

Inverse Reinforcement Learning

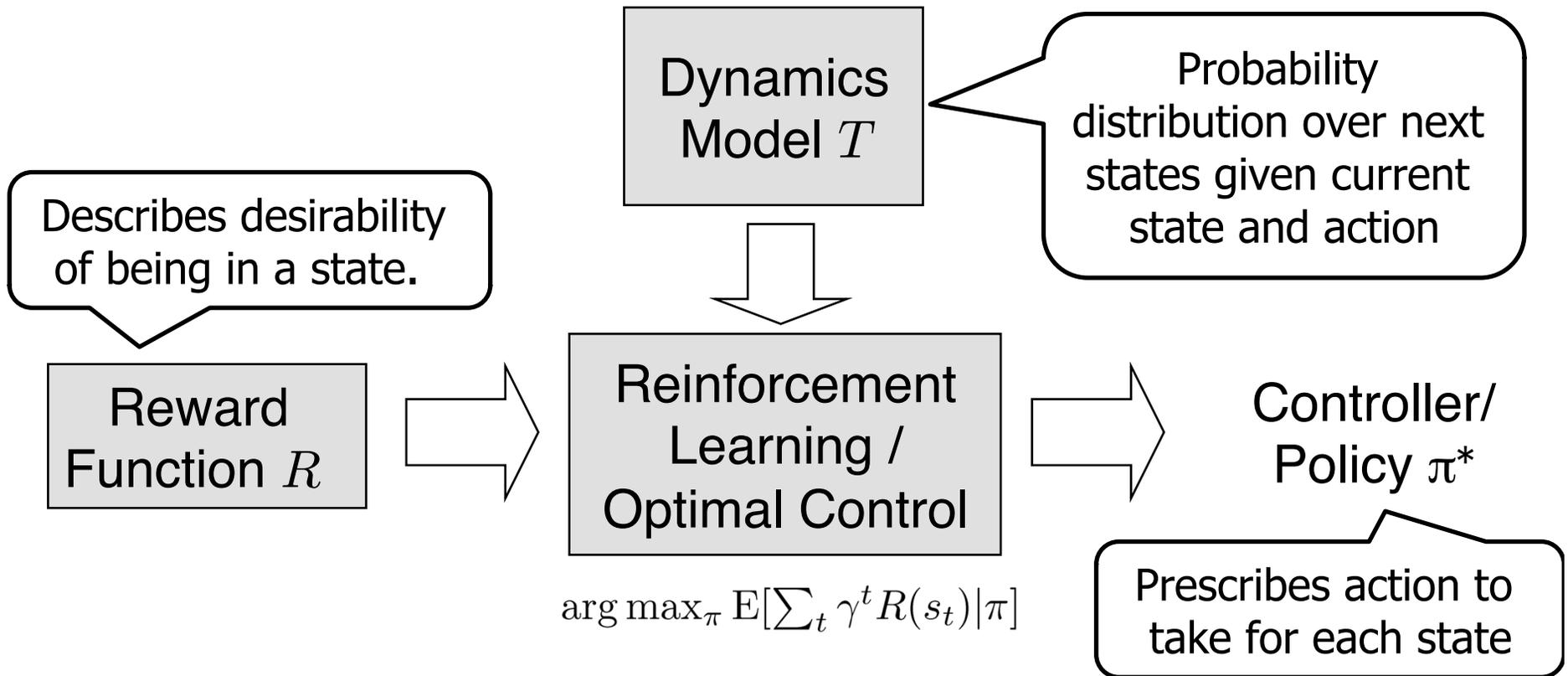
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Inverse Reinforcement Learning

**[equally good titles: Inverse Optimal Control,
Inverse Optimal Planning]**

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High-level picture



Inverse RL:

Given π^* and T , can we recover R ?

More generally, given execution traces, can we recover R ?

Motivation for inverse RL

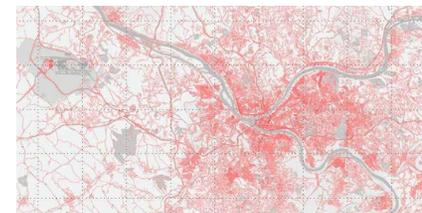
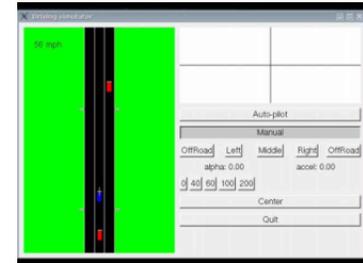
- Scientific inquiry
 - Model animal and human behavior
 - E.g., bee foraging, songbird vocalization. [See intro of Ng and Russell, 2000 for a brief overview.]
- Apprenticeship learning/Imitation learning through inverse RL
 - Presupposition: reward function provides the most succinct and transferable definition of the task
 - Has enabled advancing the state of the art in various robotic domains
- Modeling of other agents, both adversarial and cooperative

Lecture outline

- Example applications
- Inverse RL vs. behavioral cloning
- Historical sketch of inverse RL
- Mathematical formulations for inverse RL
- Case studies

Examples

- Simulated highway driving
 - Abbeel and Ng, ICML 2004,
 - Syed and Schapire, NIPS 2007
- Aerial imagery based navigation
 - Ratliff, Bagnell and Zinkevich, ICML 2006
- Parking lot navigation
 - Abbeel, Dolgov, Ng and Thrun, IROS 2008
- Urban navigation
 - Ziebart, Maas, Bagnell and Dey, AAAI 2008



Urban navigation

- Reward function for urban navigation?



→ destination prediction

Lecture outline

- Example applications
- *Inverse RL vs. behavioral cloning*
- Historical sketch of inverse RL
- Mathematical formulations for inverse RL
- Case studies

Problem setup

- Input:
 - State space, action space
 - Transition model $P_{sa}(s_{t+1} \mid s_t, a_t)$
 - *No* reward function
 - Teacher's demonstration: $s_0, a_0, s_1, a_1, s_2, a_2, \dots$
(= trace of the teacher's policy π^*)
- Inverse RL:
 - Can we recover R ?
- Apprenticeship learning via inverse RL
 - Can we then use this R to find a good policy ?
- Behavioral cloning
 - Can we directly learn the teacher's policy using supervised learning?

Behavioral cloning

- Formulate as standard machine learning problem
 - Fix a policy class
 - E.g., support vector machine, neural network, decision tree, deep belief net, ...
 - Estimate a policy (=mapping from states to actions) from the training examples $(s_0, a_0), (s_1, a_1), (s_2, a_2), \dots$
- Two of the most notable success stories:
 - Pomerleau, NIPS 1989: ALVINN
 - Sammut et al., ICML 1992: Learning to fly (flight sim)

Inverse RL vs. behavioral cloning

- **Which has the most succinct description: π^* vs. R^* ?**
- Especially in planning oriented tasks, the reward function is often much more succinct than the optimal policy.

Lecture outline

- Example applications
- Inverse RL vs. behavioral cloning
- *Historical sketch of inverse RL*
- Mathematical formulations for inverse RL
- Case studies

Inverse RL history

- 1964, Kalman posed the inverse optimal control problem and solved it in the 1D input case
- 1994, Boyd+al.: a linear matrix inequality (LMI) characterization for the general linear quadratic setting
- 2000, Ng and Russell: first MDP formulation, reward function ambiguity pointed out and a few solutions suggested
- 2004, Abbeel and Ng: inverse RL for apprenticeship learning---reward feature matching
- 2006, Ratliff+al: max margin formulation

Inverse RL history

- 2007, Ratliff+al: max margin with boosting---enables large vocabulary of reward features
- 2007, Ramachandran and Amir [R&A], and Neu and Szepesvari: reward function as characterization of policy class
- 2008, Kolter, Abbeel and Ng: hierarchical max-margin
- 2008, Syed and Schapire: feature matching + game theoretic formulation
- 2008, Ziebart+al: feature matching + max entropy
- 2008, Abbeel+al: feature matching -- application to learning parking lot navigation style
- 2009, Baker, Saxe, Tenenbaum: same formulation as [R&A], investigation of understanding of human inverse planning inference
- 2009, Mombaur, Truong, Laumond: human path planning
- Active inverse RL? Inverse RL w.r.t. minmax control, partial observability, learning stage (rather than observing optimal policy), ... ?

Lecture outline

- Example applications
- Inverse RL vs. behavioral cloning
- Historical sketch of inverse RL
- *Mathematical formulations for inverse RL*
- Case studies

Three broad categories of formalizations

- Max margin
- Feature expectation matching
- Interpret reward function as parameterization of a policy class

Basic principle

- Find a reward function R^* which explains the expert behaviour.

- Find R^* such that

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R^*(s_t) \mid \pi^*\right] \geq \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R^*(s_t) \mid \pi\right] \quad \forall \pi$$

- In fact a convex feasibility problem, but many challenges:
 - $R=0$ is a solution, more generally: reward function ambiguity
 - We typically only observe expert traces rather than the entire expert policy π^* --- how to compute left-hand side?
 - Assumes the expert is indeed optimal --- otherwise infeasible
 - Computationally: assumes we can enumerate all policies

Feature based reward function

- Let $R(s) = w^\top \phi(s)$, where $w \in \mathbb{R}^n$, and $\phi : S \rightarrow \mathbb{R}^n$.

$$\begin{aligned} \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t) | \pi\right] &= \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t w^\top \phi(s_t) | \pi\right] \\ &= w^\top \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t \phi(s_t) | \pi\right] \\ &= w^\top \underbrace{\mu(\pi)} \end{aligned}$$

Expected cumulative discounted sum of feature values or “feature expectations”

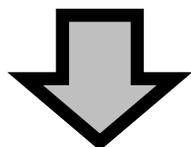
- Subbing into $\mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R^*(s_t) | \pi^*\right] \geq \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R^*(s_t) | \pi\right] \quad \forall \pi$

gives us:

$$\text{Find } w^* \text{ such that } w^{*\top} \mu(\pi^*) \geq w^{*\top} \mu(\pi) \quad \forall \pi$$

Feature based reward function

$$\mathbb{E}[\sum_{t=0}^{\infty} \gamma^t R^*(s_t) | \pi^*] \geq \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t R^*(s_t) | \pi] \quad \forall \pi$$



Let $R(s) = w^\top \phi(s)$, where $w \in \mathbb{R}^n$, and $\phi : S \rightarrow \mathbb{R}^n$.

