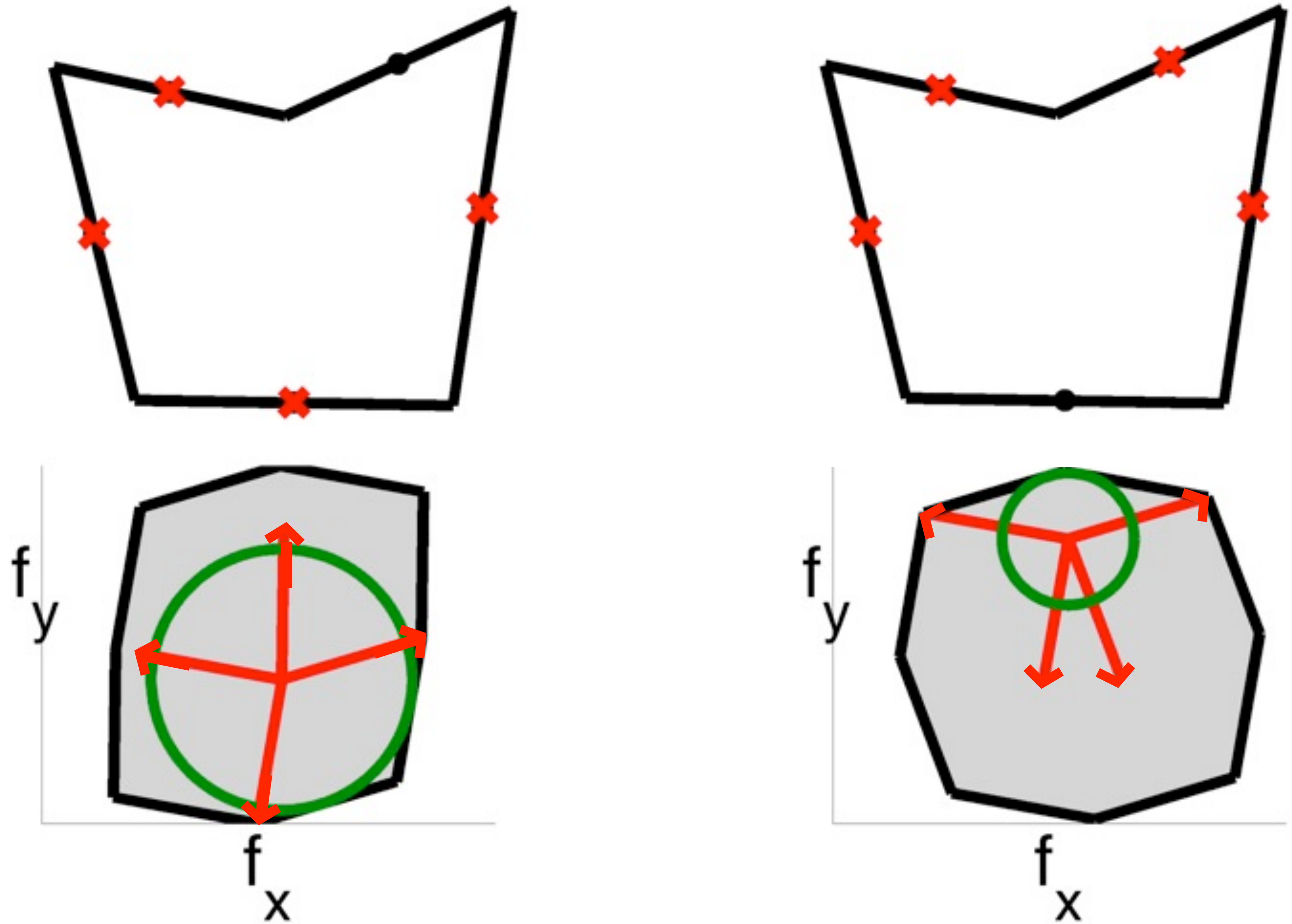
Quality metrics



Q_∞: size of smallest wrench that can't be resisted.

Schulman, Goldberg, Abbeel

Quality metrics

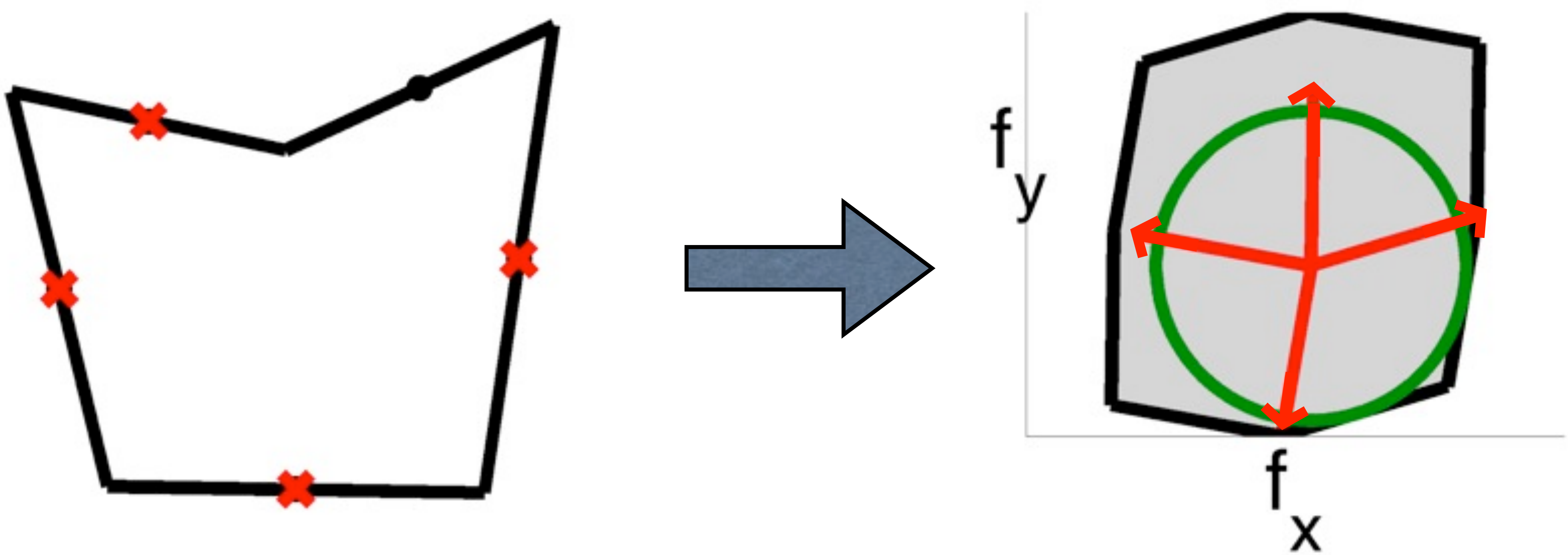


Q_∞: size of smallest wrench that can't be resisted.
residual radius of the set of achievable wrenches

Schulman, Goldberg, Abbeel

Problem formulation

Given a shape and a set of n candidate contact points, find the k-element subset that maximizes Q_∞



Schulman, Goldberg, Abbeel

Contributions

- Fast algorithm for selecting near-optimal contact points
  - New formulas for quality metrics: can evaluate with discretization
  - Exploiting submodularity to use SATURATE
  - Algorithm makes it practical to choose a large number of contact points

Related work

Foundations of force closure	Releaux (1876) Mishra, Schwartz, Sharir (1987)
Quality metrics	Ferrari, Canny (1991) Kirkpatrick, Mishra, Yap (1991)
Quality evaluation	Borst, Fischer, Hirzinger (1999)
Quality optimization	Teichman & Mishra (2000)
fixture design compliant grasps uncertainty dimensionality reduction clutter	Brost & Goldberg (1994) Burdick & Rimon (2000) Hsiao, Kaelbling, Lozano-Perez (2007) Ciocarlie, Goldfeder, Allen (2007) Berenson, Srinivasa, et. al (2007)

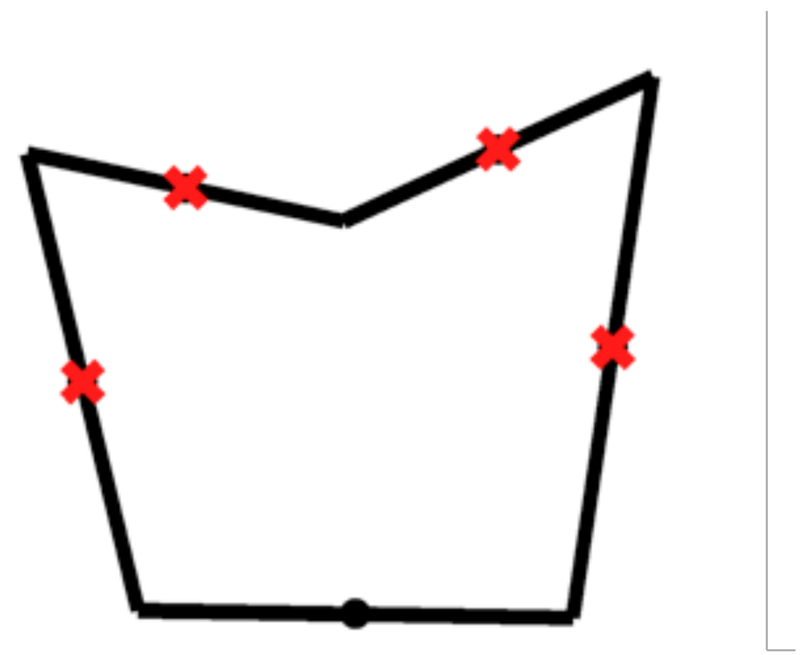
Naive search takes too long

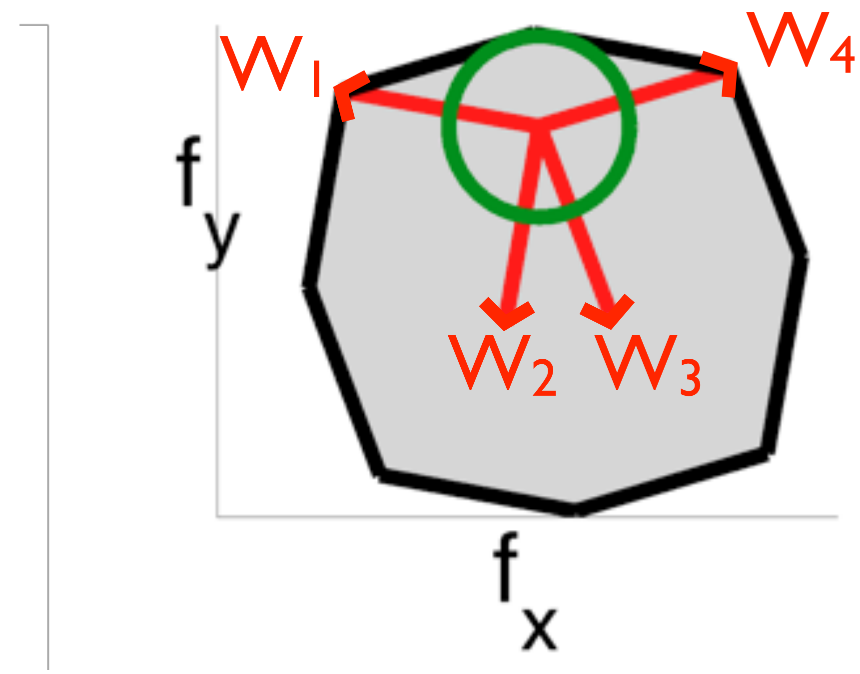
- $O(n^k)$ ways to choose k contacts from n candidates
- For each choice we must evaluate Q_∞

