SCP for trajectory optimization

- Basic problem
  - minimize_{traj} path_length + other costs
  - subject to pose constraints, joint limits, “no collisions”
- Why use optimization for planning?
  - Solve high-DOF problems
  - Smooth solutions
  - Encode preferences
  - It’s wicked fast
- Why SCP rather than some other descent method?
  - Deals with hard constraints and discontinuous costs stably and robustly
  - Solver isn’t the bottleneck anyway

SCP in general

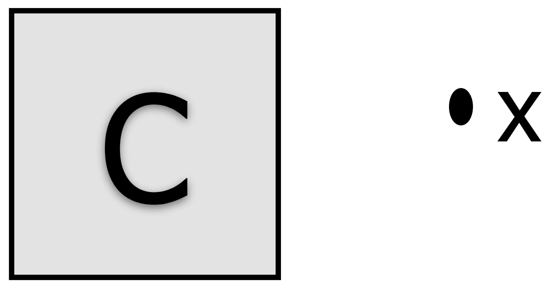
minimize f(x)
subject to $g(x) \leq 0$

where f, g, may not be convex

- repeat until convergence:
  - convexify objective and constraints
  - solve convex approximation to problem
  - recalculate actual objective
  - if objective decreased
    - shrink trust region
  - else
    - accept update

Non-overlap constraints

- Any kind of collision cost/constraint is non-convex, but we can locally approximate it as convex
  - simple example: consider constraint $x \notin C$



  - For convex C, this is an “OR” of linear constraints
  - Approximation: only impose constraint/cost from closest side to current x

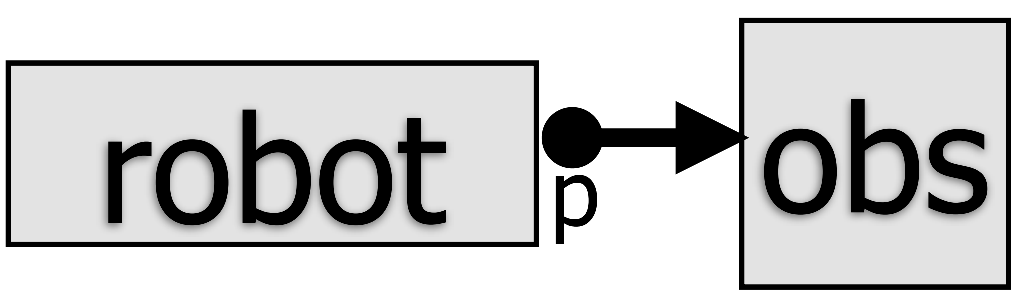
Signed distance

- distance(shape1, shape2) = length of shortest translation that puts them in contact. (for non-overlapping shapes)
- penetration_depth(shape1, shape2) = length of shortest translation that takes them out of contact (for overlapping shapes)
- signed_distance(shape1, shape2) =
  - if overlapping: - penetration_depth
  - else: + distance

- There are efficient algorithms for convex shapes, based on considering Minkowski difference
  - GJK: find if convex set contains the origin
  - EPA: find distance from origin to exterior

Collision cost

- Decompose the robot into convex parts
- Cost: $$\sum_t \sum_{i,j} |d_{safe} - \text{signeddist}(part_i, obstacle_j)|^+$$
- Convexification
  - detect all near-collisions
  - for each near-collision, linearize position of closest point using Jacobian



$$\Delta p = J \Delta \theta$$

$$\Delta d = \hat{n} \cdot \dot{J} \Delta \theta$$

Two problems

- Need to make collision cost high enough to get out of all collisions
  - solution: increate collision cost coefficient
- Need to make sure trajectory is **continuous-time** safe
  - solution: subdivide trajectory in collision intervals

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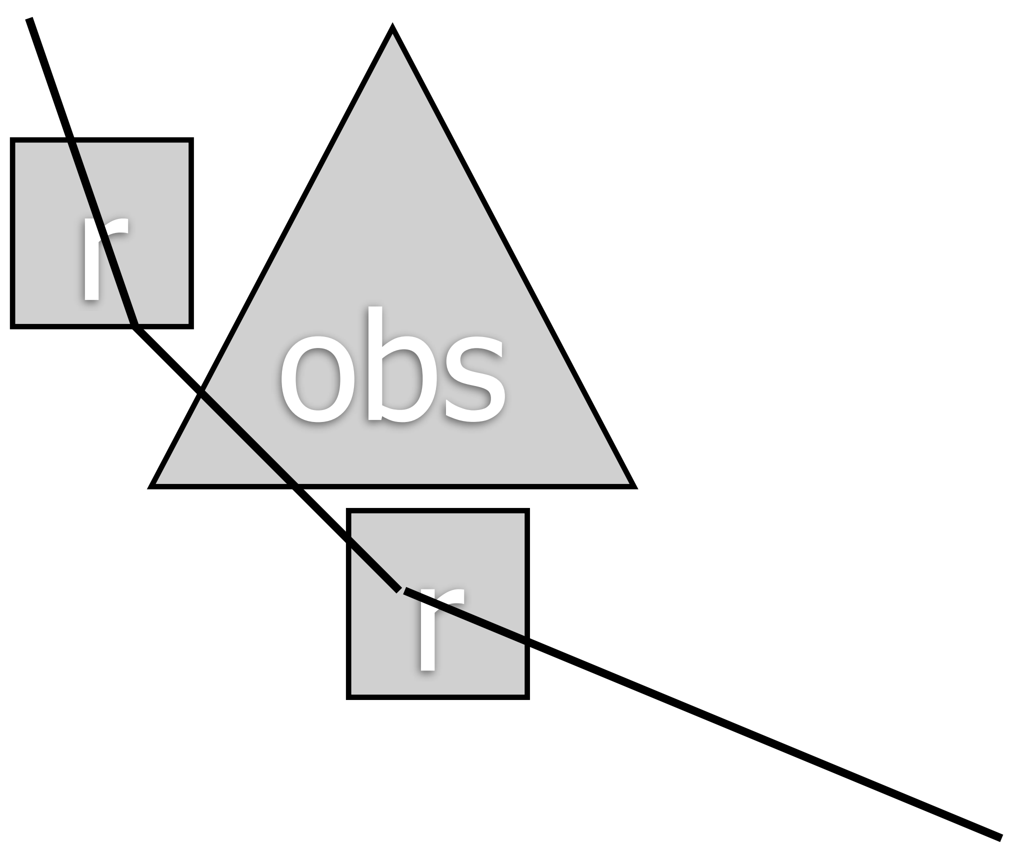