Find w^* such that $w^{*\top} \mu(\pi^*) \geq w^{*\top} \mu(\pi) \quad \forall \pi$

- Feature expectations can be readily estimated from sample trajectories.
- The number of expert demonstrations required scales with the number of features in the reward function.
- The number of expert demonstration required does *not* depend on
 - Complexity of the expert's optimal policy π^*
 - Size of the state space

Recap of challenges

Let $R(s) = w^\top \phi(s)$, where $w \in \mathbb{R}^n$, and $\phi : S \rightarrow \mathbb{R}^n$.

$$w^{*\top} \mu(\pi^*) \geq w^{*\top} \mu(\pi) \quad \forall \pi$$

- Challenges:
 - Assumes we know the entire expert policy π^* → assumes we can estimate expert feature expectations
 - $R=0$ is a solution (now: $w=0$), more generally: reward function ambiguity
 - Assumes the expert is indeed optimal---became even more of an issue with the more limited reward function expressiveness!
 - Computationally: assumes we can enumerate all policies

Ambiguity

- Standard max margin:

$$\begin{aligned} \min_w \quad & \|w\|_2^2 \\ \text{s.t.} \quad & w^\top \mu(\pi^*) \geq w^\top \mu(\pi) + 1 \quad \forall \pi \end{aligned}$$

- “Structured prediction” max margin:

$$\begin{aligned} \min_w \quad & \|w\|_2^2 \\ \text{s.t.} \quad & w^\top \mu(\pi^*) \geq w^\top \mu(\pi) + m(\pi^*, \pi) \quad \forall \pi \end{aligned}$$

- Justification: margin should be larger for policies that are very different from π^* .
- Example: $m(\pi, \pi^*) =$ number of states in which π^* was observed and in which π and π^* disagree

Expert suboptimality

- Structured prediction max margin with slack variables:

$$\begin{aligned} \min_{w, \xi} \quad & \|w\|_2^2 + C\xi \\ \text{s.t.} \quad & w^\top \mu(\pi^*) \geq w^\top \mu(\pi) + m(\pi^*, \pi) - \xi \quad \forall \pi \end{aligned}$$

- Can be generalized to multiple MDPs (could also be same MDP with different initial state)

$$\begin{aligned} \min_{w, \xi^{(i)}} \quad & \|w\|_2^2 + C \sum_i \xi^{(i)} \\ \text{s.t.} \quad & w^\top \mu(\pi^{(i)*}) \geq w^\top \mu(\pi^{(i)}) + m(\pi^{(i)*}, \pi^{(i)}) - \xi^{(i)} \quad \forall i, \pi^{(i)} \end{aligned}$$

Complete max-margin formulation

$$\begin{aligned} \min_w \quad & \|w\|_2^2 + C \sum_i \xi^{(i)} \\ \text{s.t.} \quad & w^\top \mu(\pi^{(i)*}) \geq w^\top \mu(\pi^{(i)}) + m(\pi^{(i)*}, \pi^{(i)}) - \xi^{(i)} \quad \forall i, \pi^{(i)} \end{aligned}$$

[Ratliff, Zinkevich and Bagnell, 2006]

- Resolved: access to π^* , ambiguity, expert suboptimality
- One challenge remains: very large number of constraints
 - Ratliff+al use subgradient methods.
 - In this lecture: constraint generation

Constraint generation

Initialize $\Pi^{(i)} = \{\}$ for all i and then iterate

- Solve

$$\begin{aligned} \min_w \quad & \|w\|_2^2 + C \sum_i \xi^{(i)} \\ \text{s.t.} \quad & w^\top \mu(\pi^{(i)*}) \geq w^\top \mu(\pi^{(i)}) + m(\pi^{(i)*}, \pi^{(i)}) - \xi^{(i)} \quad \forall i, \forall \pi^{(i)} \in \Pi^{(i)} \end{aligned}$$

- For current value of w , find the most violated constraint for all i by solving:

$$\max_{\pi^{(i)}} w^\top \mu(\pi^{(i)}) + m(\pi^{(i)*}, \pi^{(i)})$$

= find the optimal policy for the current estimate of the reward function (+ loss augmentation m)

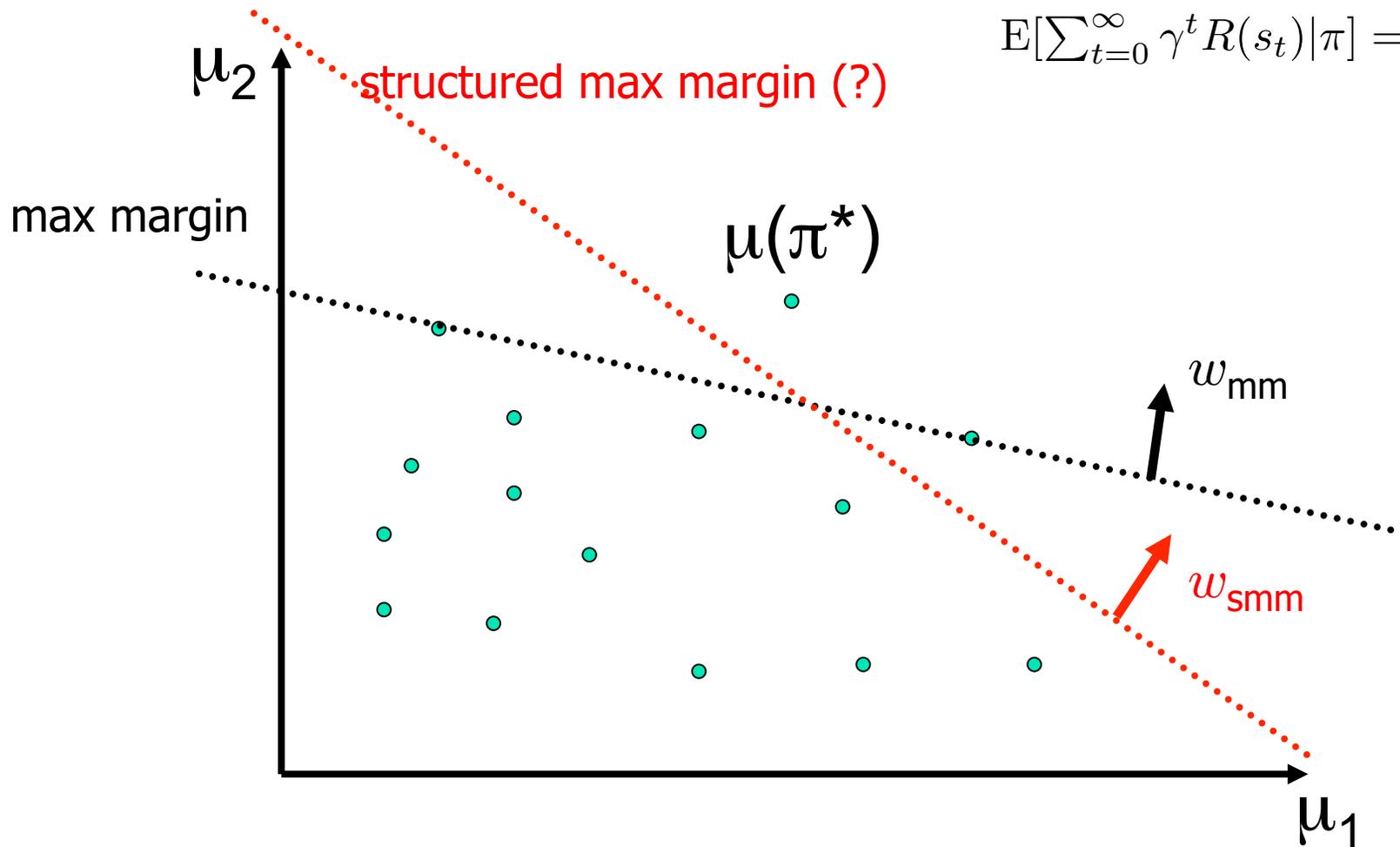
- For all i add $\pi^{(i)}$ to $\Pi^{(i)}$

- If no constraint violations were found, we are done.

Visualization in feature expectation space

- Every policy π has a corresponding feature expectation vector $\mu(\pi)$, which for visualization purposes we assume to be 2D

$$E[\sum_{t=0}^{\infty} \gamma^t R(s_t) | \pi] = w^\top \mu(\pi)$$

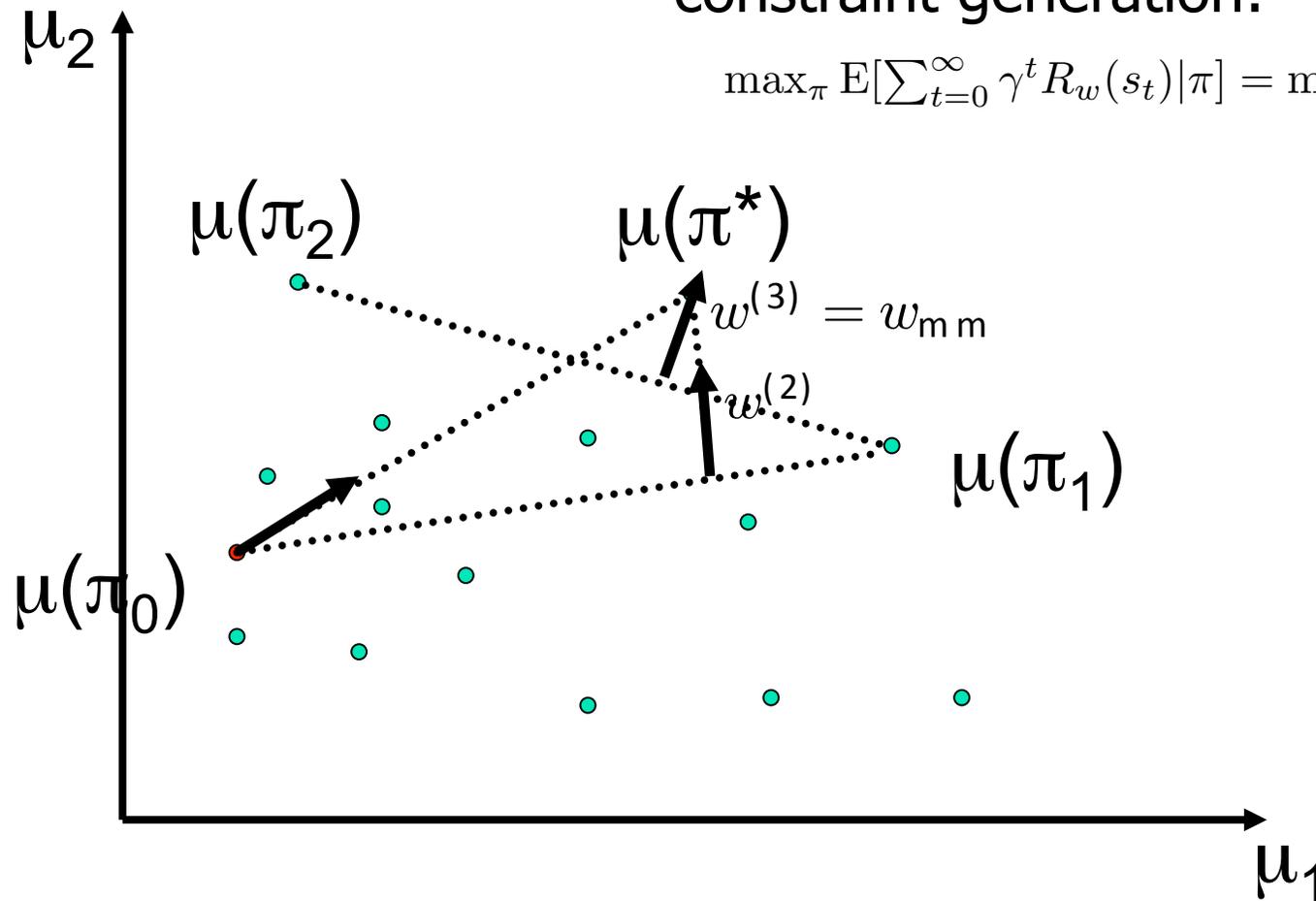


Constraint generation

- Every policy π has a corresponding feature expectation vector $\mu(\pi)$, which for visualization purposes we assume to be 2D

constraint generation:

$$\max_{\pi} E[\sum_{t=0}^{\infty} \gamma^t R_w(s_t) | \pi] = \max_{\pi} w^{\top} \mu(\pi)$$