Problem reformulation

Original formulation

$r_{\text{res}}(\text{MinkowskiSum}(W_1, W_2, \ldots, W_k))$





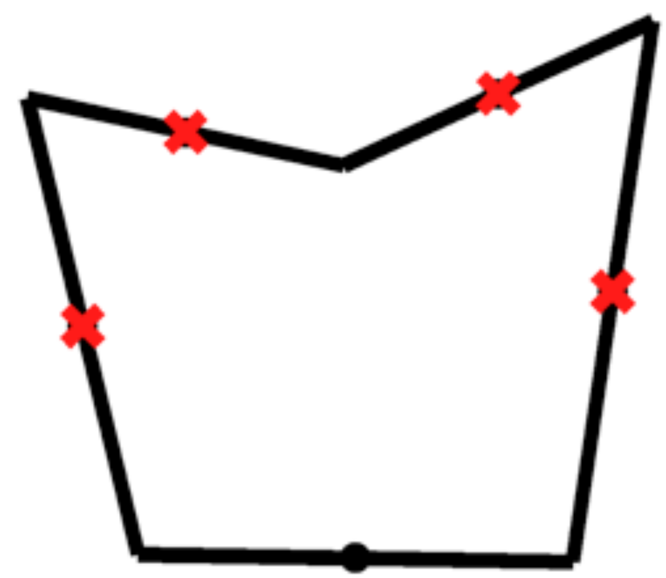
Problem reformulation

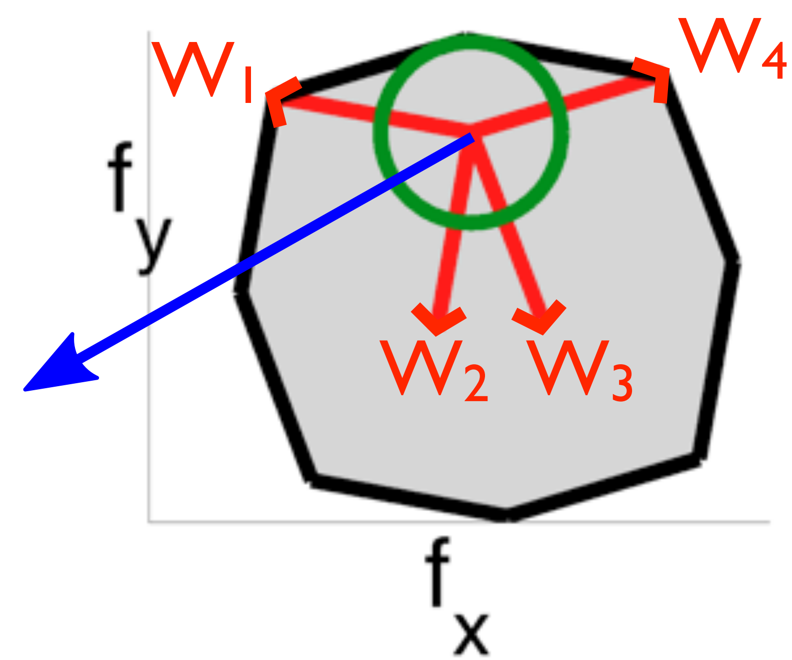
Original formulation

$$r_{\text{res}}(\text{MinkowskiSum}(W_1, W_2, \ldots, W_k))$$

Equivalent reformulation

$$\min_{\|y\|=1} \sum_{i \in S'} \sup_{\mathbf{x} \in W_i} \mathbf{x}^T \mathbf{y}$$





Schulman, Goldberg, Abbeel

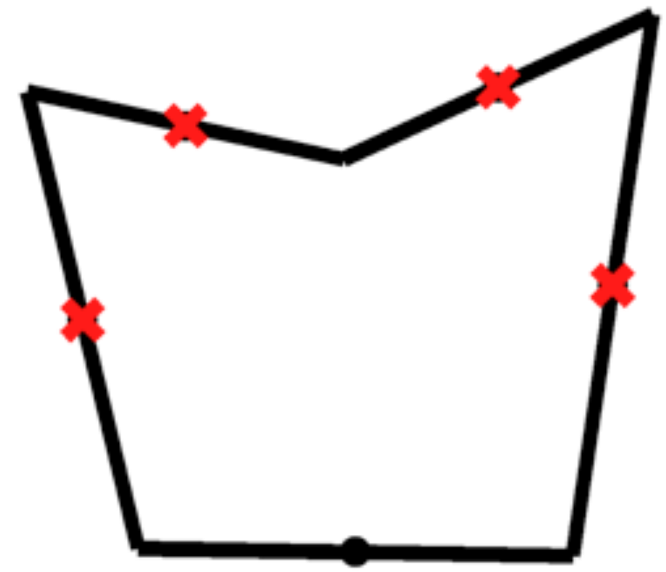
Problem reformulation

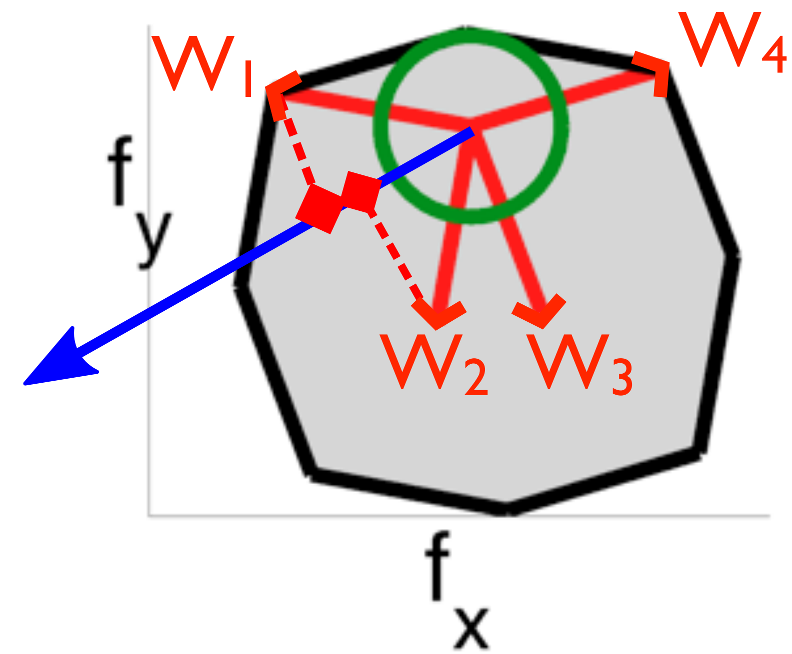
Original formulation

$$r_{\text{res}}(\text{MinkowskiSum}(W_1, W_2, \ldots, W_k))$$

Equivalent reformulation

$$\min_{\|y\|=1} \sum_{i \in S'} \sup_{\mathbf{x} \in W_i} \mathbf{x}^T \mathbf{y}$$





Schulman, Goldberg, Abbeel

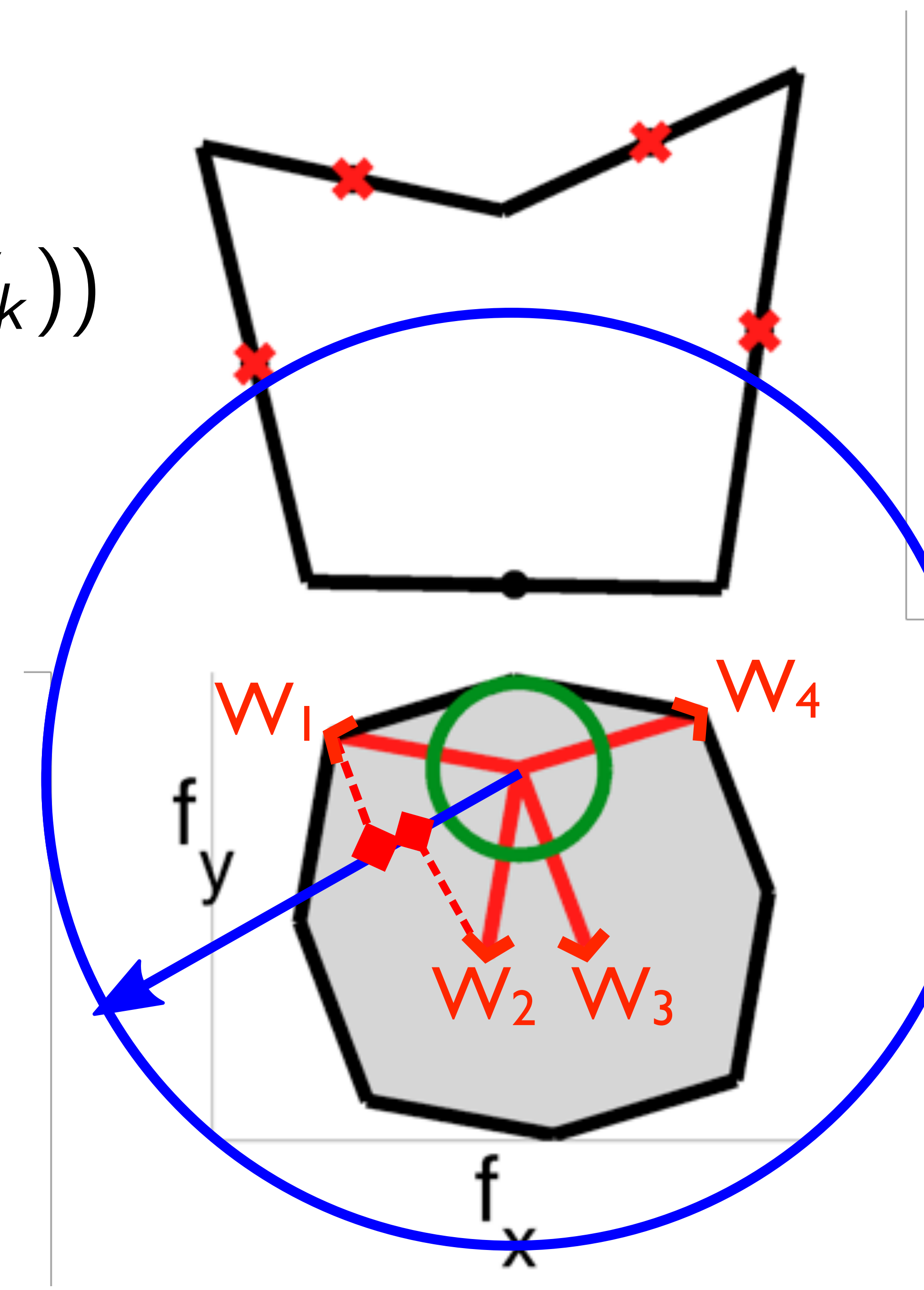
Problem reformulation

Original formulation

$$r_{\text{res}}(\text{MinkowskiSum}(W_1, W_2, \ldots, W_k))$$

Equivalent reformulation

$$\min_{\|y\|=1} \sum_{i \in S'} \sup_{\mathbf{x} \in W_i} \mathbf{x}^T \mathbf{y}$$



Schulman, Goldberg, Abbeel

Problem reformulation

Original formulation

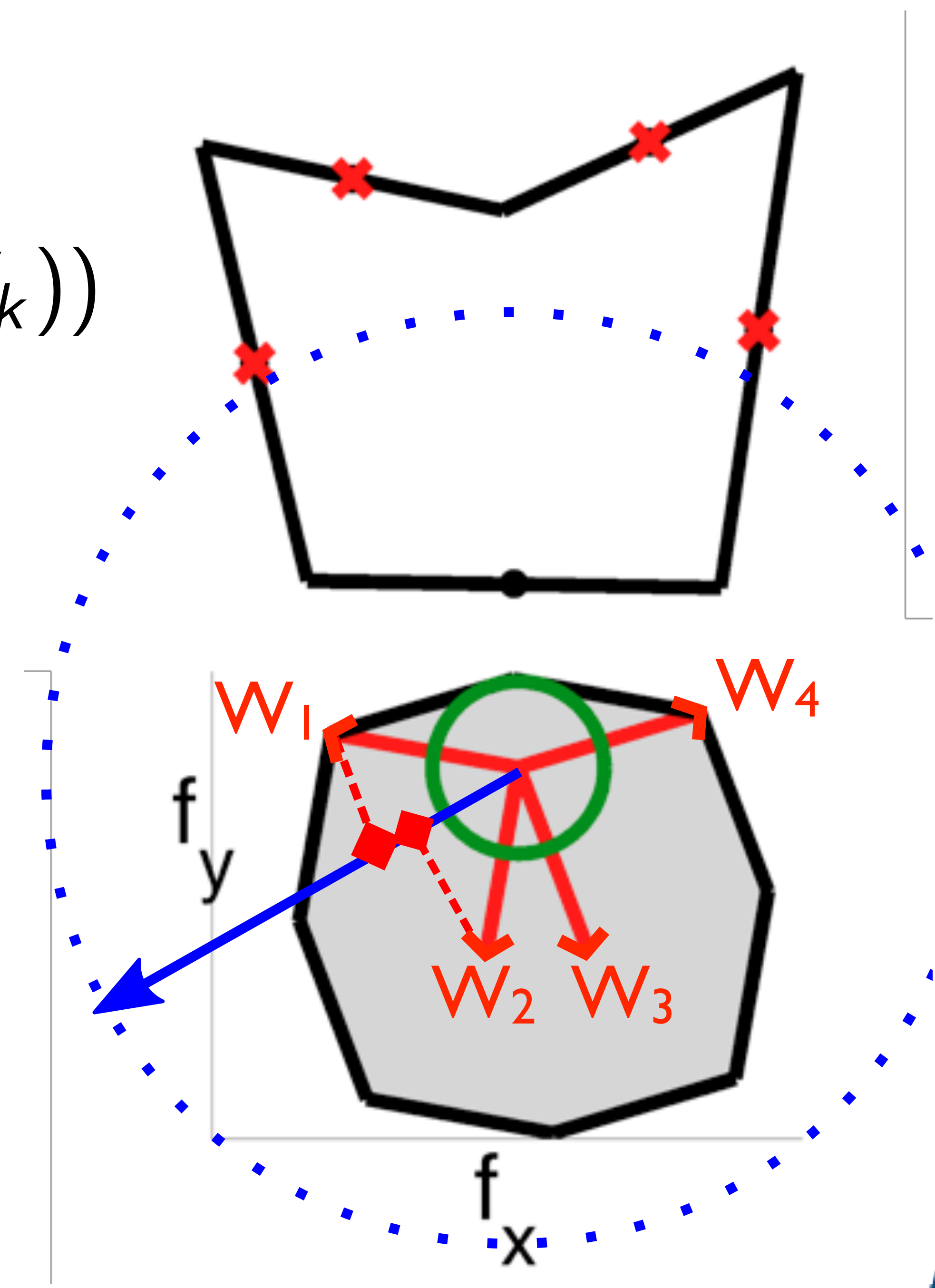
$$r_{\text{res}}(\text{MinkowskiSum}(W_1, W_2, \ldots, W_k))$$