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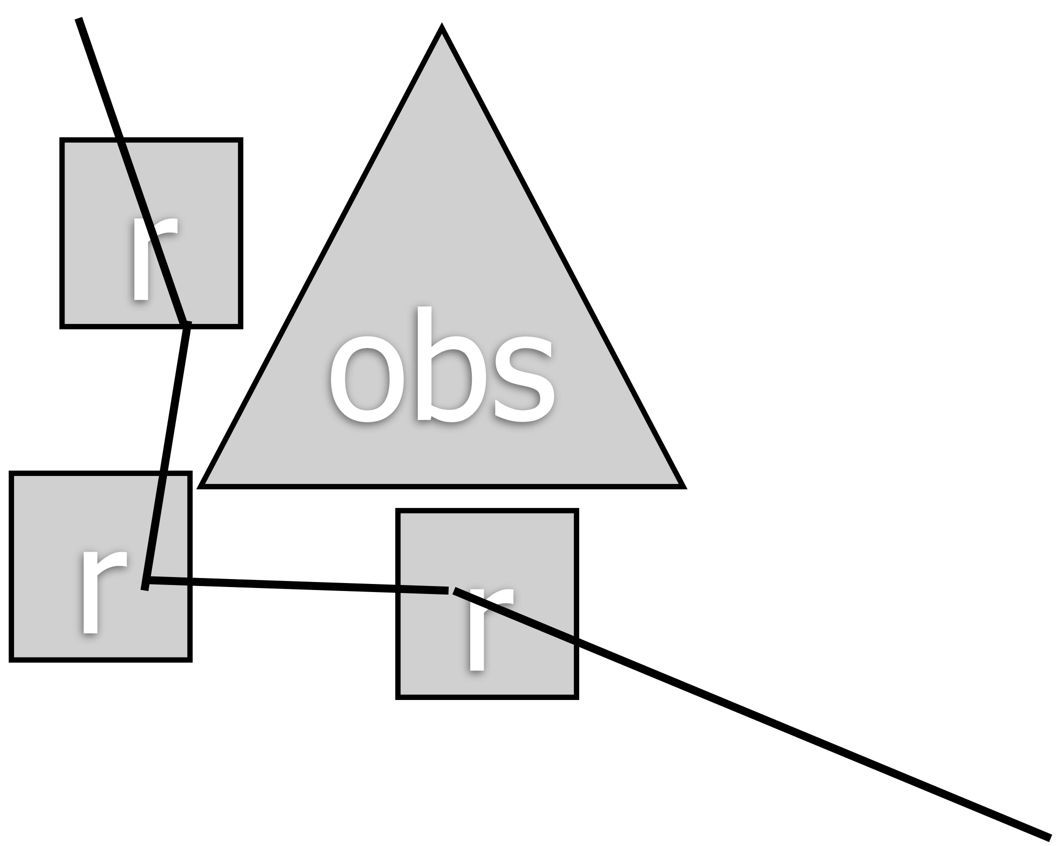


Trajectory optimization: outer loop

```
while true:
 do sqp optimization
 if trajectory is not discrete-time safe:
   increase penalty parameter
   continue
 if traj is not continuous-time safe:
   subdivide collision intervals
   continue
 break
```

Demo videos

How to make SCP fast

- Convexification
  - If func evaluation is expensive, use analytic gradients
- Solving
  - Warm-start
  - Use a fast solver that exploits sparsity (any trajectory problem has banded-diagonal structure)
- Fast convergence
  - Use adaptive trust region adjustment

```
If exact_improvement > .2 * approx_improvement:
   expand trust region
Else:
   shrink trust region
```

Robot LfD: comparison of techniques

- Inverse Optimal Control
  - Learn the objective function from human demonstrations, then do optimal control
  - e.g. Abbeel & Ng, 2004
- Trajectory learning
  - Learn a trajectory, the control inputs that achieve it, and a dynamics model
  - e.g. Abbeel, Coates, and Ng 2010
- Behavioral cloning
  - Learn mapping between states and actions
  - e.g. Calinon, Guenter, and Billard 2007
  - the following work

When can’t we use traditional planning & opt. ctrl?

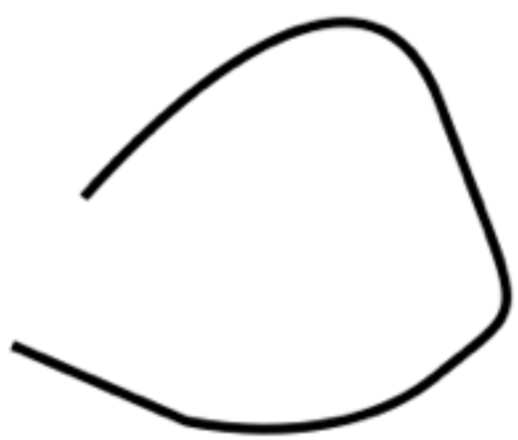
- Planning problem is hard
  - state space is big and you don’t get any gradient info
  - e.g. with deformable objects like rope or cloth
- Can’t simulate
  - e.g. we don’t want to do a fluid simulation to figure out how to pour liquid
- Can simulate, but unable to perceive the full state
  - e.g. crumpled up clothing article