Three broad categories of formalizations

- Max margin (Ratliff+al, 2006)
 - Feature boosting [Ratliff+al, 2007]
 - Hierarchical formulation [Kolter+al, 2008]
- *Feature expectation matching (Abbeel+Ng, 2004)*
 - *Two player game formulation of feature matching (Syed +Schapire, 2008)*
 - *Max entropy formulation of feature matching (Ziebart+al,2008)*
- Interpret reward function as parameterization of a policy class. (Neu +Szepesvari, 2007; Ramachandran+Amir, 2007; Baker, Saxe, Tenenbaum, 2009; Mombaur, Truong, Laumond, 2009)

Feature matching

- Inverse RL starting point: find a reward function such that the expert outperforms other policies

Let $R(s) = w^\top \phi(s)$, where $w \in \mathbb{R}^n$, and $\phi : S \rightarrow \mathbb{R}^n$.

Find w^* such that $w^{*\top} \mu(\pi^*) \geq w^{*\top} \mu(\pi) \quad \forall \pi$

- Observation in Abbeel and Ng, 2004: for a policy π to be guaranteed to perform as well as the expert policy π^* , it suffices that the feature expectations match:

$$\|\mu(\pi) - \mu(\pi^*)\|_1 \leq \epsilon$$

implies that for all w with $\|w\|_\infty \leq 1$:

$$|w^{*\top} \mu(\pi) - w^{*\top} \mu(\pi^*)| \leq \epsilon$$

Apprenticeship learning [Abbeel & Ng, 2004]

- Assume $R_w(s) = w^\top \phi(s)$ for a feature map $\phi : S \rightarrow \mathbb{R}^n$.
- Initialize: pick some controller π_0 .
- Iterate for $i = 1, 2, \dots$:

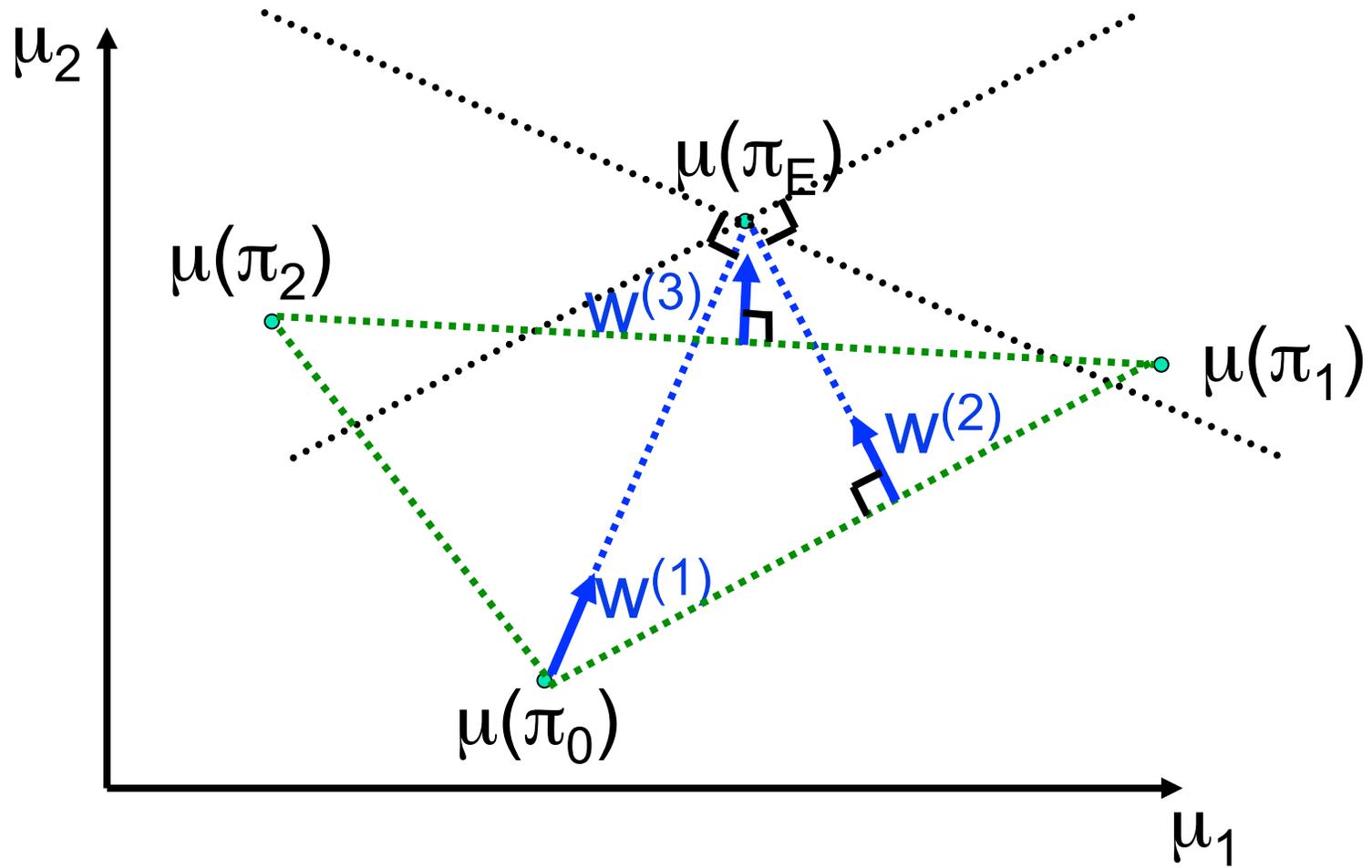
- **“Guess” the reward function:**

Find a reward function such that the teacher maximally outperforms all previously found controllers.

$$\begin{aligned} & \max_{\gamma, w: \|w\|_2 \leq 1} \gamma \\ & \text{s.t. } w^\top \mu(\pi^*) \geq w^\top \mu(\pi) + \gamma \quad \forall \pi \in \{\pi_0, \pi_1, \dots, \pi_{i-1}\} \end{aligned}$$

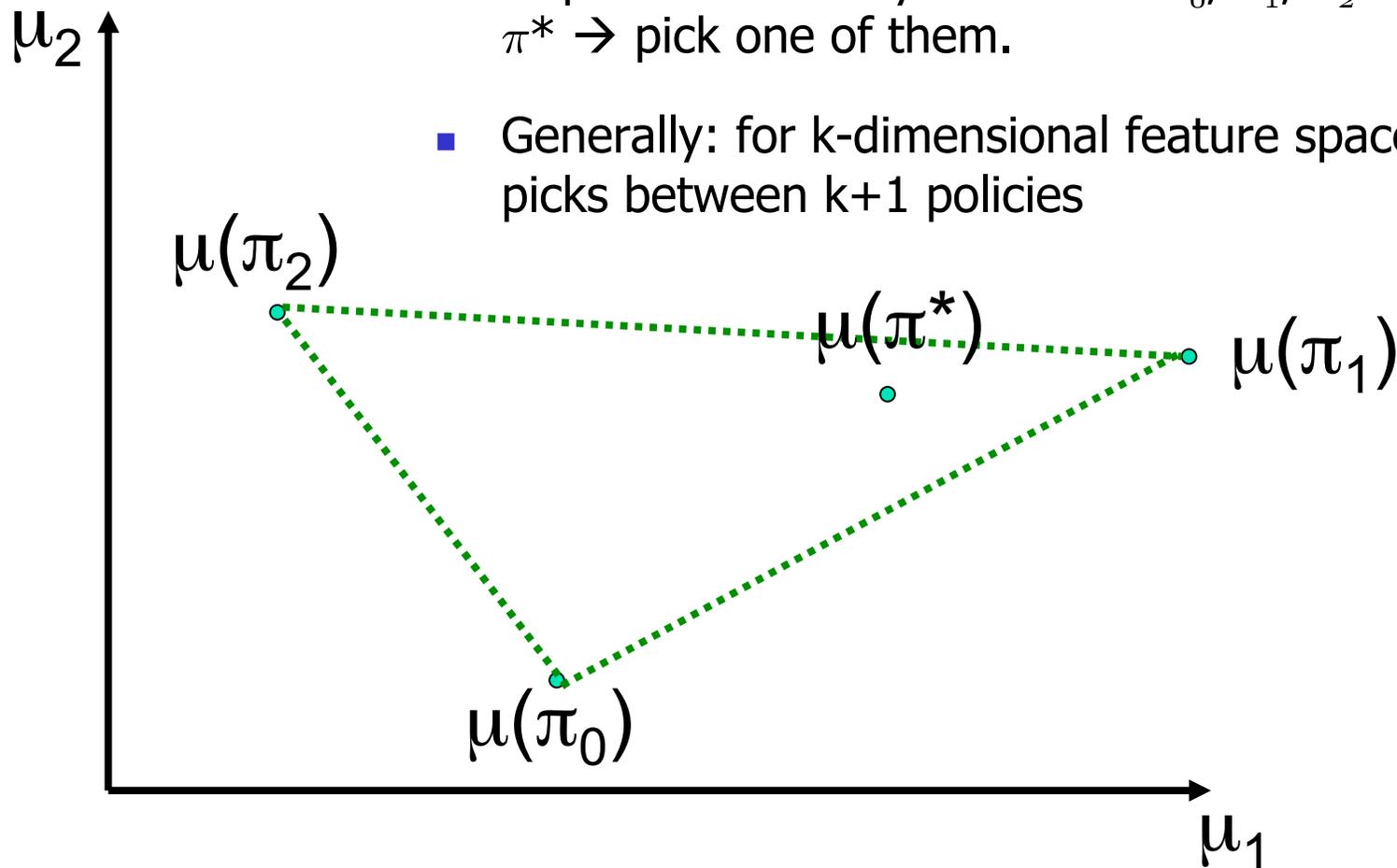
- **Find optimal control policy** π_i for the current guess of the reward function R_w .
- If $\gamma \leq \varepsilon/2$ exit the algorithm.