Equivalent reformulation

$$\min_{\|y\|=1} \sum_{i \in S'} \sup_{\mathbf{x} \in W_i} \mathbf{x}^T \mathbf{y}$$

Discretization

$$\min_{j} \sum_{i \in S'} \sup_{\mathbf{x} \in W_i} \mathbf{x}^T \mathbf{y}_j$$



Schulman, Goldberg, Abbeel

Submodular coverage problems

Submodularity: diminishing returns

$$F(A \cup \{e\}) - F(A) \geq F(A \cup B \cup \{e\}) - F(A \cup B)$$

Submodular coverage problems: minimum of a collection of submodular objectives

$$\max_{|S'| \leq k} \min_j F_j(S')$$

Quality function optimization problem

$$\max_{|S'| \leq k} \min_j \sum_{i \in S'} \sup_{\mathbf{x} \in W_i} \mathbf{x}^T \mathbf{y}_j$$

Schulman, Goldberg, Abbeel

Submodular coverage problems

Submodularity: diminishing returns

$$F(A \cup \{e\}) - F(A) \geq F(A \cup B \cup \{e\}) - F(A \cup B)$$

Submodular coverage problems: minimum of a collection of submodular objectives

$$\max_{|S'| \leq k} \min_j F_j(S')$$

Quality function optimization problem

$$\max_{|S'| \leq k} \min_j \underbrace{\sum_{i \in S'} \sup_{\mathbf{x} \in W_i} \mathbf{x}^T \mathbf{y}_j}_{F_j(S')}$$

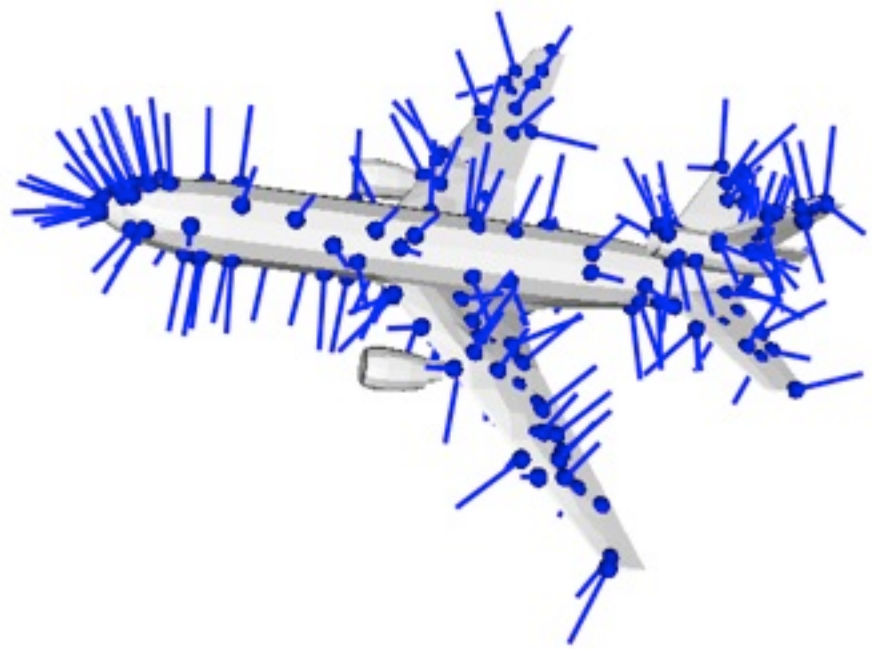
Schulman, Goldberg, Abbeel

Solving submodular coverage problems

- SATURATE: Krause et. al, 2007
- $O(n \cdot k)$ (versus $O(n^k)$ for enumeration)

Complete algorithm

- Select candidate contact points, and calculate contact wrenches



- Precompute table of values

$$A_{ij} = \sup_{\mathbf{x} \in W_i} \mathbf{y}_j^T \mathbf{x}_i$$

- Optimize with SATURATE

$$\max_{|S'|=k} \min_j \sum_{i \in S'} A_{ij}$$

Schulman, Goldberg, Abbeel

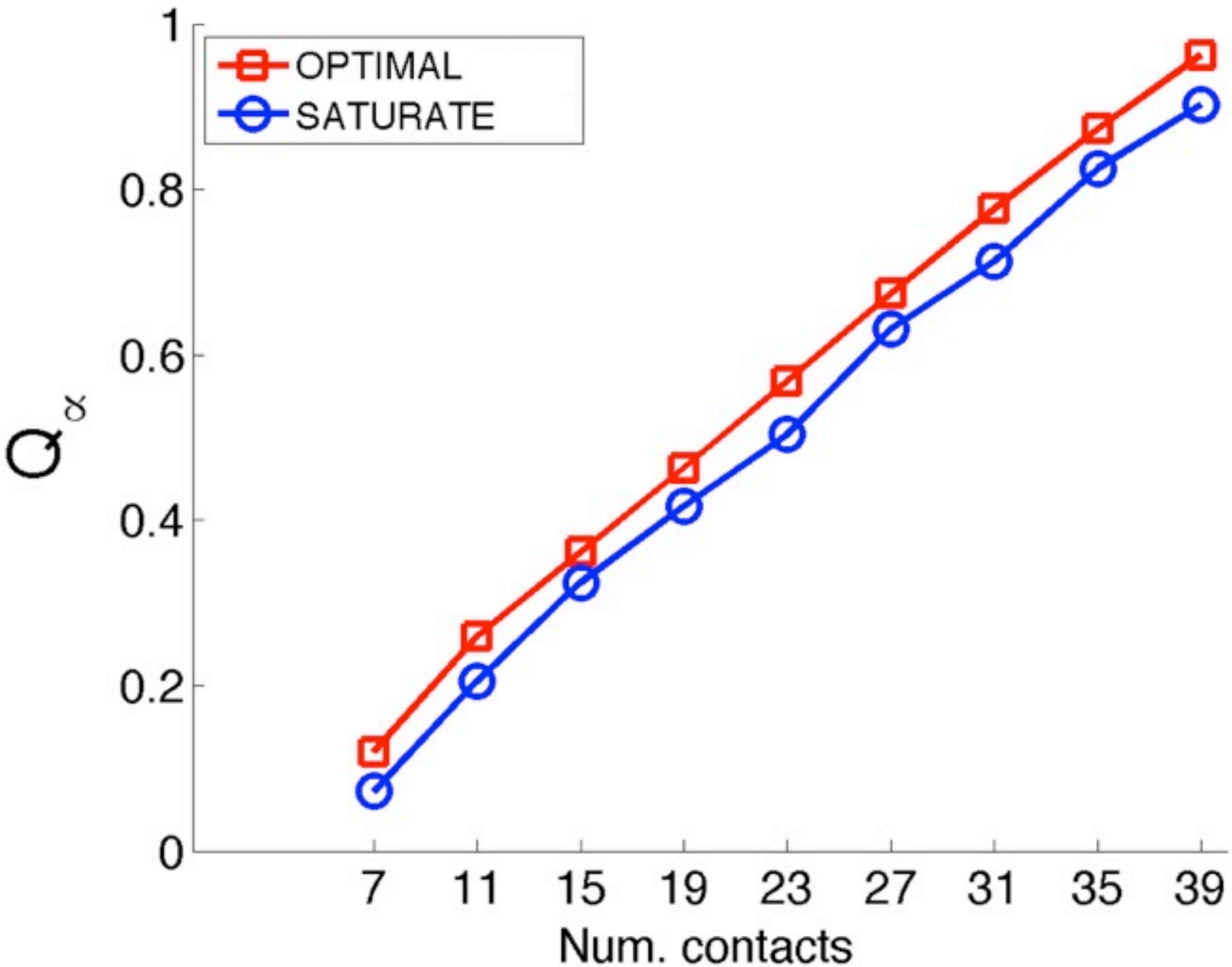
Generality of method

- Various contact models: coulomb friction, soft finger, surface & line contacts
- Q_1 quality metric where the *sum* of the forces is constrained
- Disturbance set is known
- Disturbances aren't centered around the origin (gravity)

Numerical experiments

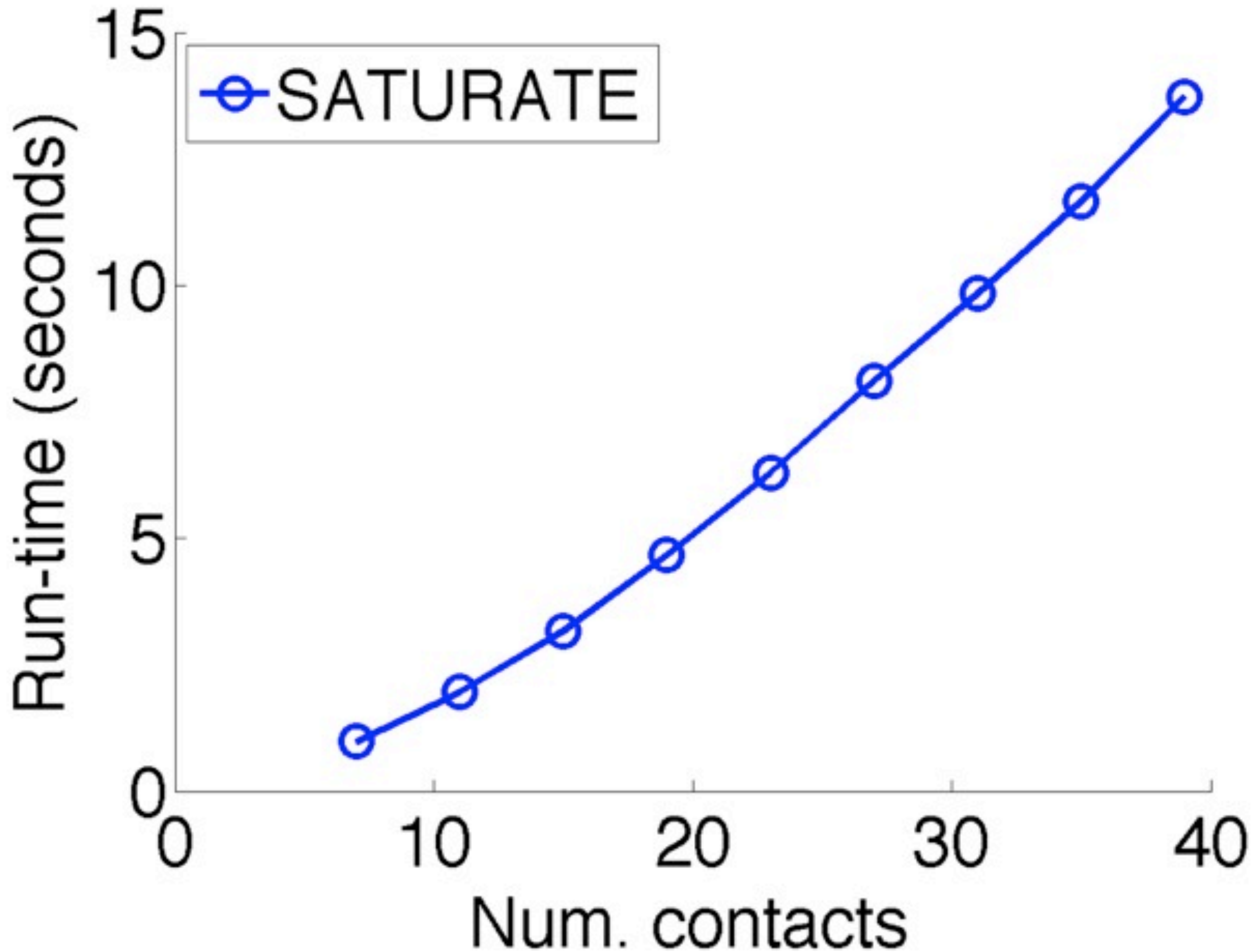
Princeton shape database
(Shilane, Min, et al., 2004)