Generalizing trajectories

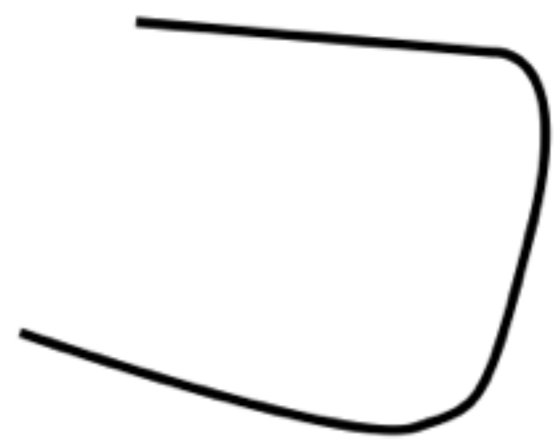
- Abstract problem: given a bunch of demonstrations of a task, (scene_1, traj_1), (scene_2, traj_2) ..., learn to generate a correct trajectory given a new scene

Knot tying

- very hard to program
- To my knowledge, no one has gotten a robot to autonomously and robustly tie knots with a closed-loop procedure
- The most basic problem:

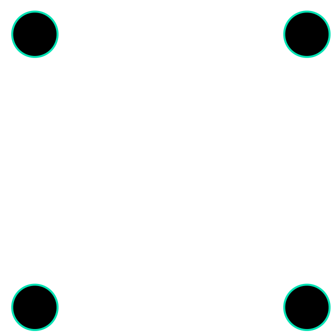


given a demonstrated
motion
on this rope...



generate an
appropriate motion
for this rope

Cartoon Problem Setting



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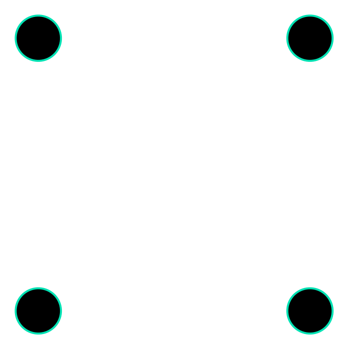
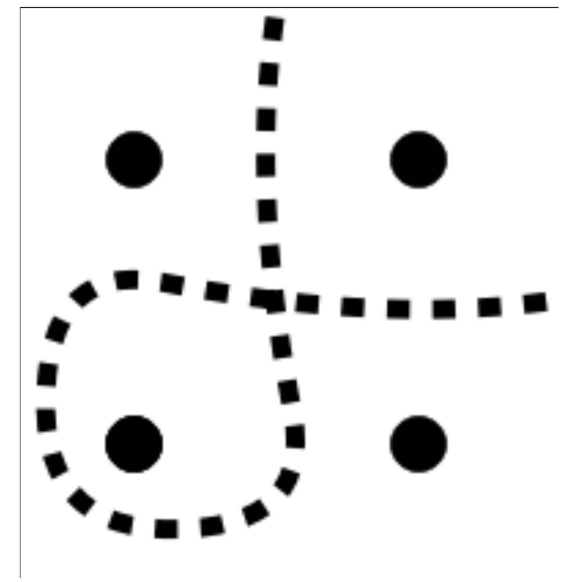
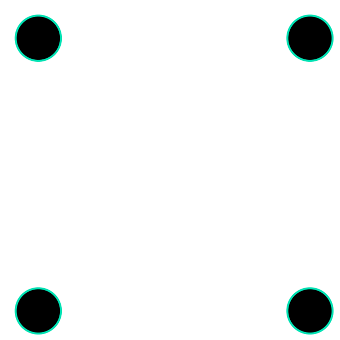


demonstration: --- trajectory



Cartoon Problem Setting

Train situation:



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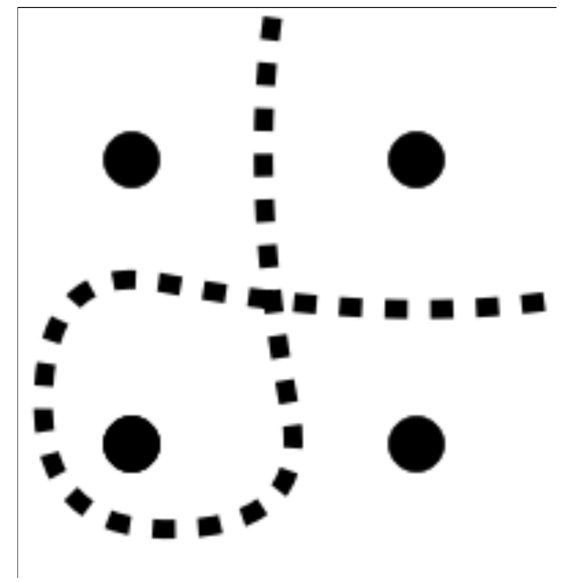

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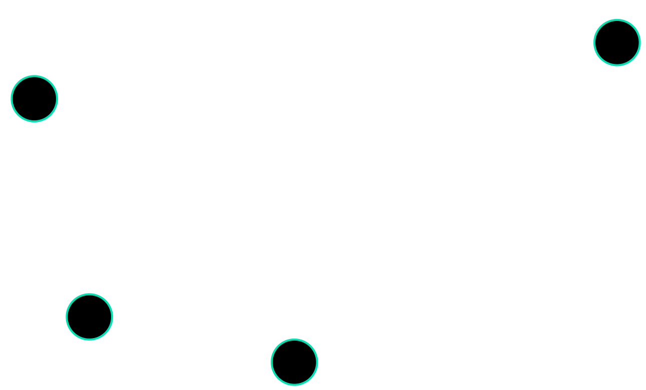

How to perform action here?