Algorithm example run



Suboptimal expert case

- Can match expert by stochastically mixing between 3 policies
- In practice: for any w^* one of π_0, π_1, π_2 outperforms $\pi^* \rightarrow$ pick one of them.
- Generally: for k -dimensional feature space the user picks between $k+1$ policies

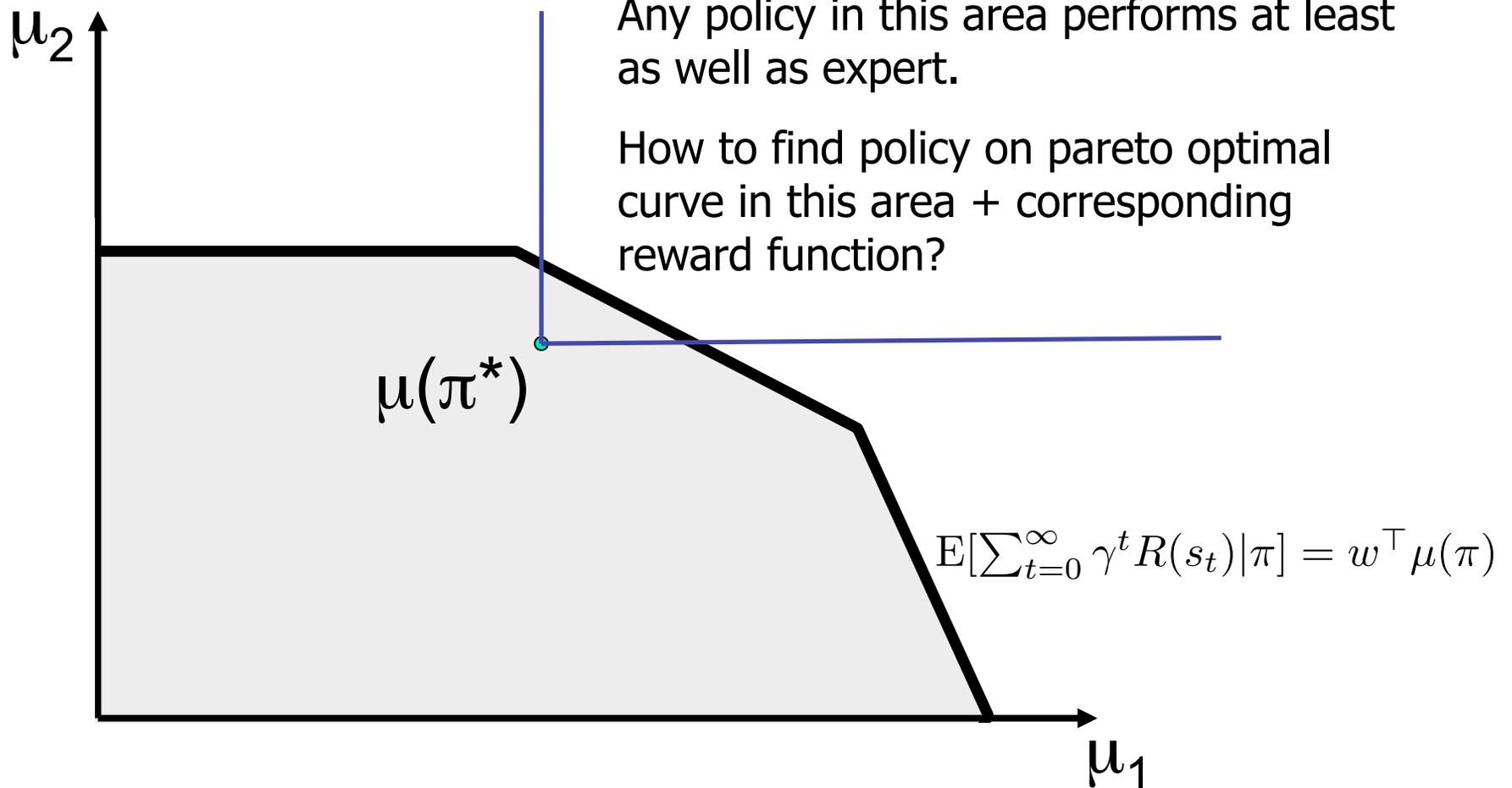


Feature expectation matching

- If expert suboptimal then the resulting policy is a mixture of somewhat arbitrary policies which have expert in their convex hull.
- In practice: pick the best one of this set and pick the corresponding reward function.
- Next:
 - Syed and Schapire, 2008.
 - Ziebart+al, 2008.

Min-Max feature expectation matching Syed and Schapire (2008)

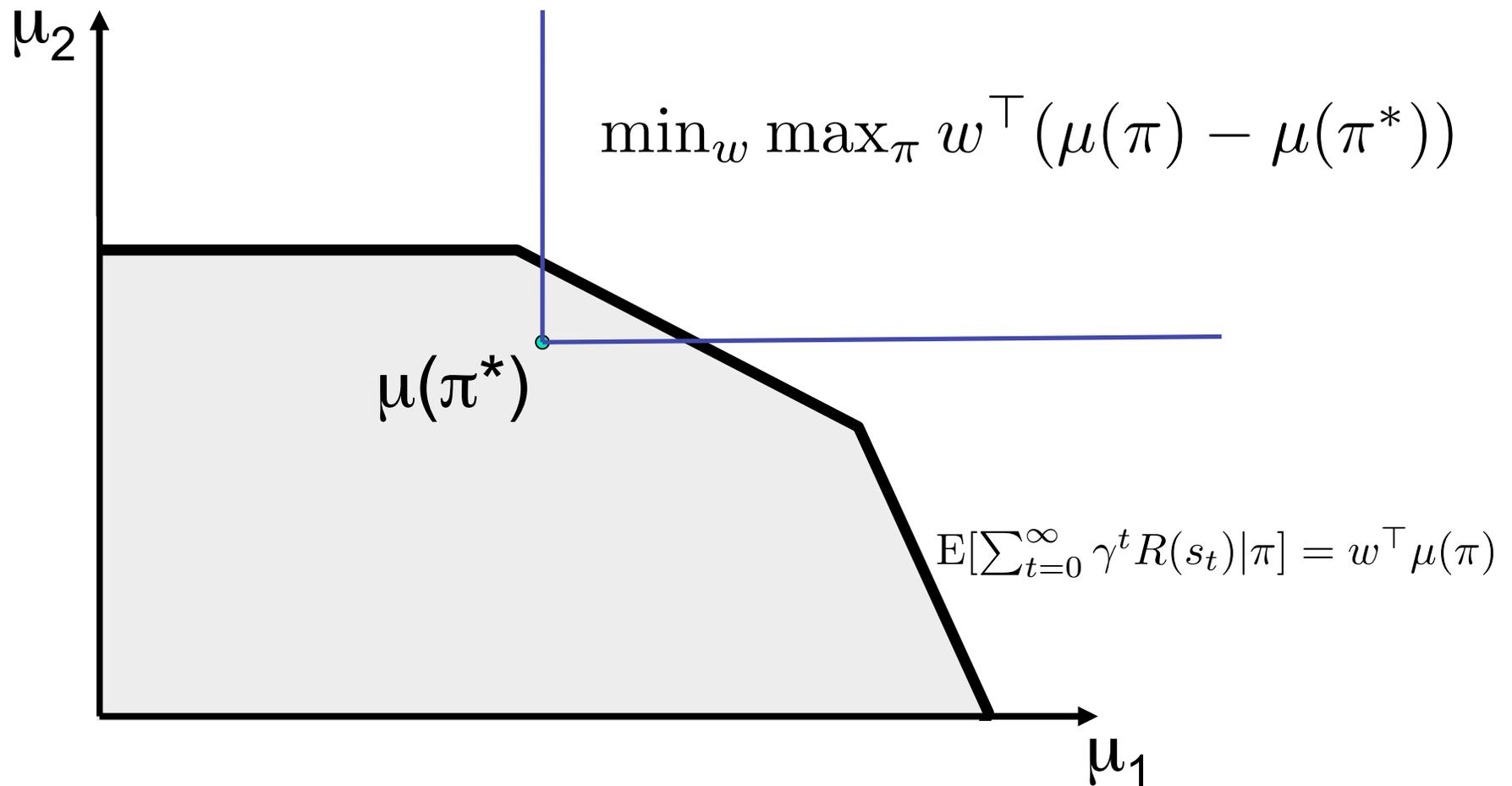
Additional assumption: $w \geq 0$, $\sum_i w_i = 1$.



Min-Max feature expectation matching

Syed and Schapire (2008)

Additional assumption: $w \geq 0$, $\sum_i w_i = 1$.



Min max games

- Example of standard min-max game setting:

rock-paper-scissors pay-off matrix:

		<i>maximizer</i>		
		rock	paper	scissors
<i>minimizer</i>	rock	0	1	-1
	paper	-1	0	1
	scissors	1	-1	0

pay-off matrix G

$$\min_{w_m: w_m \geq 0, \|w_m\|_1=1} \max_{w_M: w_M \geq 0, \|w_M\|_1=1} w_m^\top G w_M$$

Nash equilibrium solution is mixed strategy: $(1/3, 1/3, 1/3)$ for both players

Min-Max feature expectation matching

Syed and Schapire (2008)

- Standard min-max game:

$$\min_{w_m: w_m \geq 0, \|w_m\|_1=1} \max_{w_M: w_M \geq 0, \|w_M\|_1=1} w_m^\top G w_M$$

- Min-max inverse RL:

$$\min_{w: \|w\|_1=1, w \geq 0} \max_{\pi} w^\top (\mu(\pi) - \mu(\pi^*))$$

- Solution: maximize over weights λ which weigh the contribution of all policies $\pi_1, \pi_2, \dots, \pi_N$ to the mixed policy.