Results

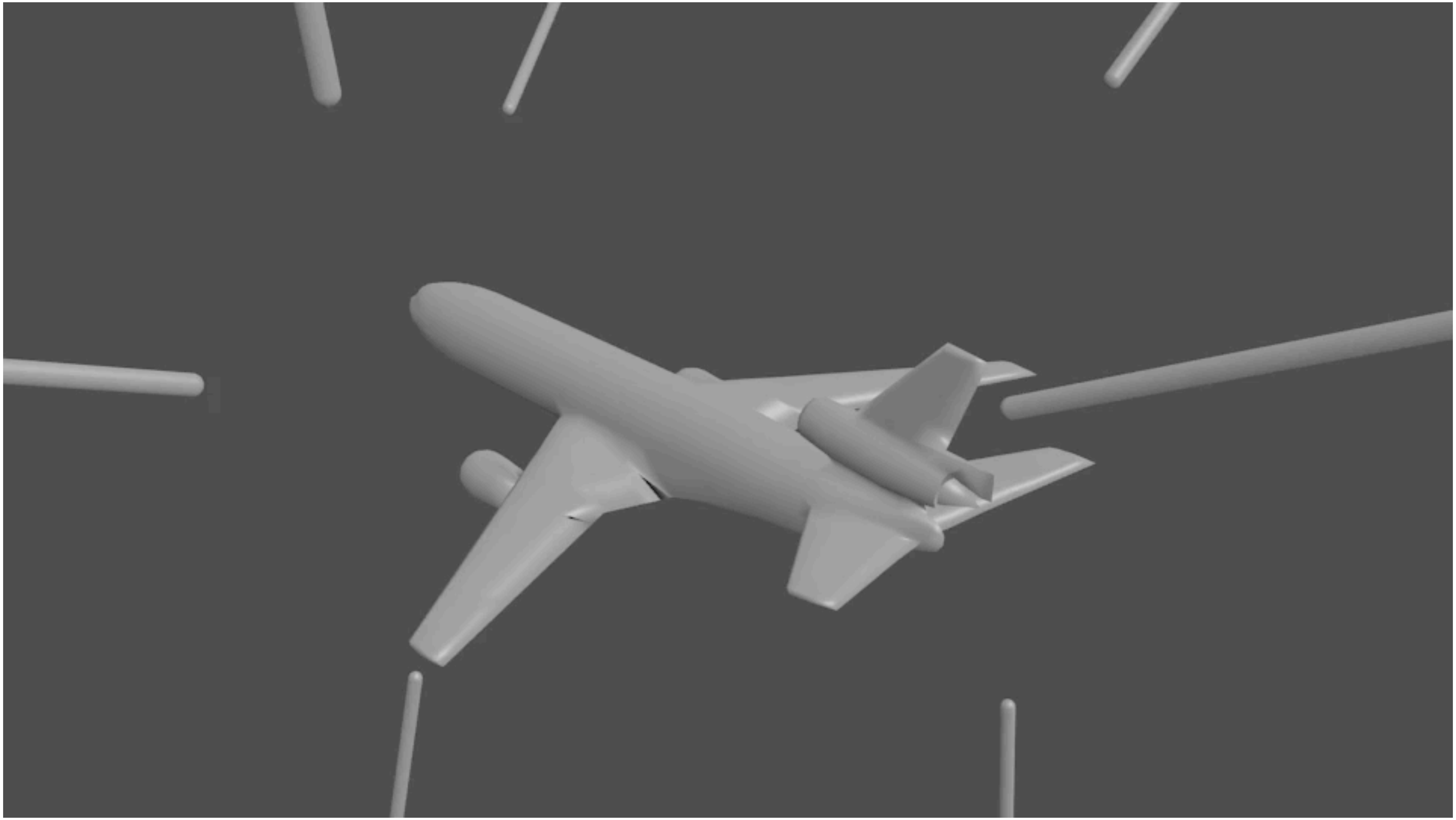


Schulman, Goldberg, Abbeel

Run-time

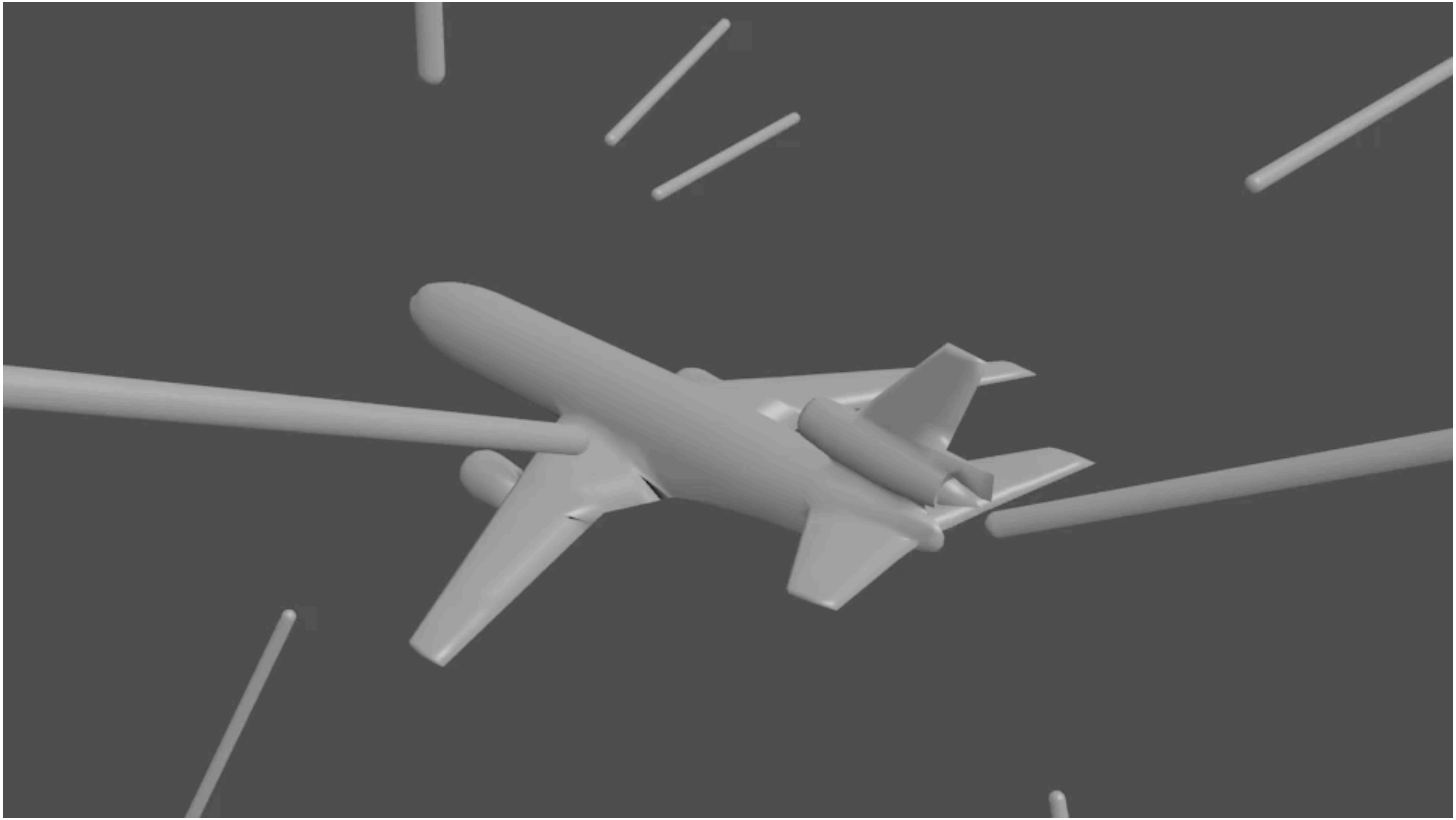


Schulman, Goldberg, Abbeel



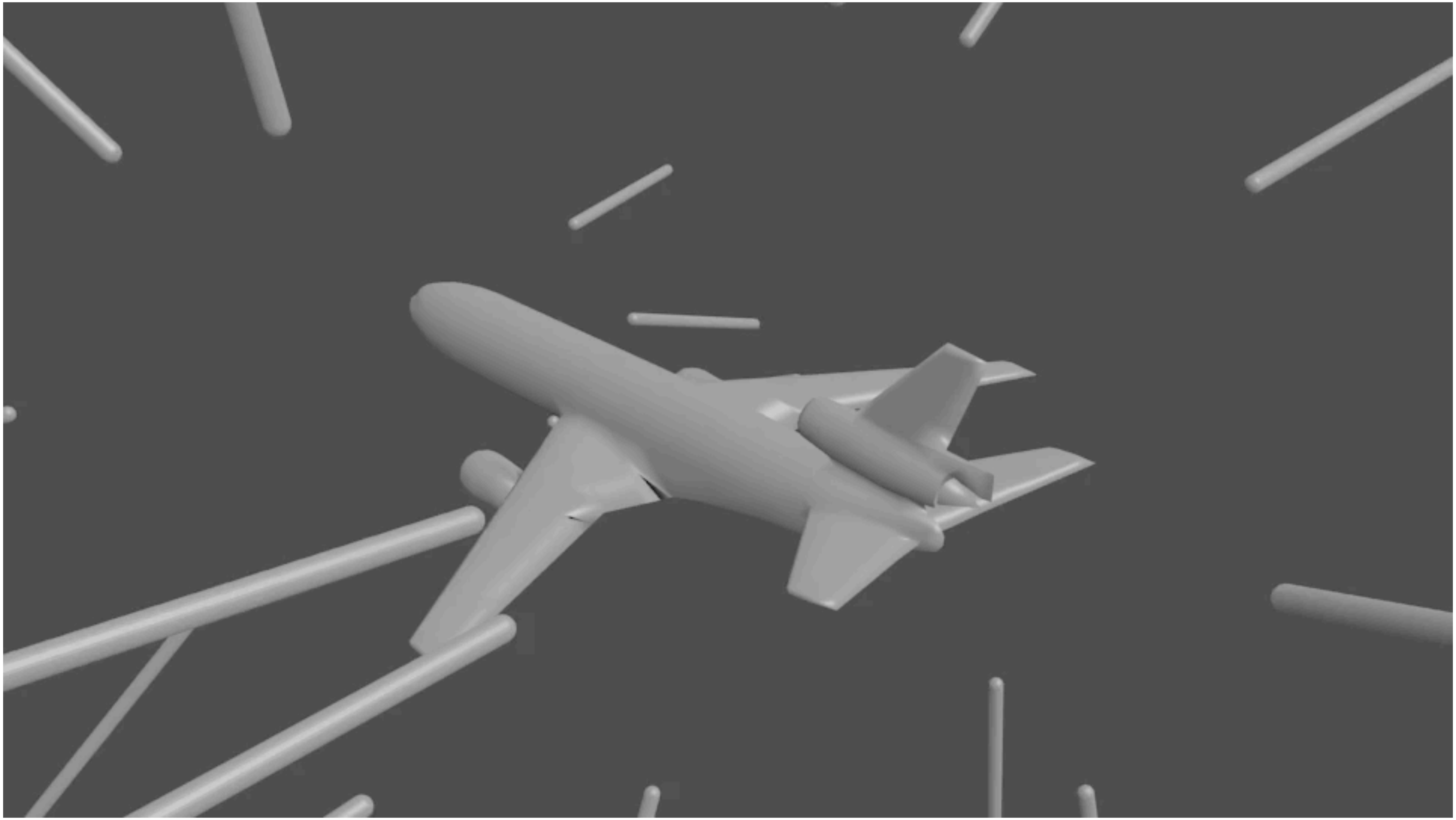
plane: 10

Schulman, Goldberg, Abbeel



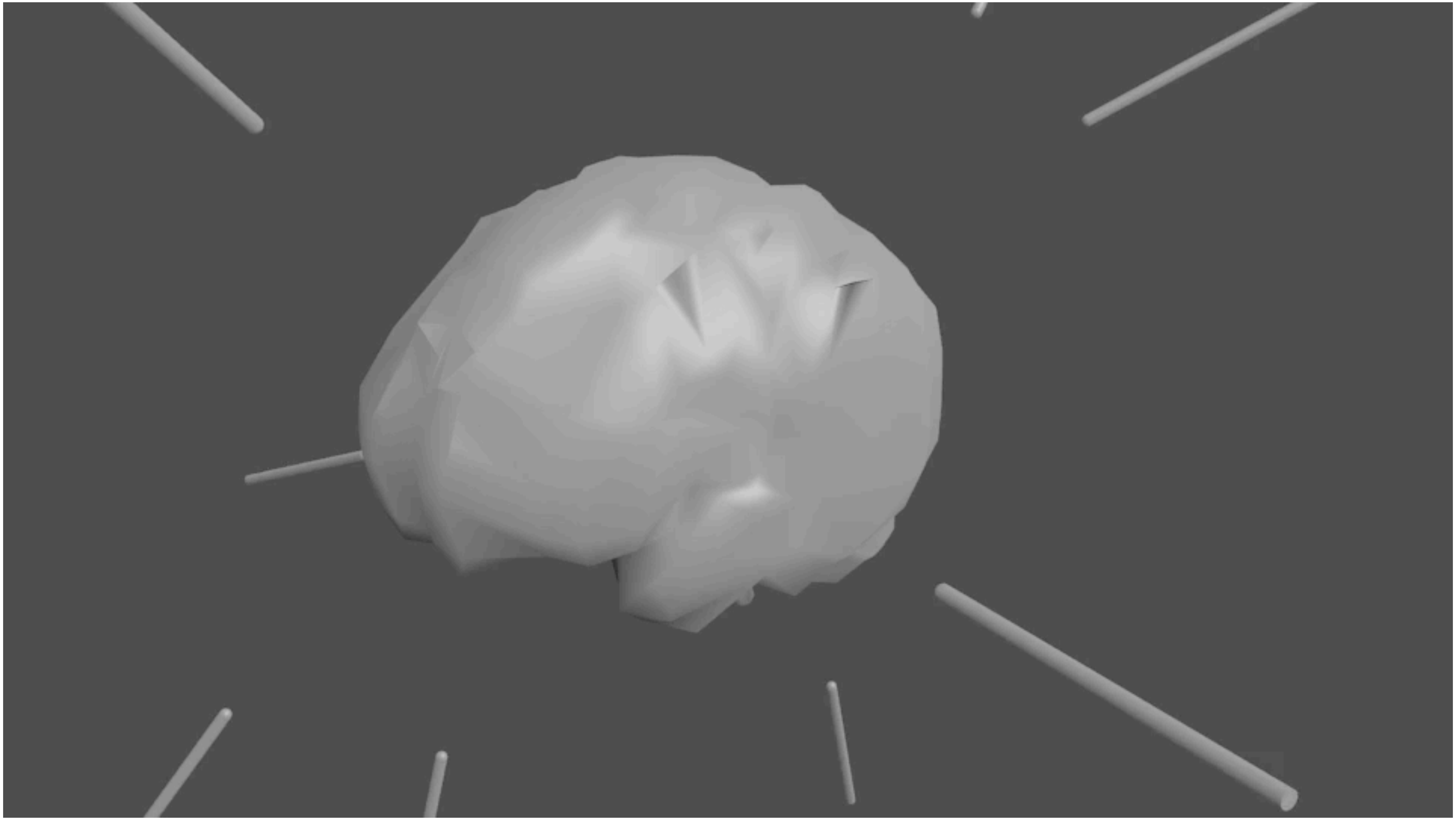
plane: 20

Schulman, Goldberg, Abbeel



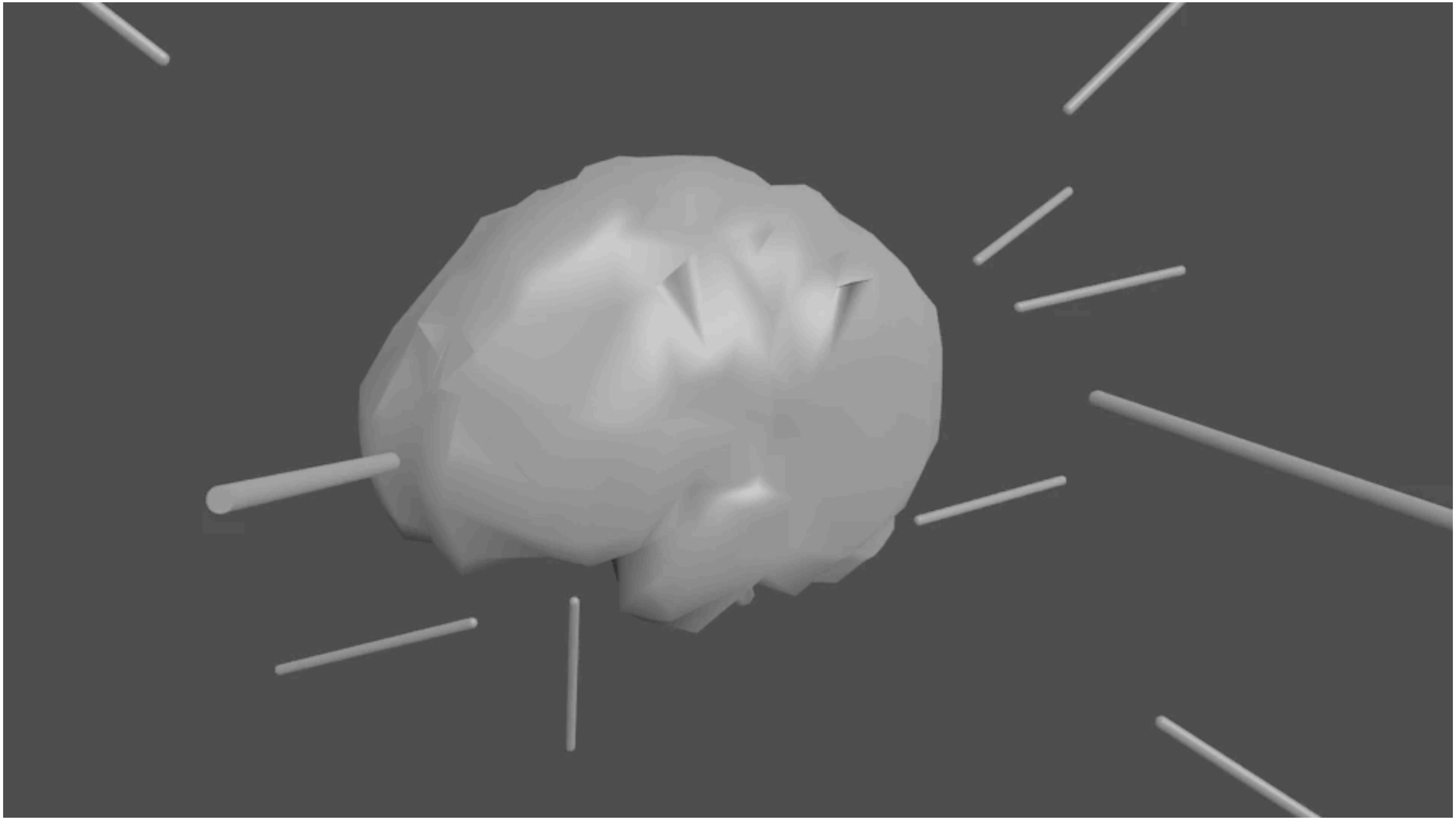
plane: 30

Schulman, Goldberg, Abbeel



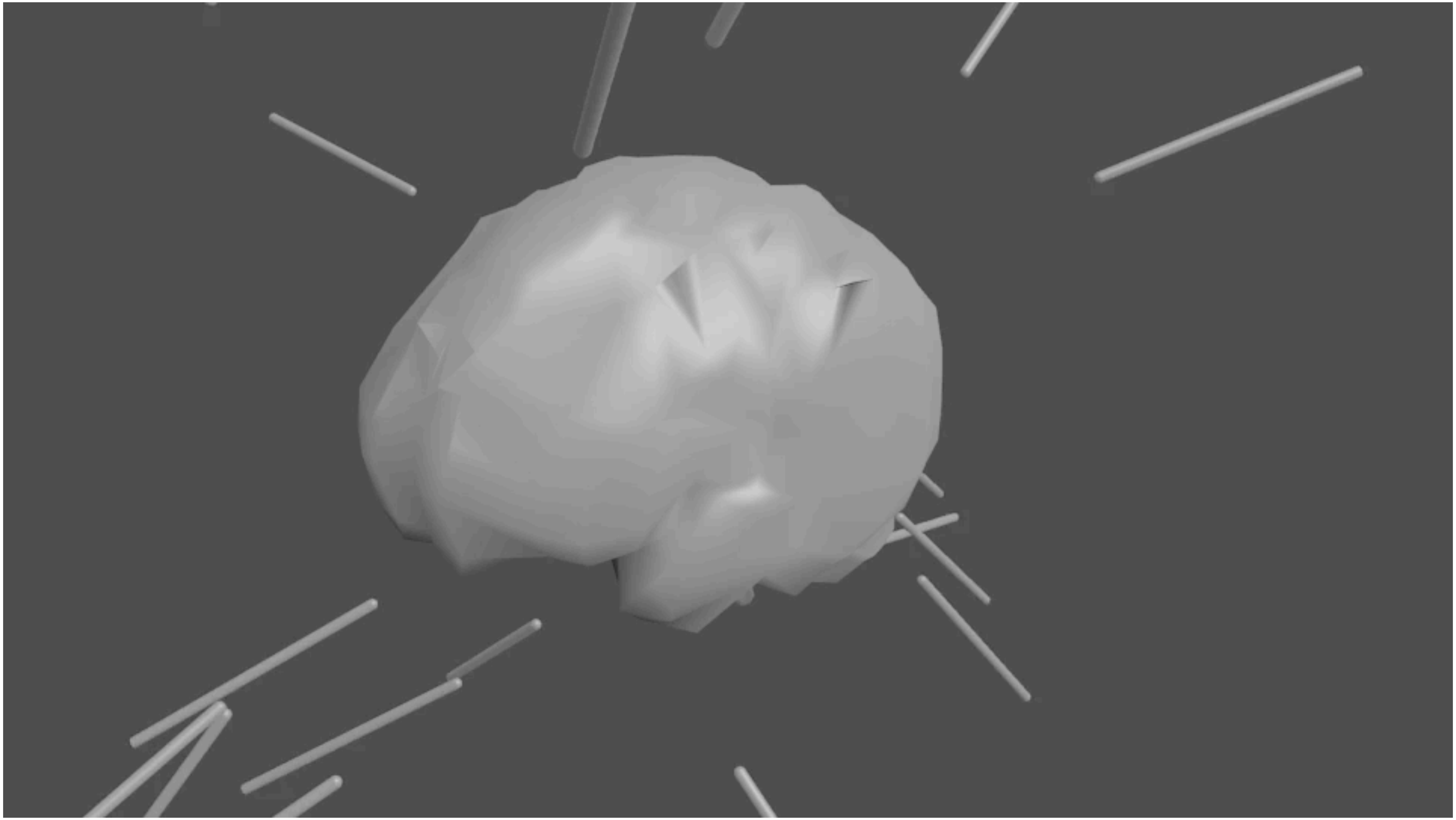
brain: 10

Schulman, Goldberg, Abbeel



brain: 20

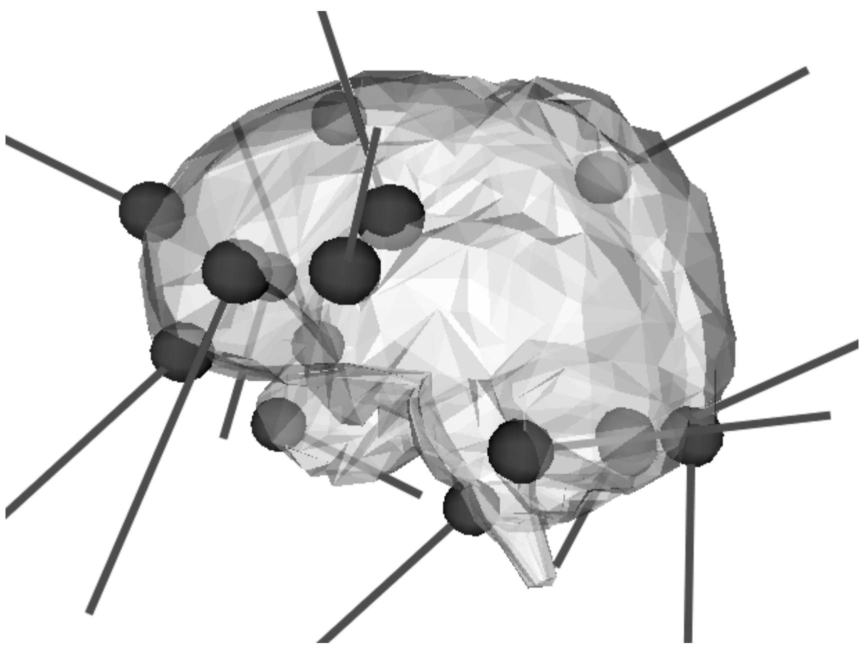
Schulman, Goldberg, Abbeel