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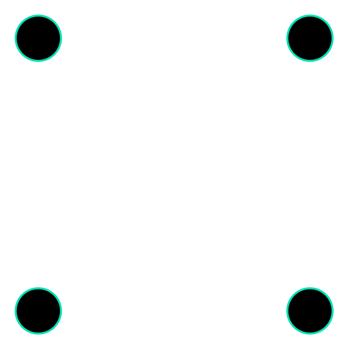

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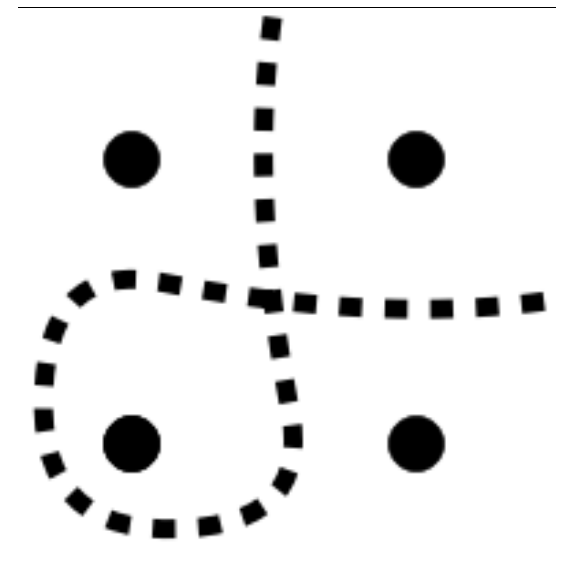

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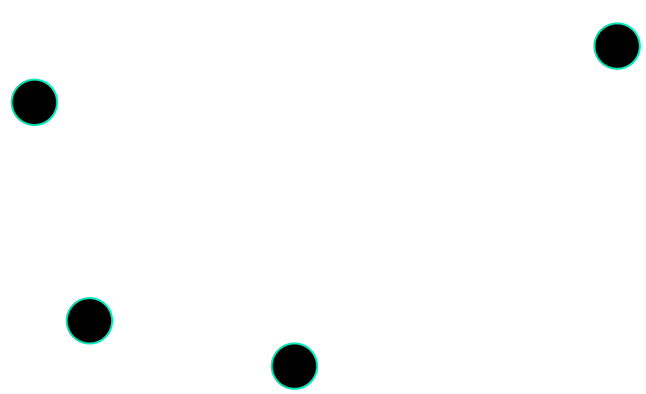

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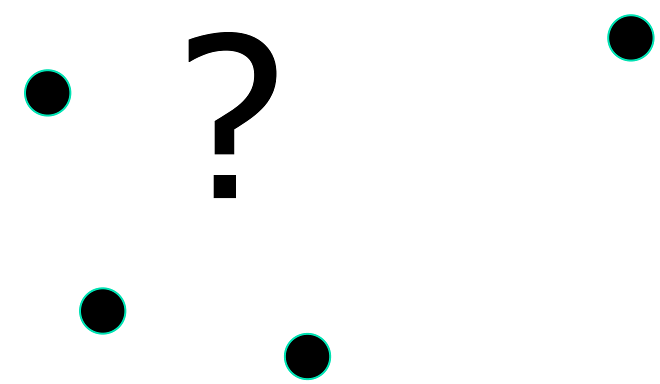

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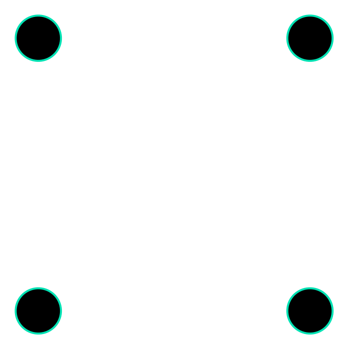

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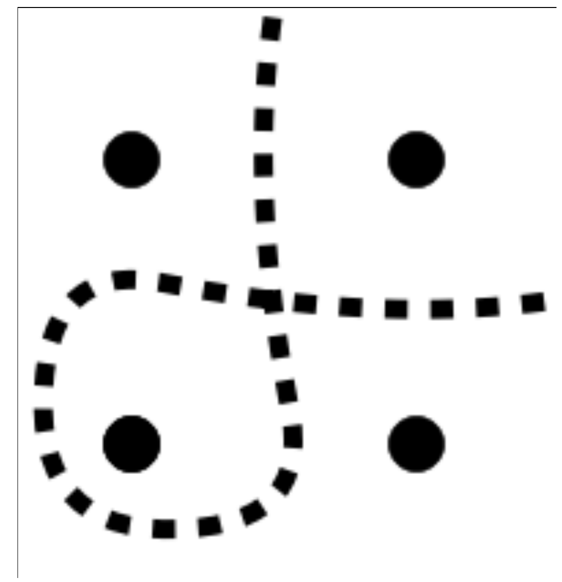

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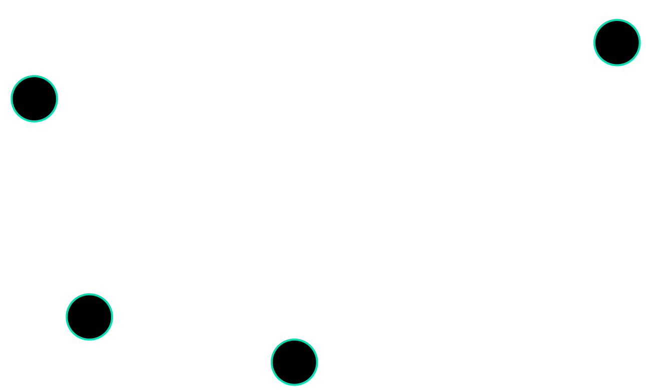

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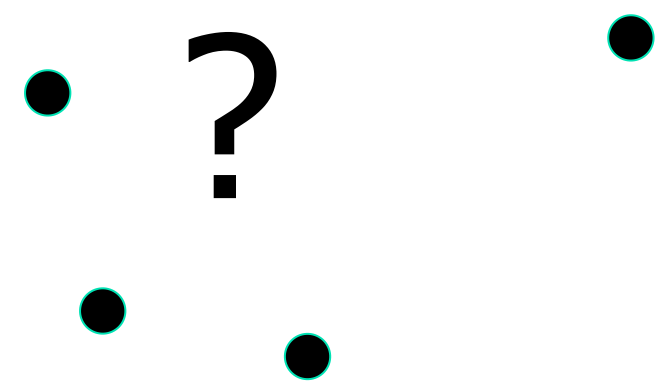