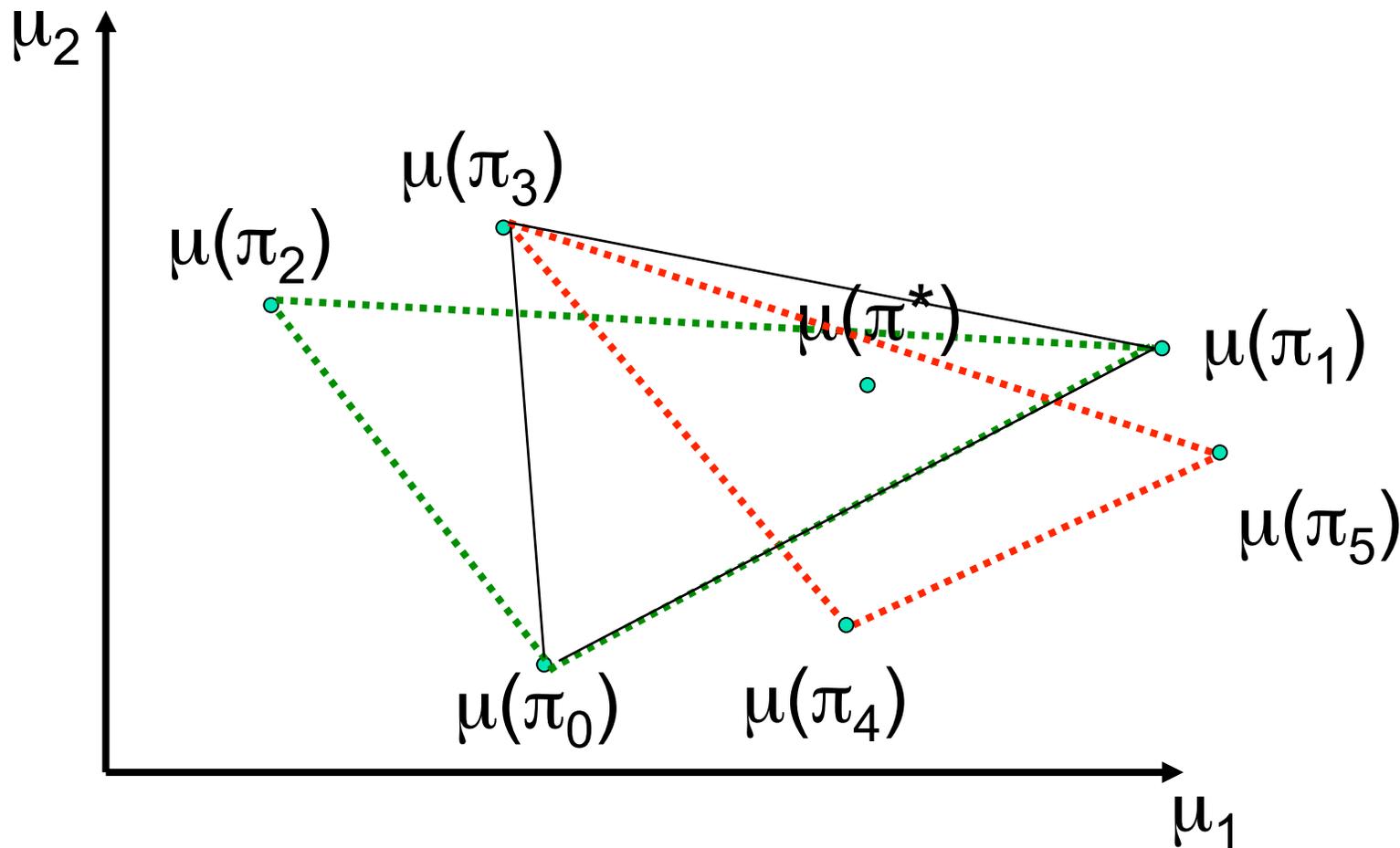
- Formally:

$$\min_w \max_{\lambda} w^\top G \lambda \quad G_{ij} = (\mu(\pi_j) - \mu(\pi^*))_i$$

- Remaining challenge: G very large! See paper for algorithm that only uses relevant parts of G . [Strong similarity with constraint generation schemes we have seen.]

Maximum-entropy feature expectation matching --- Ziebart+al, 2008

- Recall feature matching in suboptimal expert case:



Maximum-entropy feature expectation matching --- Ziebart+al, 2008

- Maximize entropy of distributions over paths followed while satisfying the constraint of feature expectation matching:

$$\begin{aligned} \max_P \quad & - \sum_{\zeta} P(\zeta) \log P(\zeta) \\ \text{s.t.} \quad & \sum_{\zeta} P(\zeta) \mu(\zeta) = \mu(\pi^*) \end{aligned}$$

- This turns out to imply that P is of the form:

$$P(\zeta) = \frac{1}{Z(w)} \exp(w^\top \mu(\zeta))$$

- See paper for algorithmic details.

Feature expectation matching

- If expert suboptimal:
 - *Abbeel and Ng, 2004*: resulting policy is a mixture of policies which have expert in their convex hull---In practice: pick the best one of this set and pick the corresponding reward function.
 - *Syed and Schapire, 2008* recast the same problem in game theoretic form which, at cost of adding in some prior knowledge, results in having a unique solution for policy and reward function.
 - *Ziebart+al, 2008* assume the expert stochastically chooses between paths where each path's log probability is given by its expected sum of rewards.

Lecture outline

- Example applications
- Inverse RL vs. behavioral cloning
- Historical sketch of inverse RL
- Mathematical formulations for inverse RL
 - Max-margin
 - Feature matching
 - *Reward function parameterizing the policy class*
- Case studies

Reward function parameterizing the policy class

- Recall:

$$V^*(s; R) = R(s) + \gamma \max_a \sum_{s'} P(s'|s, a) V^*(s; R)$$

$$Q^*(s, a; R) = R(s) + \gamma \sum_{s'} P(s'|s, a) V^*(s; R)$$

- Let's assume our expert acts according to:

$$\pi(a|s; R, \alpha) = \frac{1}{Z(s; R, \alpha)} \exp(\alpha Q^*(s, a; R))$$

- Then for any R and α , we can evaluate the likelihood of seeing a set of state-action pairs as follows:

$$P((s_1, a_1)) \dots P((s_m, a_m)) = \frac{1}{Z(s_1; R, \alpha)} \exp(\alpha Q^*(s_1, a_1; R)) \dots \frac{1}{Z(s_m; R, \alpha)} \exp(\alpha Q^*(s_m, a_m; R))$$

Reward function parameterizing the policy class

- Assume our expert acts according to:

$$\pi(a|s; R, \alpha) = \frac{1}{Z(s; R, \alpha)} \exp(\alpha Q^*(s, a; R))$$

- Then for any R and α , we can evaluate the likelihood of seeing a set of state-action pairs as follows:

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- Ramachandran and Amir, AAI2007: MCMC method to sample from this distribution
- Neu and Szepesvari, UAI2007: gradient method to optimize the likelihood [MAP]
- Baker, Saxe and Tenenbaum, Cognition 2009: only 3 possible reward functions → tractable exact Bayesian inference

Reward function parameterizing the policy class --- deterministic systems

- Assume deterministic system $x_{t+1} = f(x_t, u_t)$ and an observed trajectory $(x_0^*, x_1^*, \dots, x_T^*)$
- Find reward function by solving:

$$\min_w \sum_{t=0}^T \|x_t^* - x_t^w\|_2$$

s.t. x^w is the solution of:

$$\max_x \sum_{t=0}^T \sum_i w_i \phi_i(x_t)$$

$$\text{s.t. } x_{t+1} = f(x_t, u_t)$$

$$x_0 = x_0^*, \quad x_T = x_T^*$$

Lecture outline

- Example applications
- Inverse RL vs. behavioral cloning
- History of inverse RL
- Mathematical formulations for inverse RL
- *Case studies: (1) Highway driving, (2) Crusher, (3) Parking lot navigation, (4) Route inference, (5) Human path planning, (6) Human inverse planning, (7) Quadruped locomotion*

Simulated highway driving



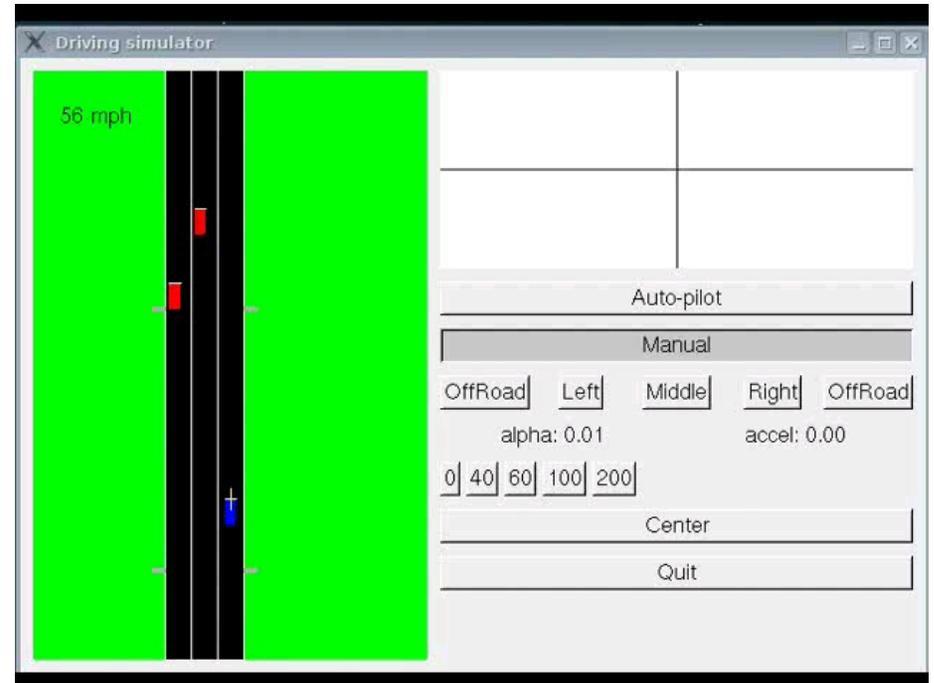
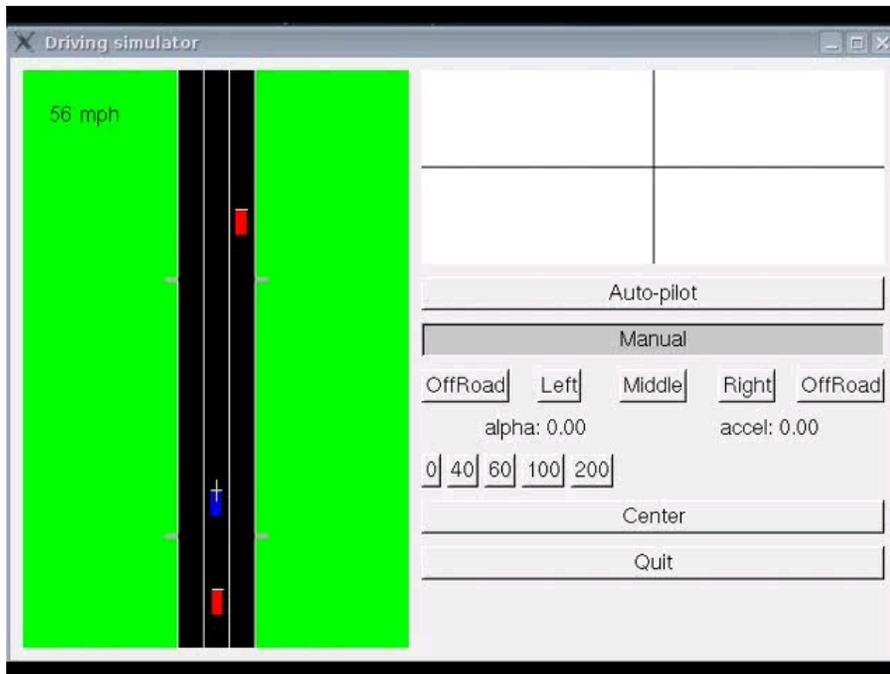
Abbeel and Ng, ICML 2004; Syed and Schapire, NIPS 2007