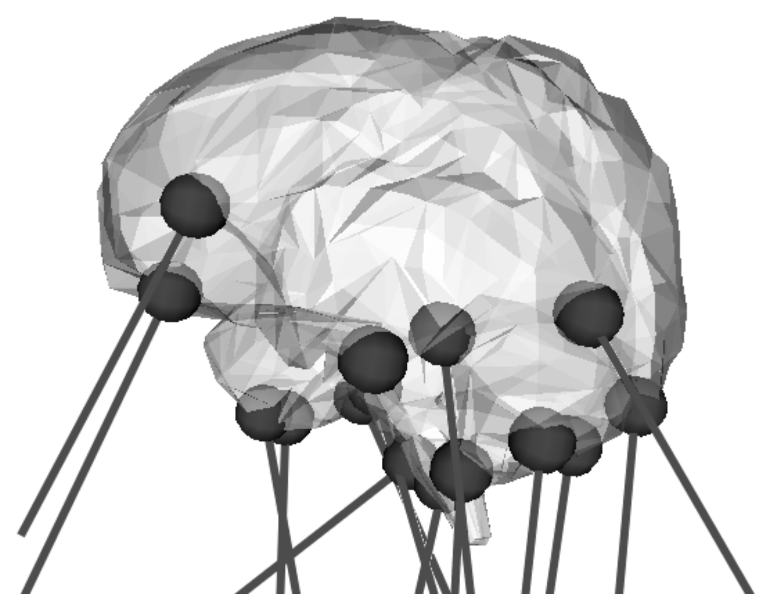
brain: 30

Schulman, Goldberg, Abbeel

Gravity



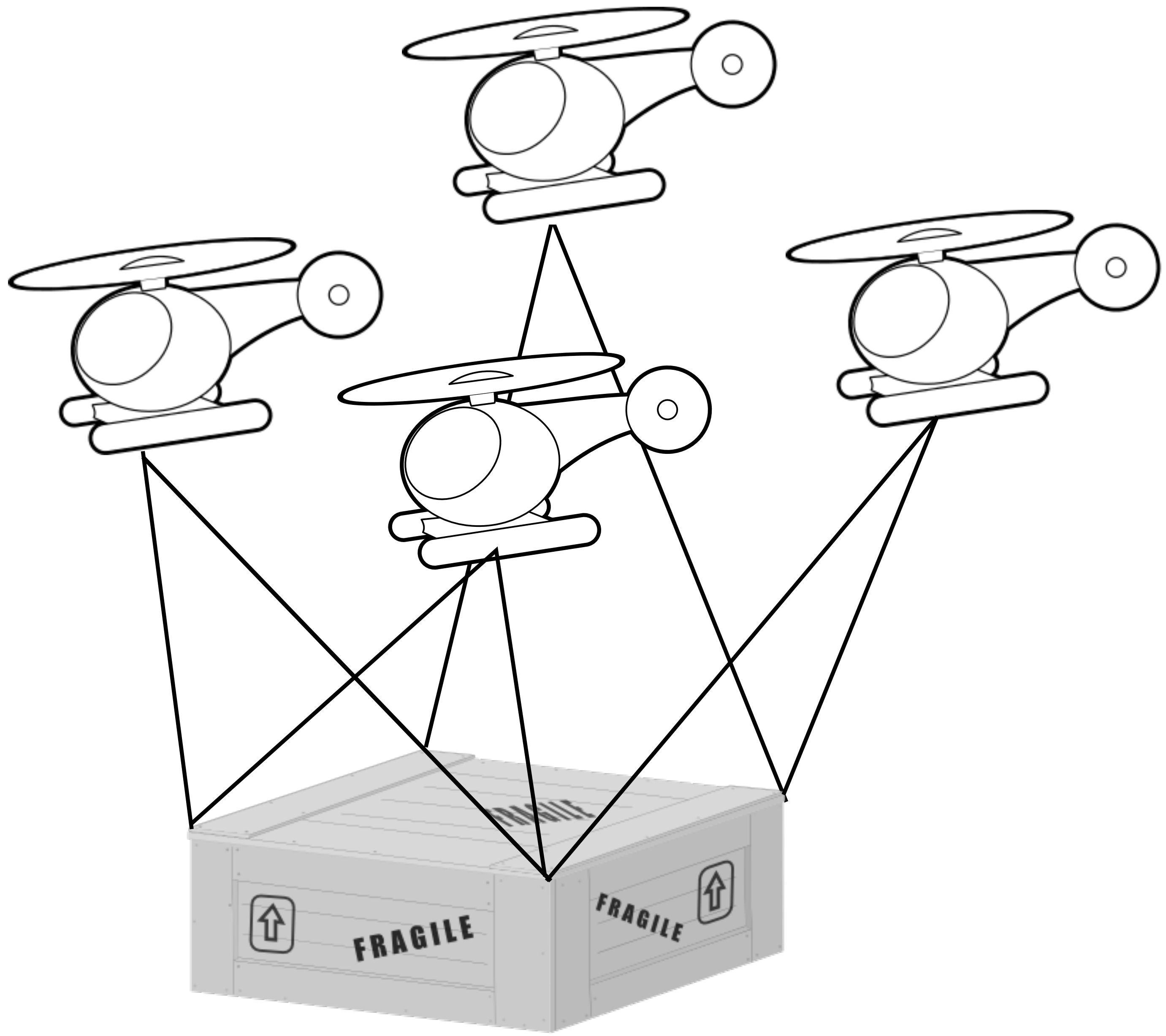
(a) no gravity



(b) high gravity

Schulman, Goldberg, Abbeel

Towing



Optimize helicopter positions based on imminent object wrenches

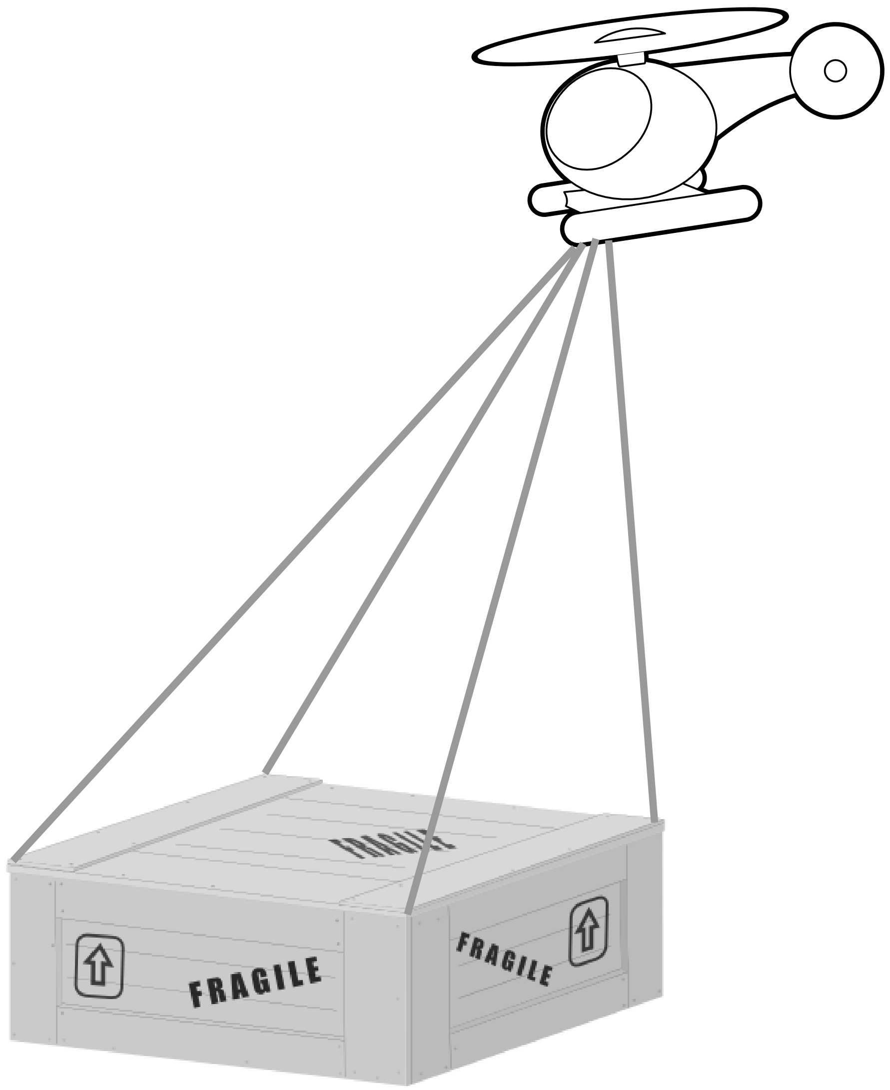
Schulman, Goldberg, Abbeel

Wrenches achievable by one helicopter

- Helicopter only applies force along ropes
- Lift is bounded

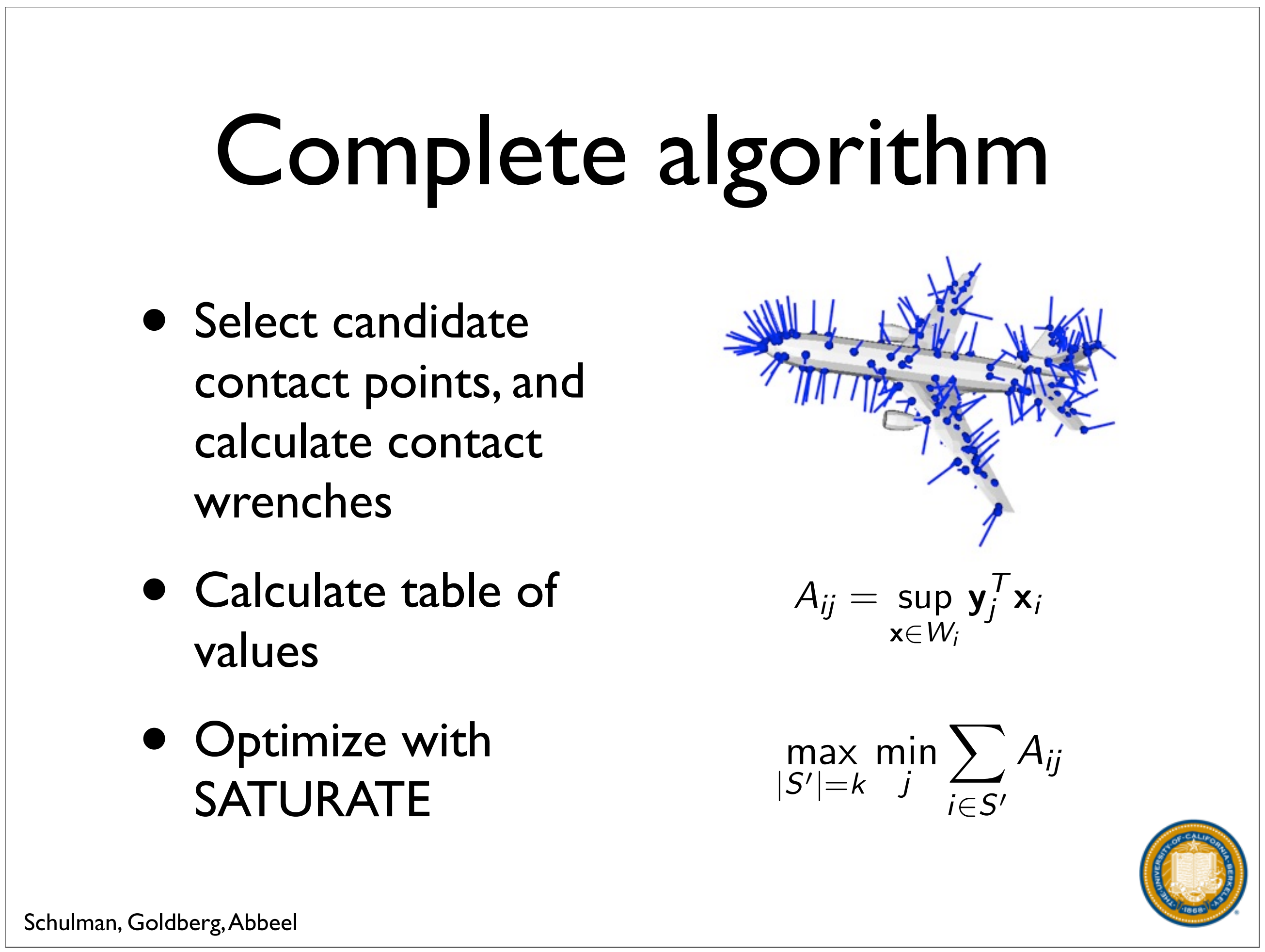
↓

Convex constraint on applied wrenches



Schulman, Goldberg, Abbeel