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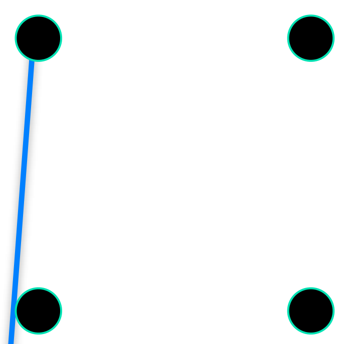

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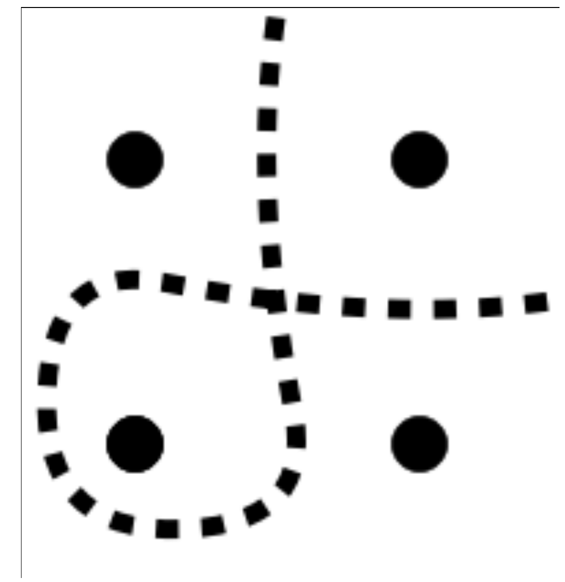

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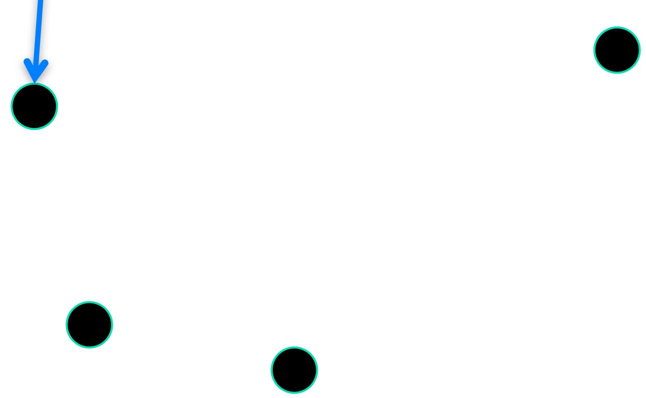

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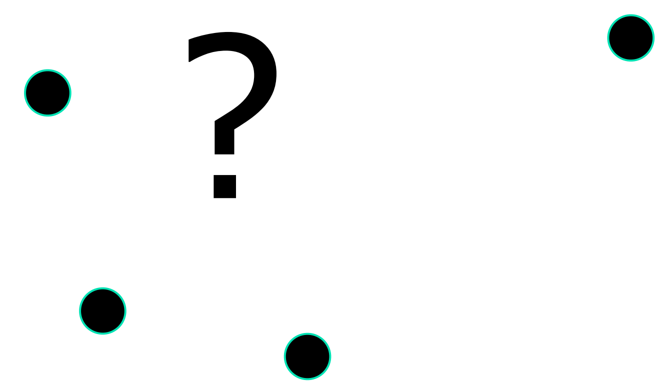

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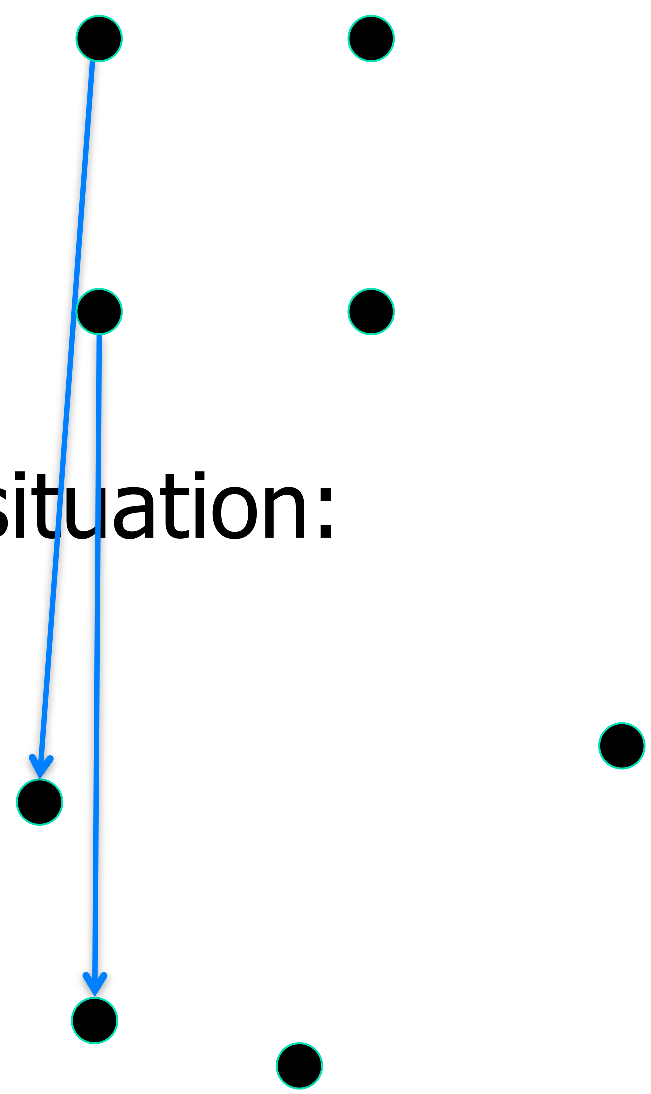