Highway driving

[Abbeel and Ng 2004]

Teacher in Training World

Learned Policy in Testing World



■ Input:

- Dynamics model / Simulator $P_{sa}(s_{t+1} | s_t, a_t)$
- Teacher's demonstration: 1 minute in "training world"
- Note: R^* is unknown.
- Reward features: 5 features corresponding to lanes/shoulders; 10 features corresponding to presence of other car in current lane at different distances

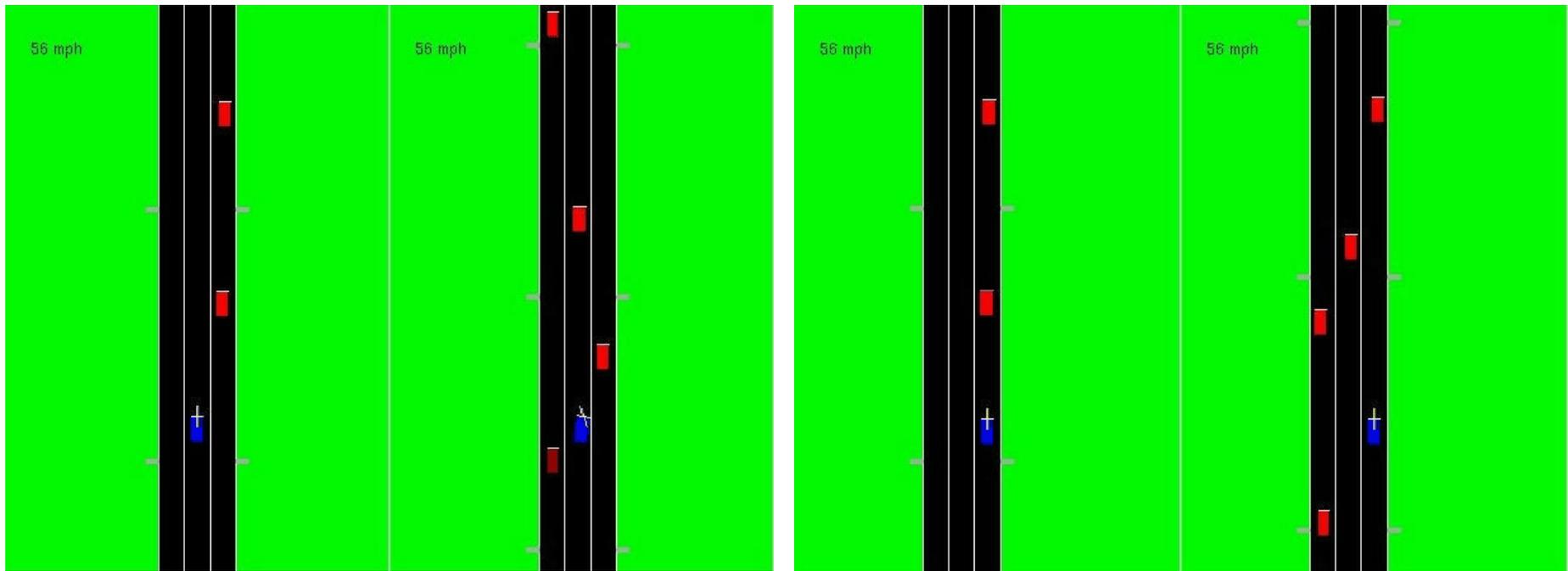
More driving examples [Abbeel and Ng 2004]

Driving demonstration

Learned behavior

Driving demonstration

Learned behavior



In each video, the left sub-panel shows a demonstration of a different driving “style”, and the right sub-panel shows the behavior learned from watching the demonstration.

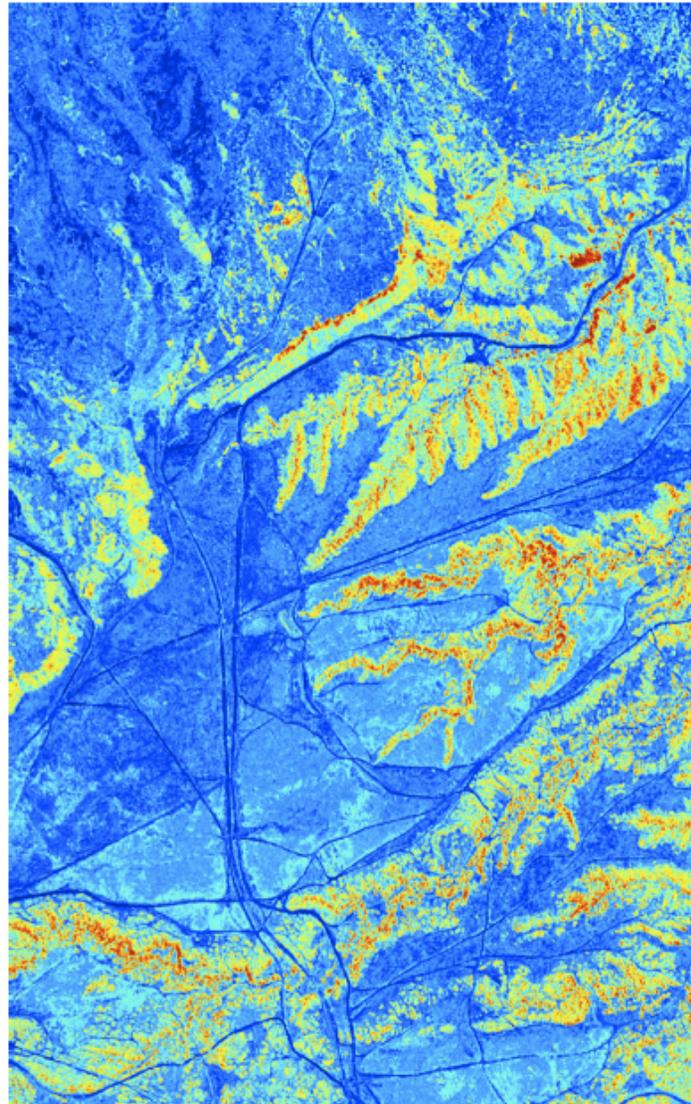


RSS 2008: Dave Silver and Drew Bagnell



example path

Max margin



[Ratliff + al, 2006/7/8]

Parking lot navigation

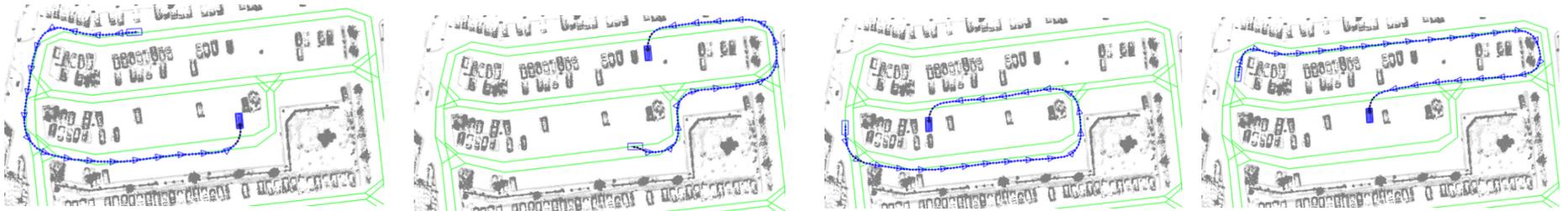


- Reward function trades off:
 - Staying "on-road,"
 - Forward vs. reverse driving,
 - Amount of switching between forward and reverse,
 - Lane keeping,
 - On-road vs. off-road,
 - Curvature of paths.

[Abbeel et al., IROS 08]

Experimental setup

- Demonstrate parking lot navigation on “train parking lots.”



- Run our apprenticeship learning algorithm to find the reward function.
- Receive “test parking lot” map + starting point and destination.
- Find the trajectory that maximizes the *learned reward function* for navigating the test parking lot.