Optimization procedure



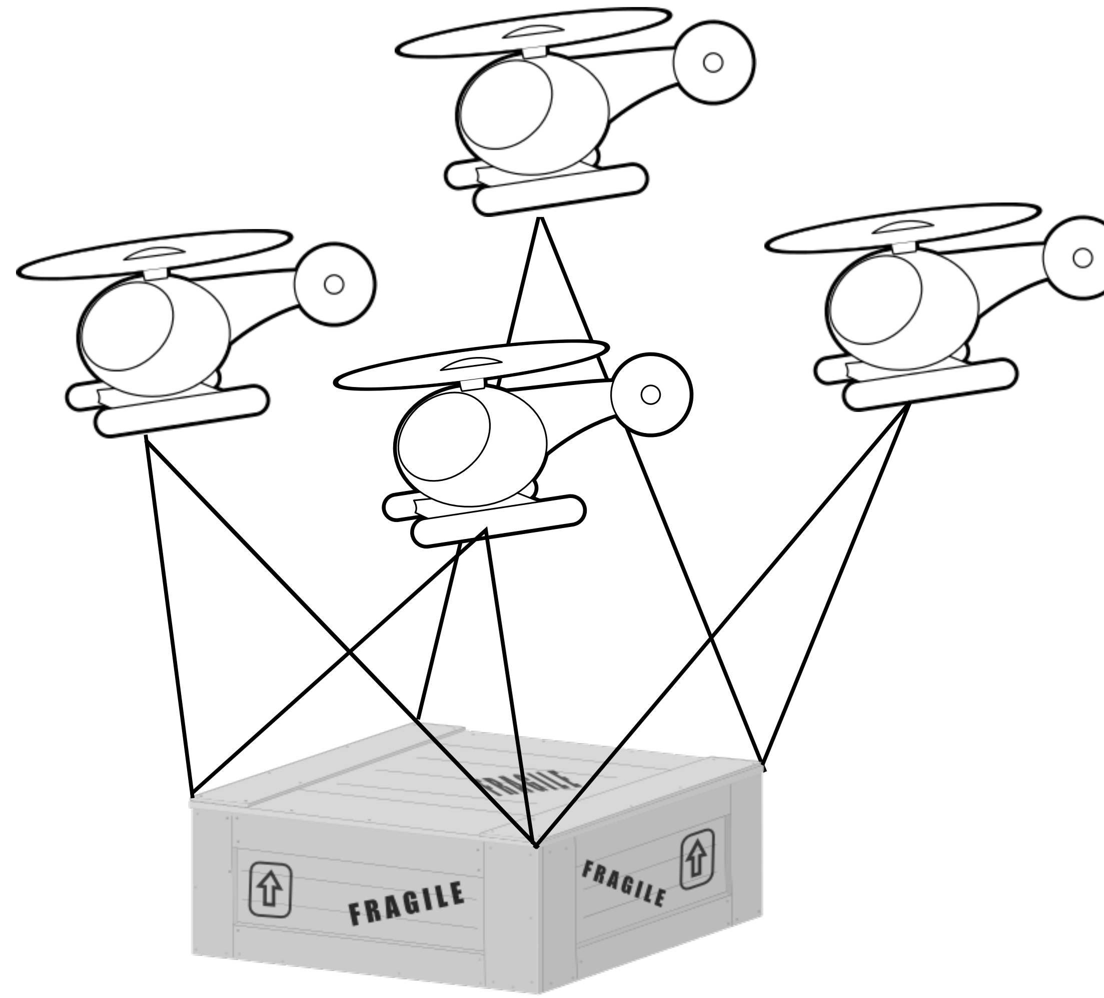
Complete algorithm

- Select candidate contact points, and calculate contact wrenches
- Calculate table of values
- Optimize with SATURATE

$$A_{ij} = \sup_{\mathbf{x} \in W_i} \mathbf{y}_j^T \mathbf{x}_i$$

$$\max_{|S'|=k} \min_j \sum_{i \in S'} A_{ij}$$

Schulman, Goldberg, Abbeel



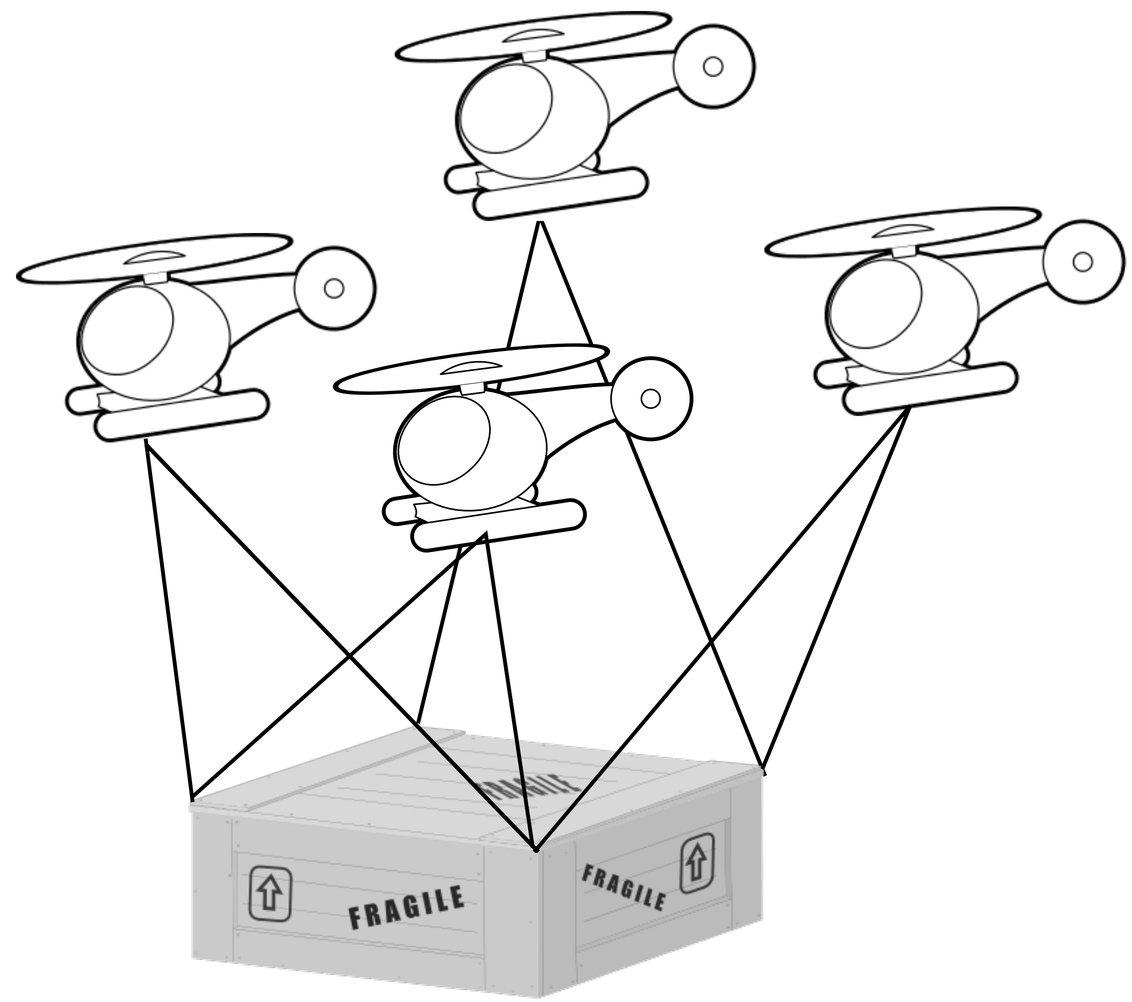
Schulman, Goldberg, Abbeel

Optimization procedure

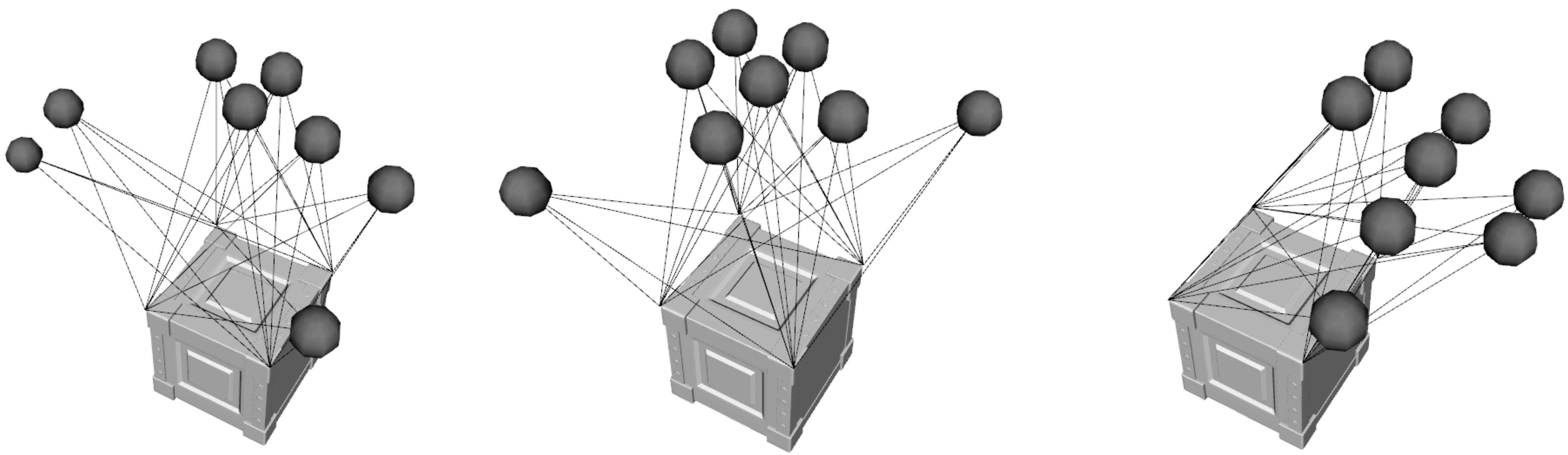
Complete algorithm

- Calculate achievable wrenches W_i for each candidate helicopter position
  - Calculate table of values $$A_{ij} = \sup_{\mathbf{x} \in W_i} \mathbf{y}_j^T \mathbf{x}_i$$
  - Optimize with SATURATE $$\max_{|S'|=k} \min_j \sum_{i \in S'} A_{ij}$$

Schulman, Goldberg, Abbeel



Optimal towing configurations



(a) light load

(b) heavy load

(c) medium load with a horizontal component

Schulman, Goldberg, Abbeel

Summary

- Goal: optimize quality metric Q_∞
- Our approach:
  - discretized formula
  - optimize in linear time with SATURATE
- Works for various contact models
- Application to towing