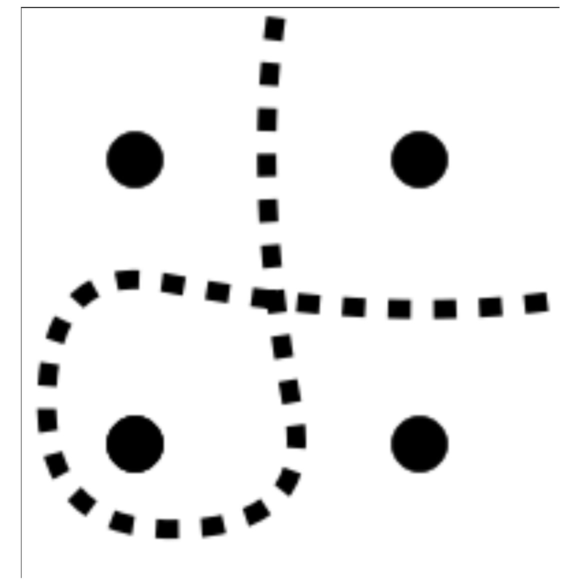

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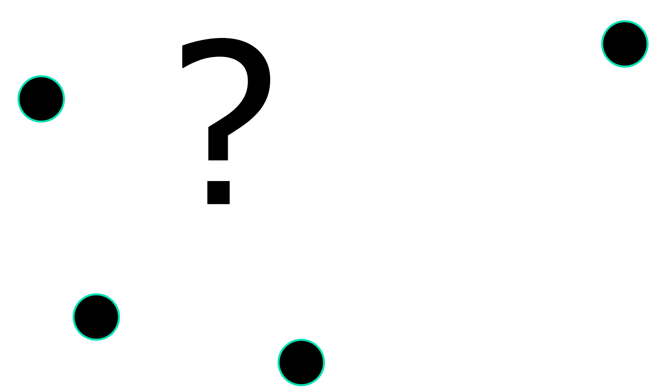

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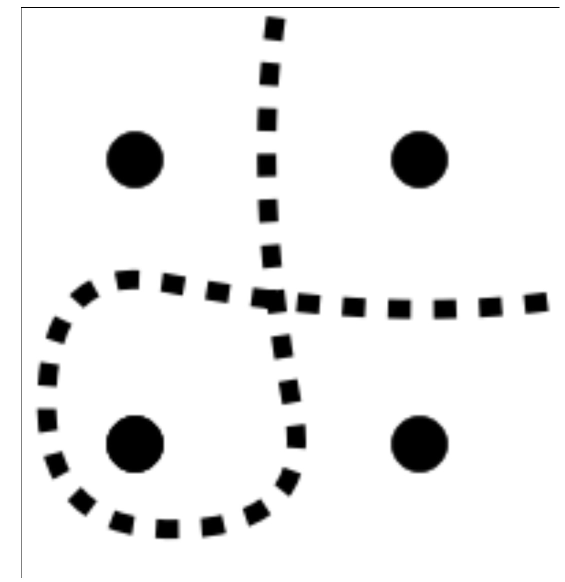

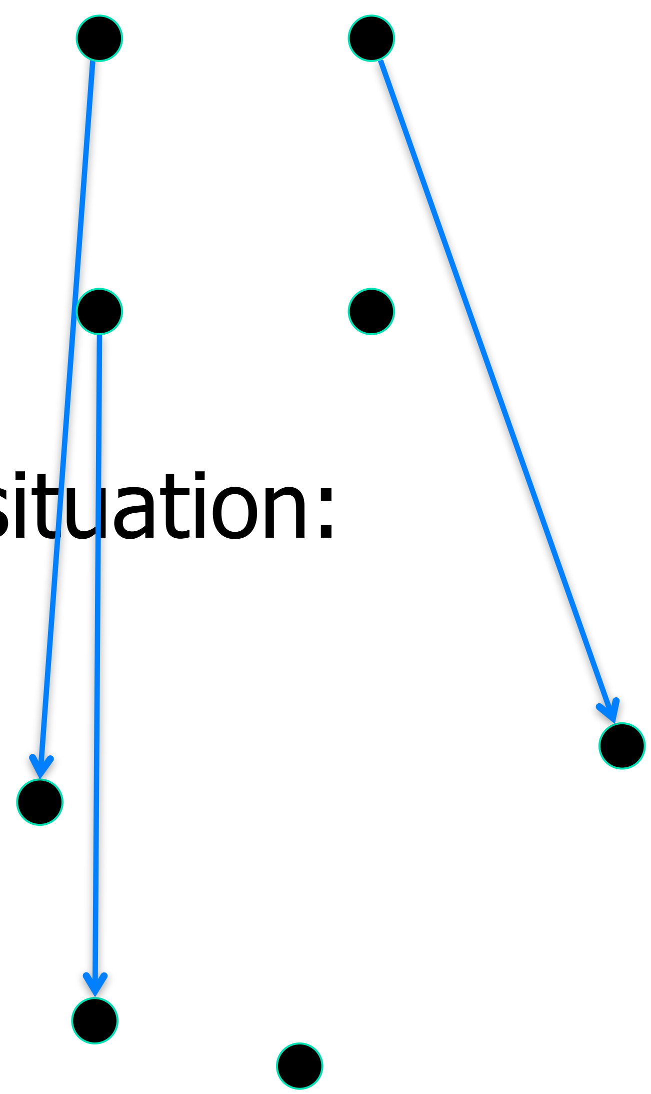

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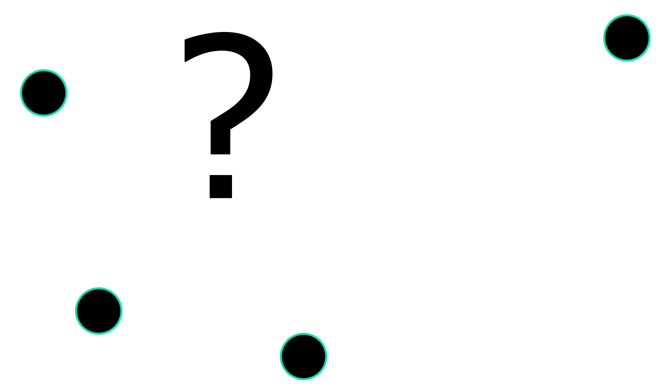