Nice driving style



Sloppy driving-style



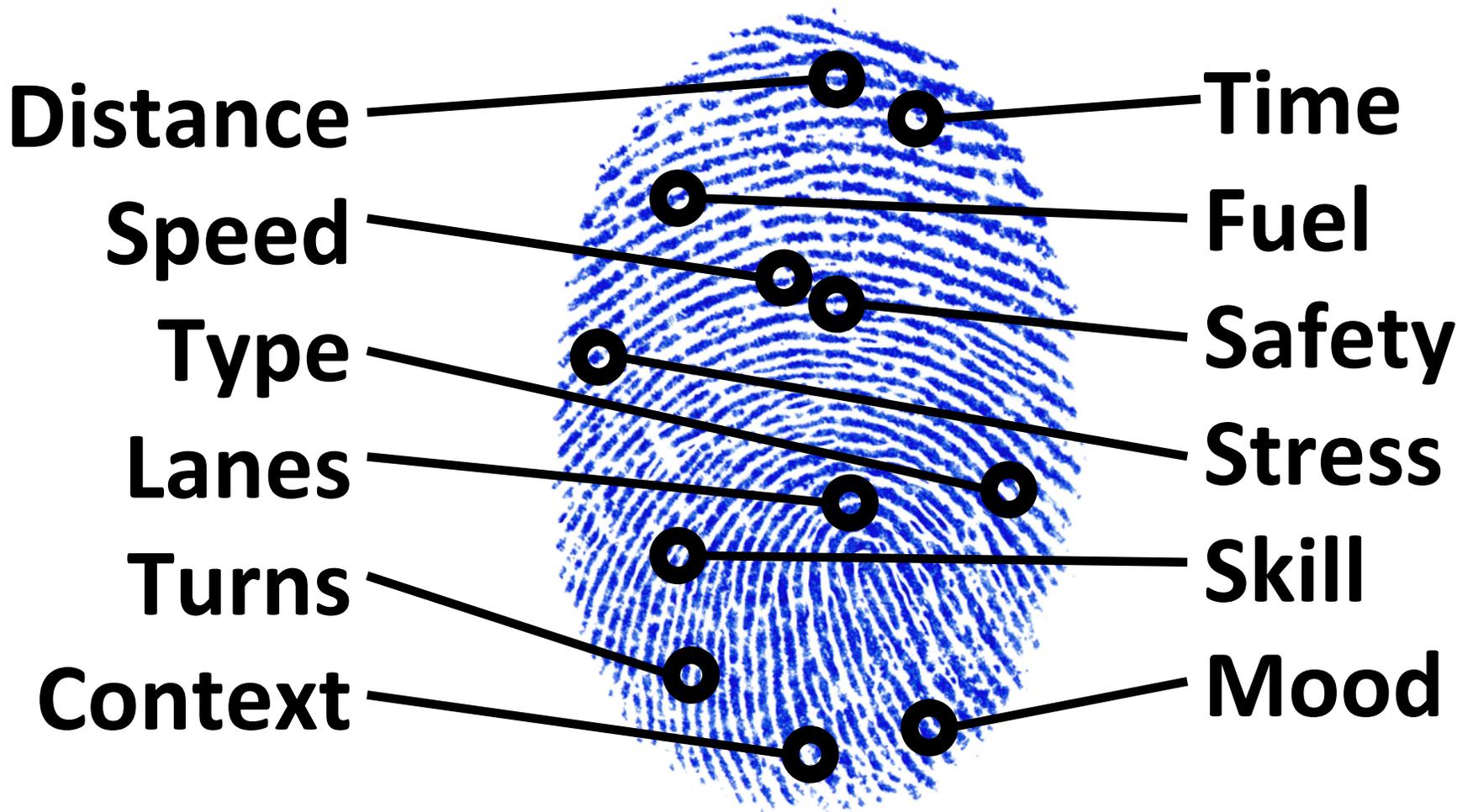
“Don't mind reverse” driving-style





**Only 35% of routes are
“fastest”** (Letchner, Krumm, &
Horvitz 2006)





Data Collection

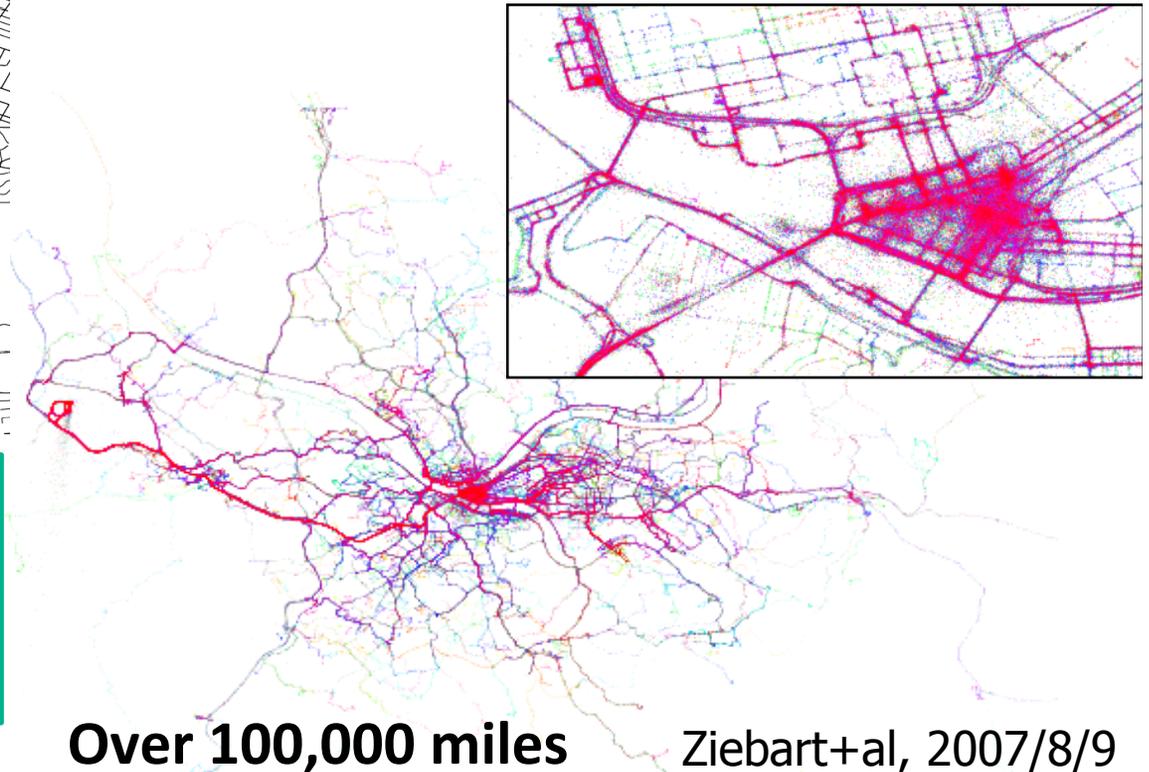


25 Taxi Drivers



Length
Speed
Road
Type
Lanes

Accidents
Construction
Congestion
Time of day



Over 100,000 miles

Ziebart+al, 2007/8/9

Destination Prediction



Human path planning

- Reward features:
 - Time to destination
 - $(\text{Forward acceleration})^2$
 - $(\text{Sideways acceleration})^2$
 - $(\text{Rotational acceleration})^2$
 - $\text{Integral}(\text{angular error})^2$



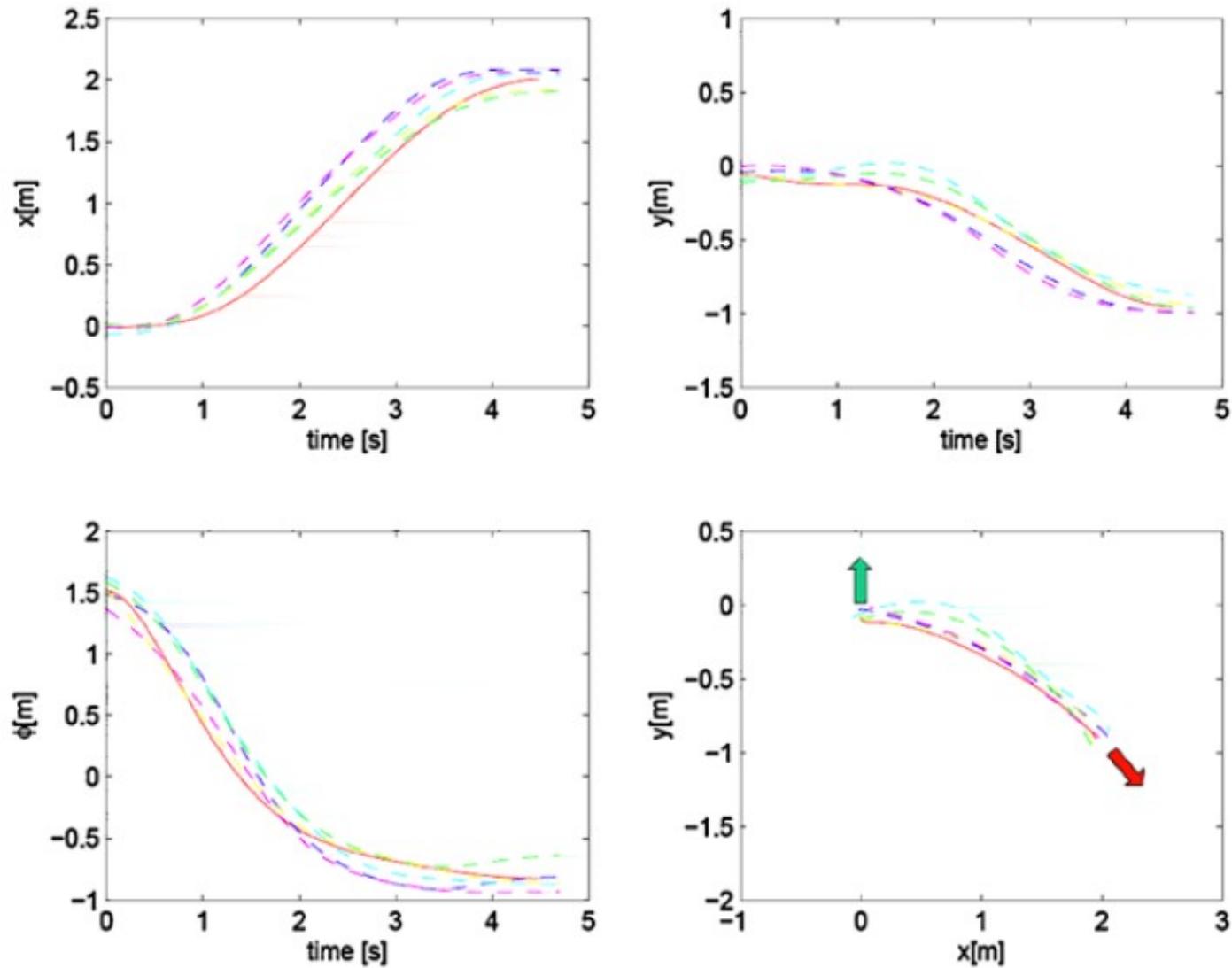
Human path planning



- Result:

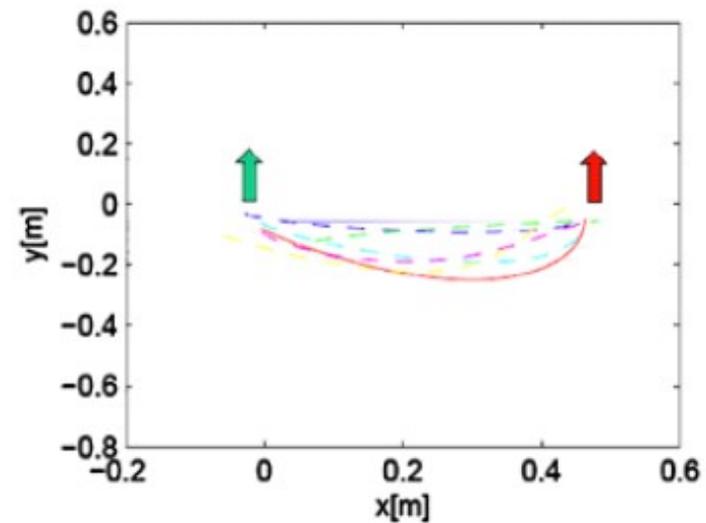
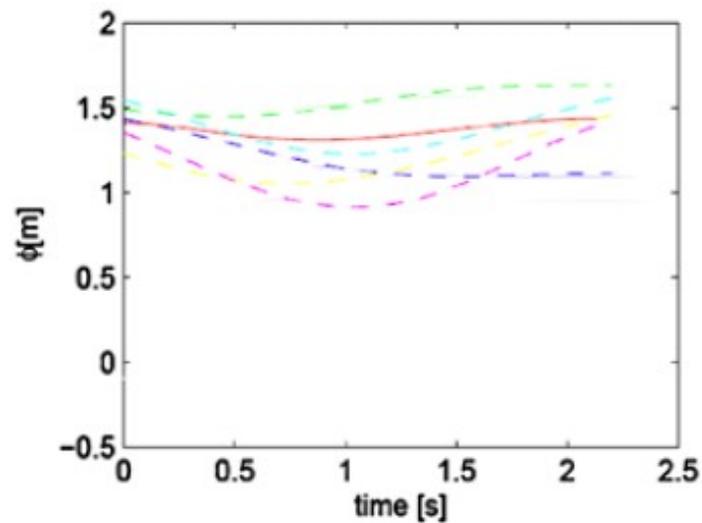
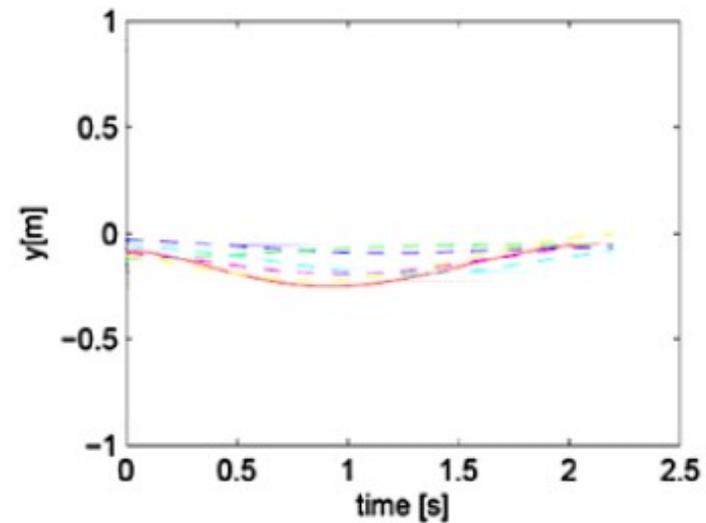
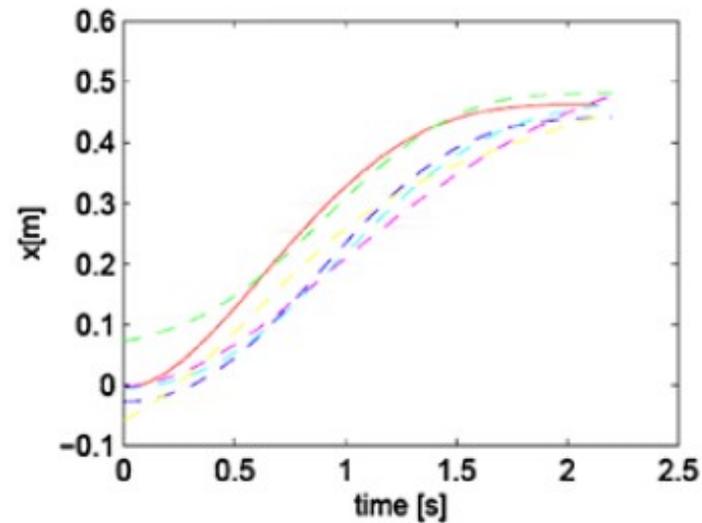
- Time to destination: 1
- (Forward acceleration)² 1.2
- (Sideways acceleration)² 1.7
- (Rotational acceleration)² 0.7
- Integral (angular error)² 5.2

