Optimization for Locally Optimal Control

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Optimal Control (Open Loop)

- Optimal control problem:

$$\min_{x, u} \sum_{t=0}^{H} c_t(x_t, u_t)$$

s.t.  
$$x_0 = \bar{x}_0$$

$$x_{t+1} = f(x_t, u_t) \quad t = 0, \ldots, H - 1$$

- Solution:
  - = Sequence of controls $u$ and resulting state sequence $x$
  - If no noise, sufficient to just execute $u$
  - In general non-convex optimization problem, can be solved with sequential convex programming (SCP)
Optimal Control (Closed Loop)

Given: \( \bar{x}_0 \)

For \( t=0, 1, 2, ..., T \)

- Solve
  \[
  \min_{x,u} \sum_{k=t}^{T} c_k(x_k, u_k)
  \]
  \[
  \text{s.t.} \quad x_{k+1} = f(x_k, u_k), \quad \forall k \in \{t, t+1, \ldots, T-1\}
  \]
  \[
  x_t = \bar{x}_t
  \]

- Execute \( u_t \)

- Observe resulting state, \( \bar{x}_{t+1} \)

= “Model Predictive Control”

Initialize with solution from \( t - 1 \) to solve fast at time \( t \)
Collocation versus Shooting

- What we considered thus far is a collocation method
  - It considers both x and u simultaneously, optimizes over both of them, and re-linearizes (inside the SCP loop) based on both x and u from the previous round

- Shooting methods
  - Optimize over u directly
  - This can be done as every u results (following the dynamics) in a state sequence x, for which in turn the cost can be computed
  - Upside: Improve sequence of controls over time
    - Versus: collocation might converge to a local optimum that’s infeasible
  - Downsides:
    - Derivatives with respect to u as well as the cost for a given u can be numerically unstable to compute (especially in case of unstable dynamical systems)
      [x provides decoupling between time-steps, making computation stable]
    - Not clear how to initialize in a way that nudges towards a goal state