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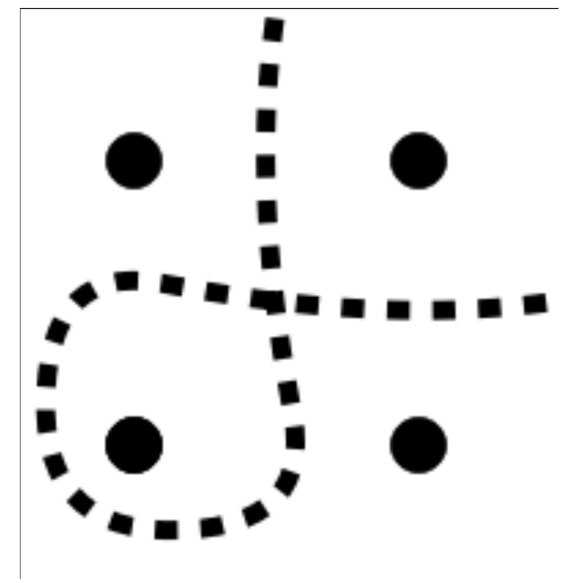

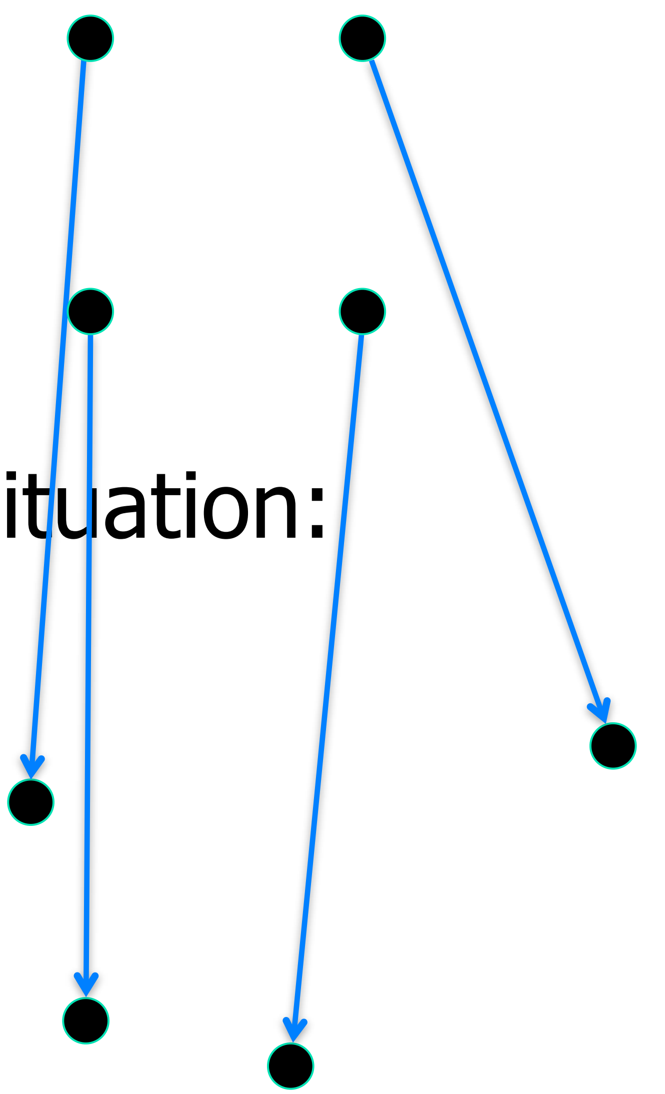

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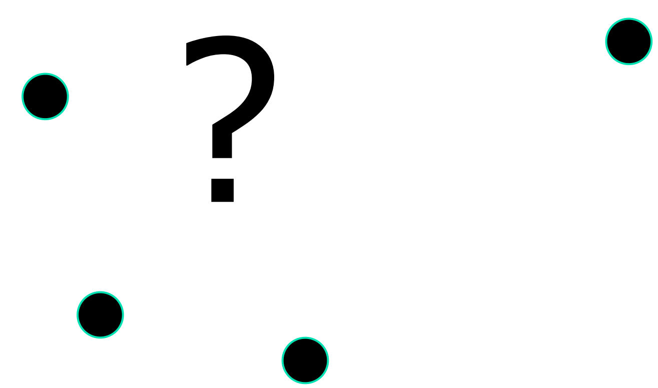

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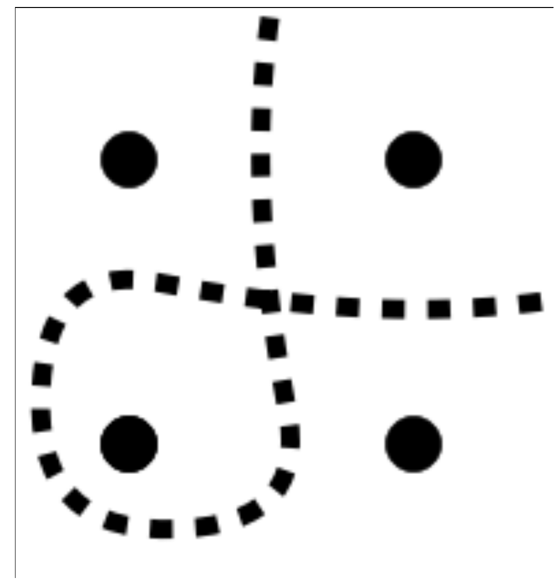