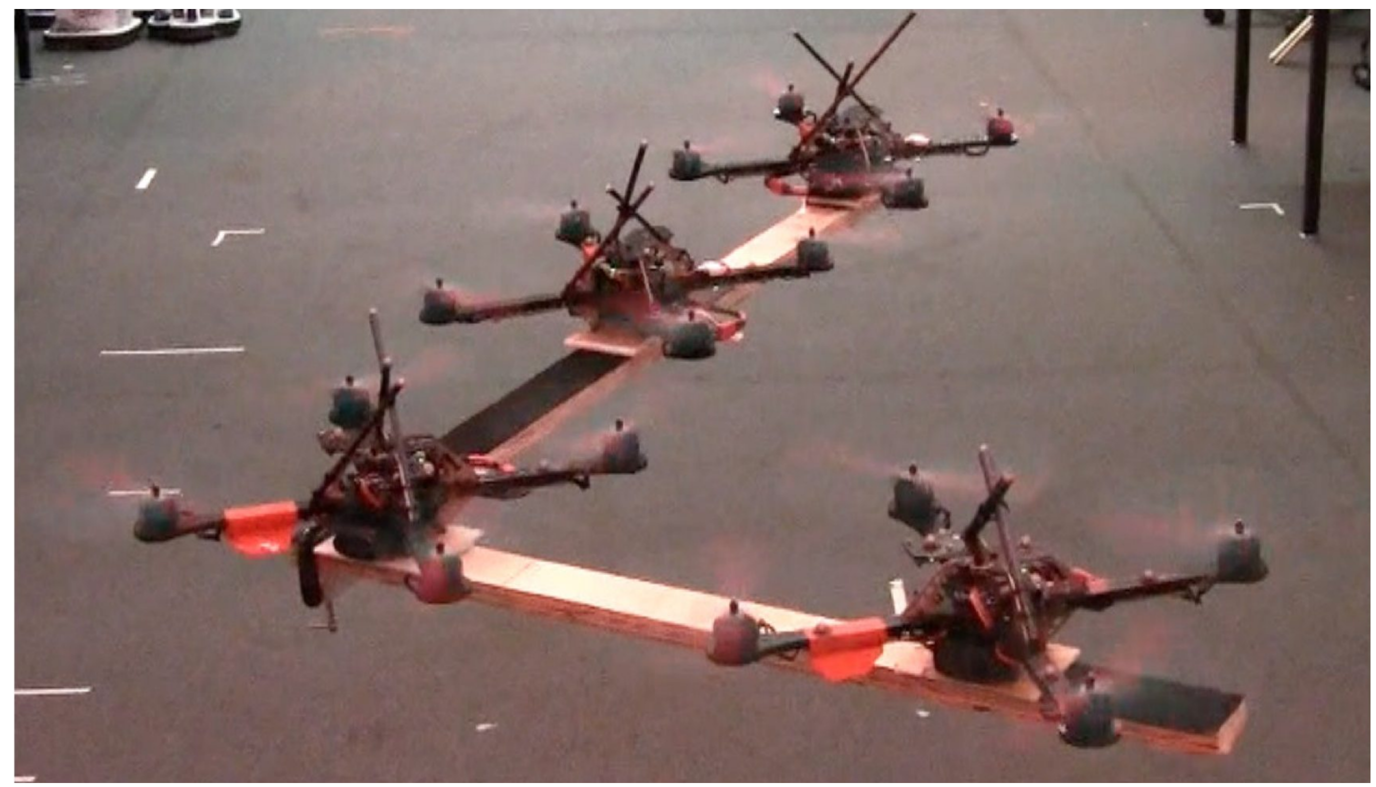
Thank you

Optimal contact point selection

Carrying

© Alex Wild

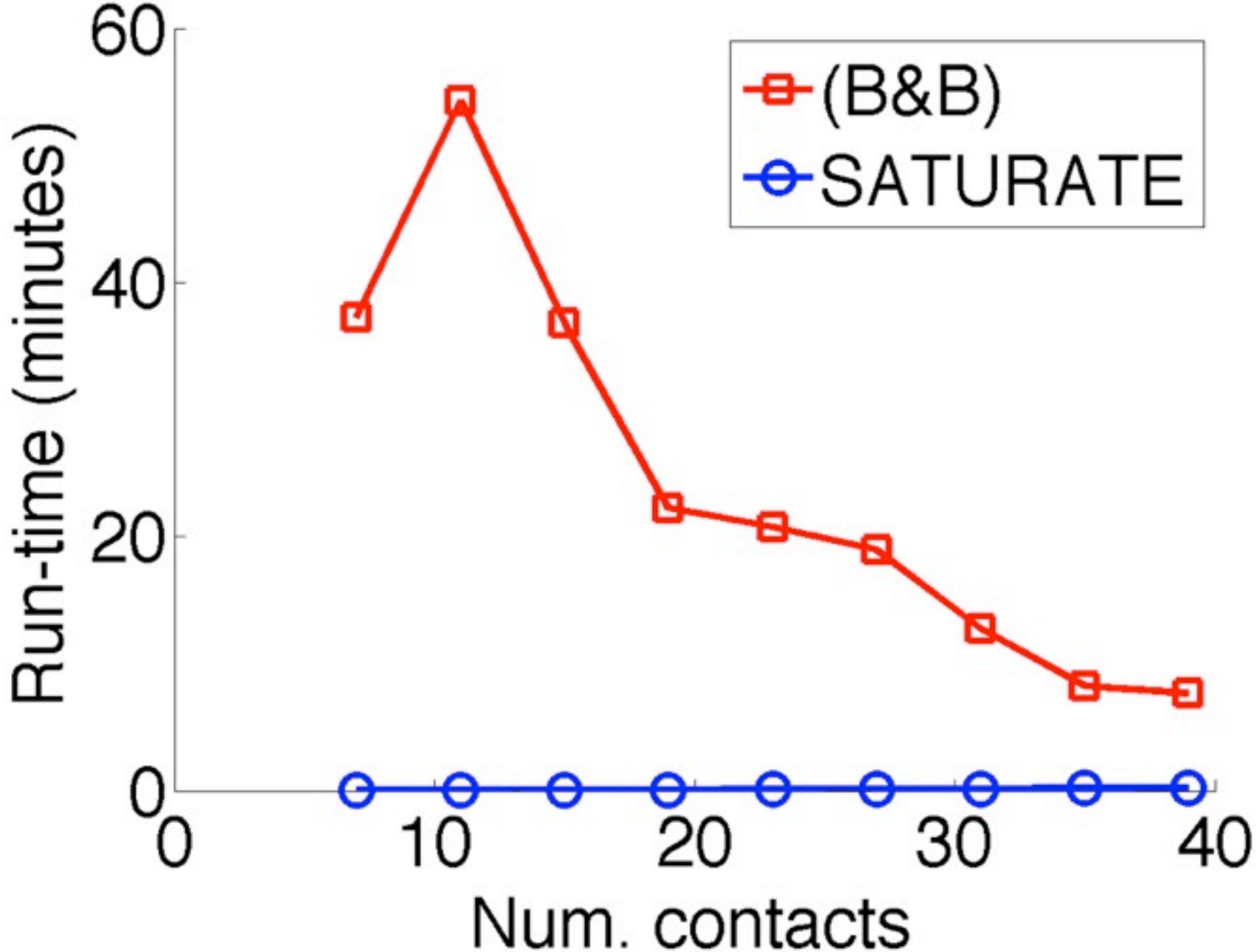
Towing



Mellinger, Shomin, Michael, Kumar (2010)

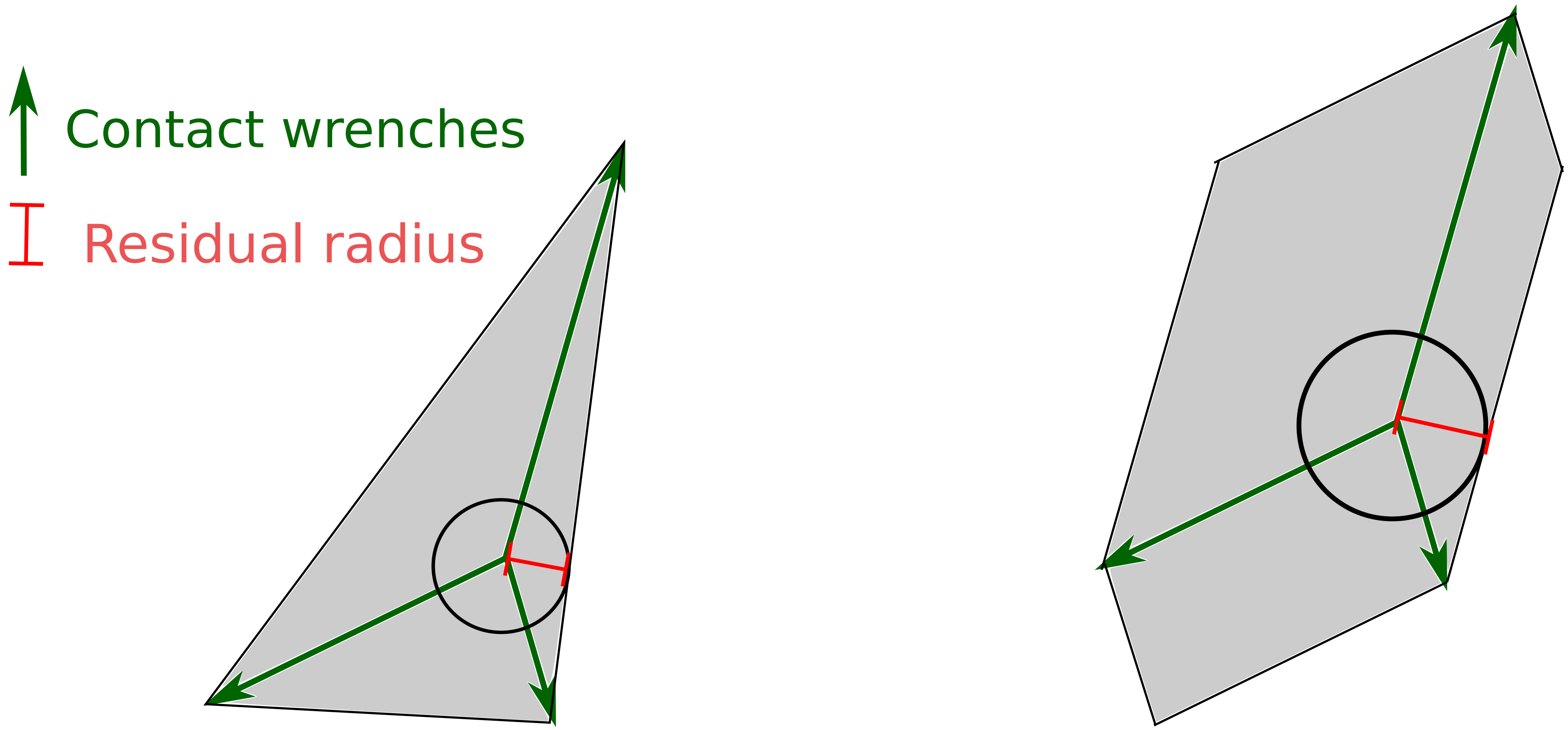
Schulman, Goldberg, Abbeel

Run-time



Schulman, Goldberg, Abbeel

Quality metrics--geometry

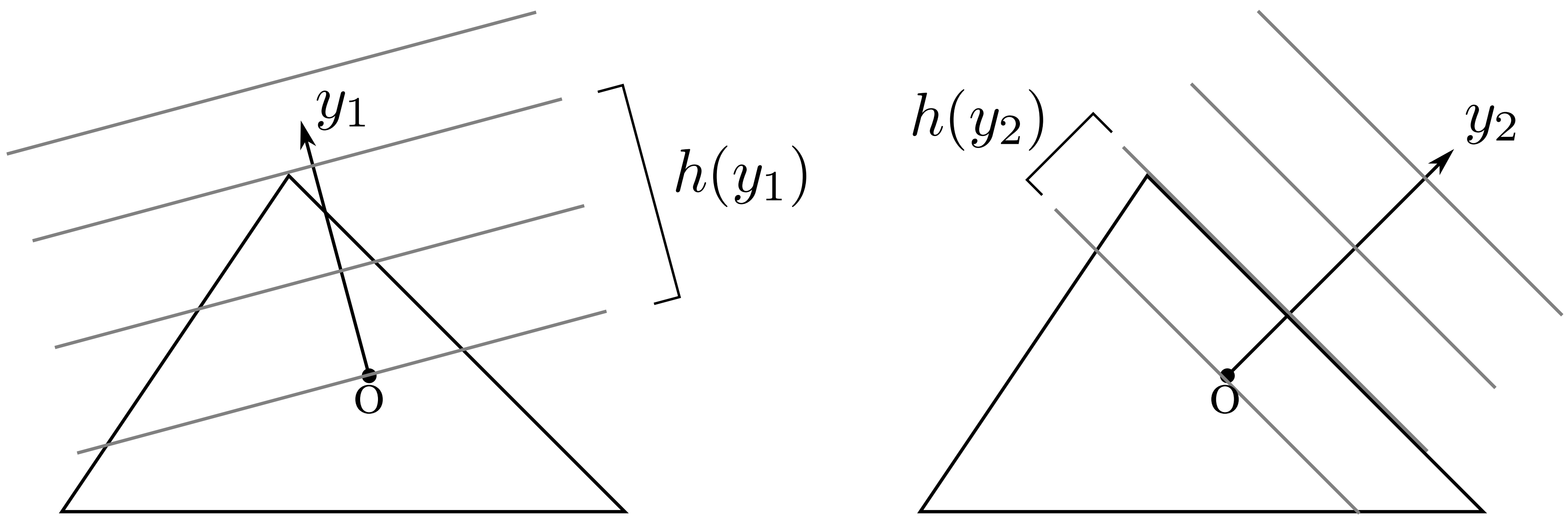


Q_1: sum of contact forces ≤ 1

Q_∞: max contact force ≤ 1

Schulman, Goldberg, Abbeel

Support function



$$h_{W_i}(\mathbf{y}) = \sup_{\mathbf{x} \in W_i} \mathbf{x}^T \mathbf{y}$$

Schulman, Goldberg, Abbeel

Optimization problem

From candidate contacts S, select $S' \subset S$

$$\max_{|S'|=k} Q_1 = \max_{|S'|=k} r_{\text{res}}(\text{ConvexHull}\{W_i \mid i \in S'\})$$

$$\max_{|S'|=k} Q_\infty = \max_{|S'|=k} r_{\text{res}}(\text{MinkowskiSum}\{W_i \mid i \in S'\})$$

where W_i is the truncated friction cone.

Equivalent formula

$$Q_\infty = r_{\text{res}}(\text{MinkowskiSum}(W_1, W_2, \ldots, W_k)) = \min_{\|\mathbf{y}\|=1} \sum_i h_{W_i}(\mathbf{y})$$

$$\approx \min_j \sum_i h_{W_i}(\mathbf{y})$$

Approximation error: $1 - \frac{\tilde{Q}}{Q} \leq \Delta\theta$

Schulman, Goldberg, Abbeel

New optimization problem

$$\max_{|S'|=k} Q_\infty \approx \max_{|S'|=k} \min_j \sum_{i \in S'} h_{W_i}(\mathbf{y}_j) = \max_{|S'|=k} \min_j \sum_{i \in S'} A_{ij} + b_j$$

$$\text{where } A_{ij} = h_{W_i}(\mathbf{y}_j)$$

$$b_j = y_j^T \mathbf{c}$$

Schulman, Goldberg, Abbeel

Branch and Bound

$$\max_{\mathbf{z}} \min_{j} \sum_{i} z_i A_{ij} + b_j \quad \text{subject to} \quad \sum_{i} z_i = k, \left[z \in \{0,1\} \right]$$

↓

$$0 \leq \mathbf{z} \leq 1$$