Human path planning



[Mombaur, Truong, Laumond, 2009]

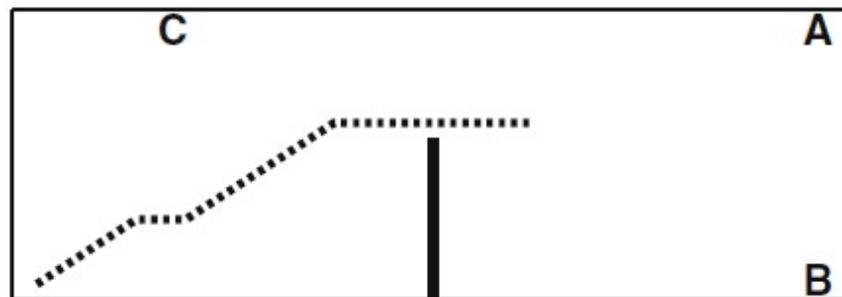
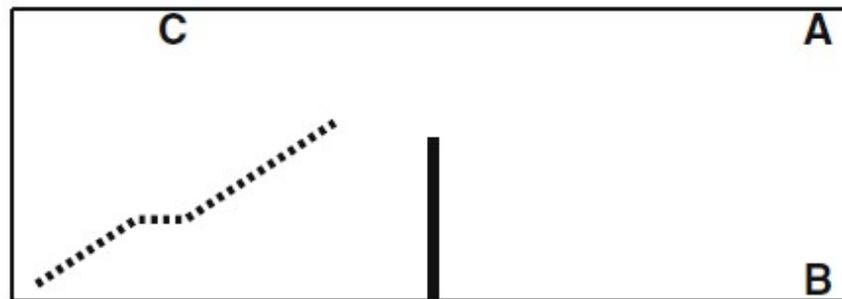
Human path planning



[Mombaur, Truong, Laumond, 2009]

Goal inference

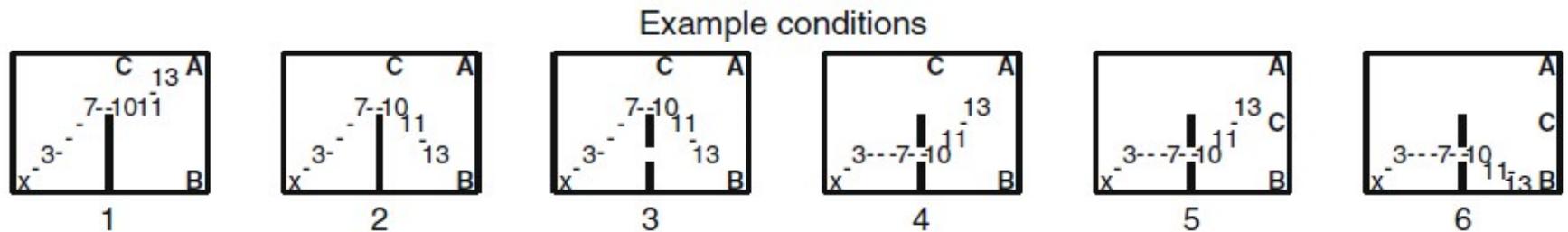
- Observe partial paths, predict goal. Goal could be either A, B, or C.
- + HMM-like extension: goal can change (with some probability over time).



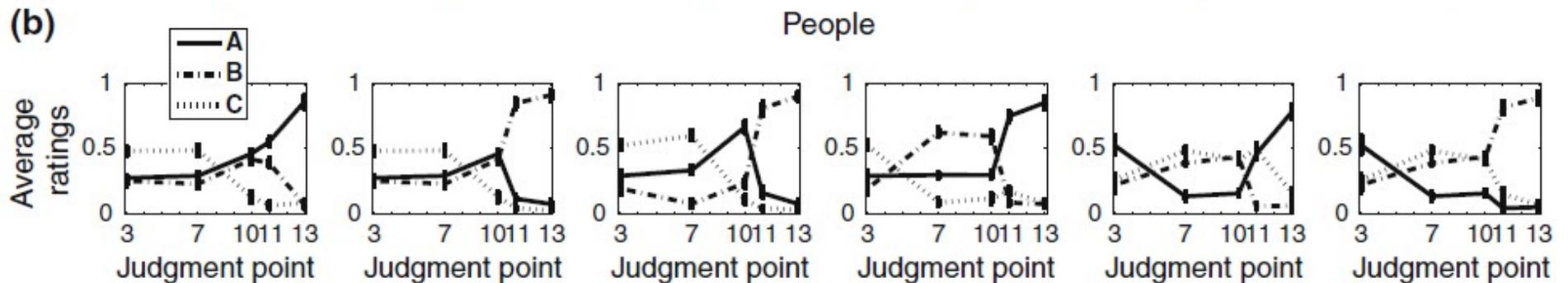
[Baker, Saxe, Tenenbaum, 2009]

Goal inference

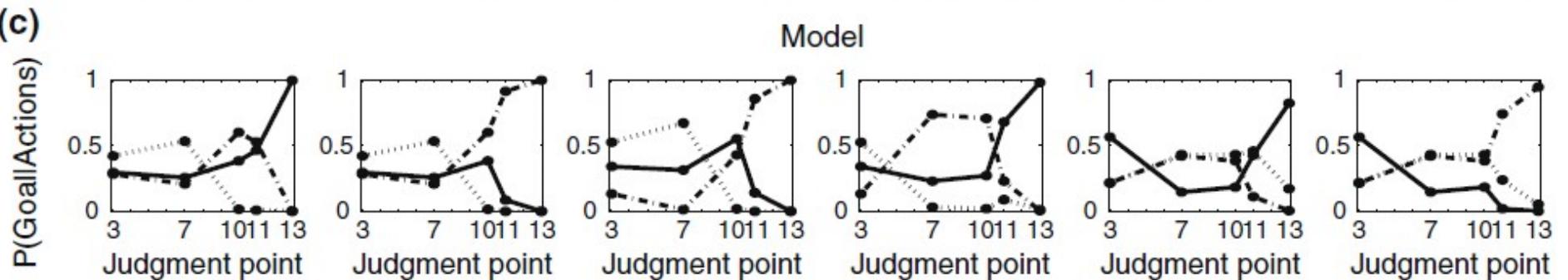
(a)



(b)



(c)



Quadruped

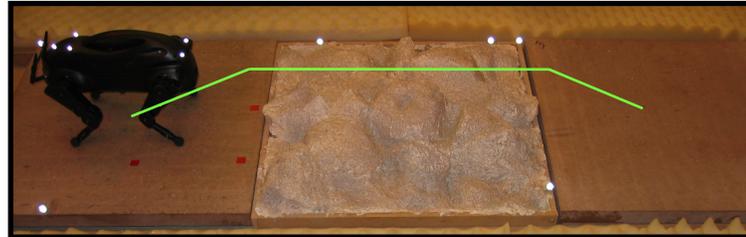


- Reward function trades off 25 features.

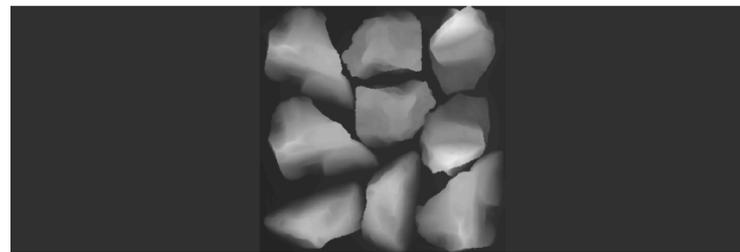
Hierarchical max margin [Kolter, Abbeel & Ng, 2008]

Experimental setup

- Demonstrate path across the “training terrain”



- Run our apprenticeship learning algorithm to find the reward function
- Receive “testing terrain”---height map.



- Find the optimal policy with respect to the *learned reward function* for crossing the testing terrain.

Without learning



With learned reward function



Quadruped: Ratliff + al, 2007

- Run footstep planner as expert (slow!)
- Run boosted max margin to find a reward function that explains the center of gravity path of the robot (smaller state space)
- At control time: use the learned reward function as a heuristic for A^* search when performing footstep-level planning

Summary

- Example applications
- Inverse RL vs. behavioral cloning
- Sketch of history of inverse RL
- Mathematical formulations for inverse RL
- Case studies

- Open directions: Active inverse RL, Inverse RL w.r.t. minmax control, partial observability, learning stage (rather than observing optimal policy), ... ?