Beam Sensor Models

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Many slides adapted from Thrun, Burgard and Fox, Probabilistic Robotics
The central task is to determine $P(z/x)$, i.e., the probability of a measurement $z$ given that the robot is at position $x$.

**Question**: Where do the probabilities come from?

**Approach**: Let’s try to explain a measurement.

**Proximity Sensors**
Beam-based Sensor Model

- Scan $z$ consists of $K$ measurements.

$$z = \{z_1, z_2, ..., z_K\}$$

- Individual measurements are independent given the robot position.

$$P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m)$$
Beam-based Sensor Model

\[ P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m) \]
Typical Measurement Errors in Range Measurements

1. Beams reflected by obstacles
2. Beams reflected by persons / caused by crosstalk
3. Random measurements
4. Maximum range measurements
Beam-based Proximity Model

Measurement noise

$$P_{hit}(z \mid x, m) = \eta \frac{1}{\sqrt{2\pi b}} e^{-\frac{1}{2} \left(\frac{z - z_{exp}}{b}\right)^2}$$

Unexpected obstacles

$$P_{unexp}(z \mid x, m) = \begin{cases} 
\eta \lambda e^{-\lambda z} & \text{if } z < z_{exp} \\
0 & \text{otherwise}
\end{cases}$$
Beam-based Proximity Model

Random measurement

Max range

\[ P_{\text{rand}}(z \mid x, m) = \eta \frac{1}{z_{\text{max}}} \]

\[ P_{\text{max}}(z \mid x, m) = \eta \frac{1}{z_{\text{small}}} \]
Resulting Mixture Density

\[
P(z \mid x, m) = \begin{pmatrix} \alpha_{\text{hit}} \\ \alpha_{\text{unexp}} \\ \alpha_{\text{max}} \\ \alpha_{\text{rand}} \end{pmatrix}^T \begin{pmatrix} P_{\text{hit}}(z \mid x, m) \\ P_{\text{unexp}}(z \mid x, m) \\ P_{\text{max}}(z \mid x, m) \\ P_{\text{rand}}(z \mid x, m) \end{pmatrix}
\]

How can we determine the model parameters?
Raw Sensor Data

Measured distances for expected distance of 300 cm.

Sonar

Laser
Approximation

- Maximize log likelihood of the data \( P(z \mid z_{\text{exp}}) \)
- Search space of n-1 parameters.
  - Hill climbing
  - Gradient descent
  - Genetic algorithms
  - ...
- Deterministically compute the n-th parameter to satisfy normalization constraint.
Approximation Results

Laser

Sonar

300cm

400cm
Approximation Results

Laser

Sonar
Influence of Angle to Obstacle
Influence of Angle to Obstacle
Influence of Angle to Obstacle
Influence of Angle to Obstacle
Summary Beam-based Model

- Assumes independence between beams.
  - Justification?
  - Overconfident!

- Models physical causes for measurements.
  - Mixture of densities for these causes.
  - Assumes independence between causes. Problem?

- Implementation
  - Learn parameters based on real data.
  - Different models should be learned for different angles at which the sensor beam hits the obstacle.
  - Determine expected distances by ray-tracing.
  - Expected distances can be pre-processed.