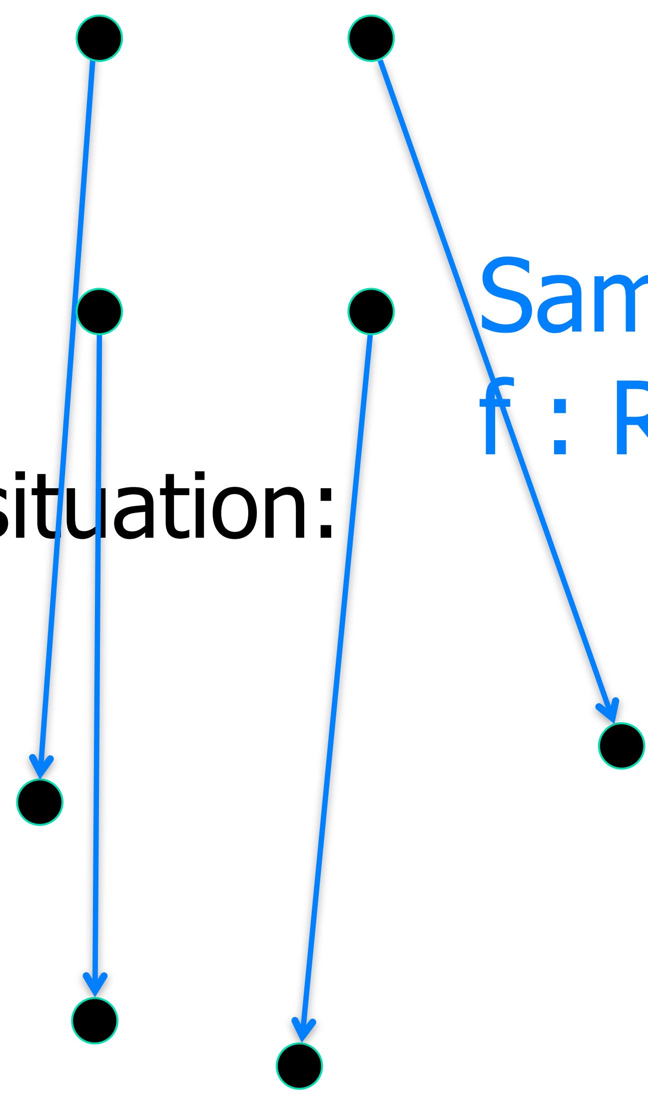

Samples of
$f : R^2 \rightarrow R^2$

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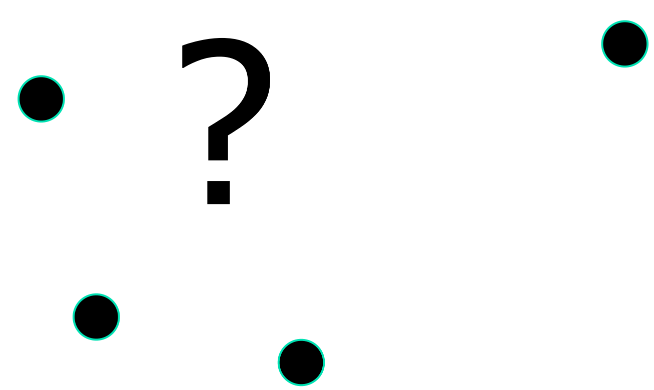

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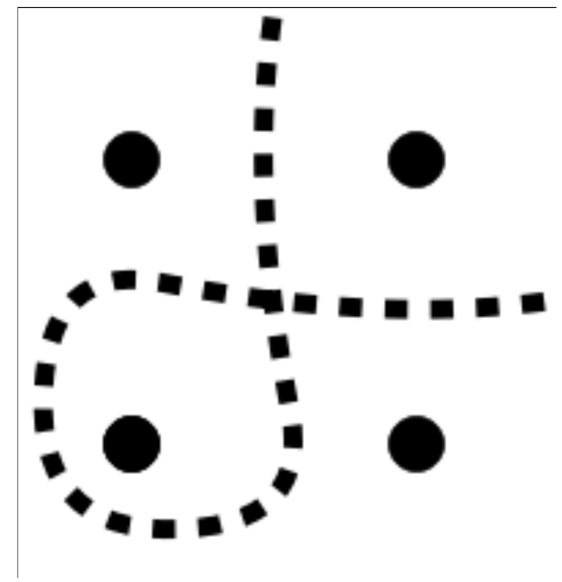

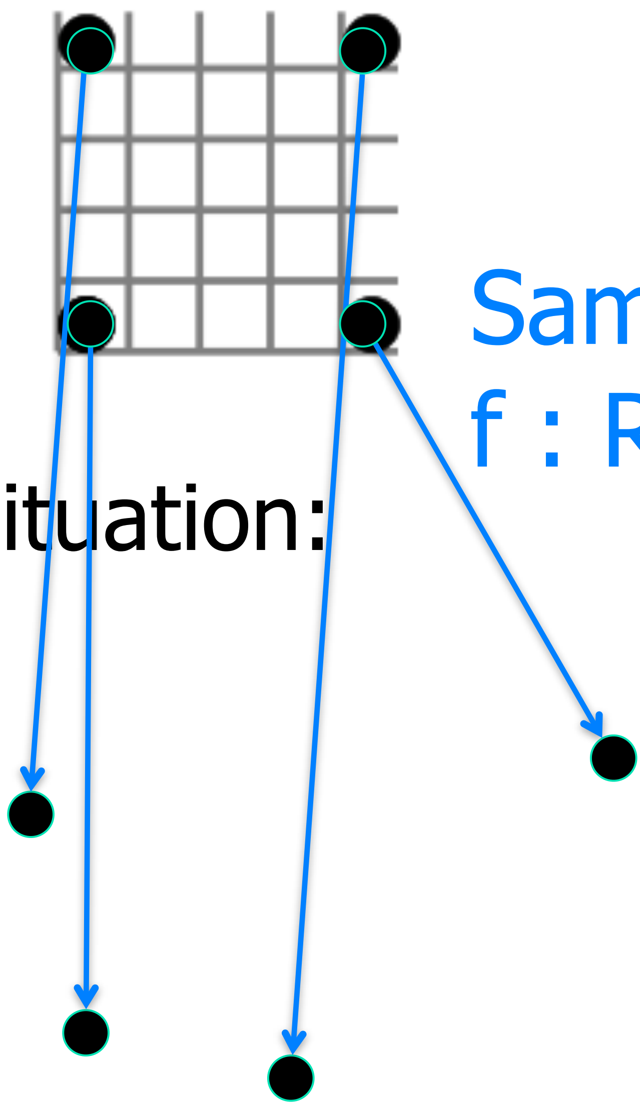

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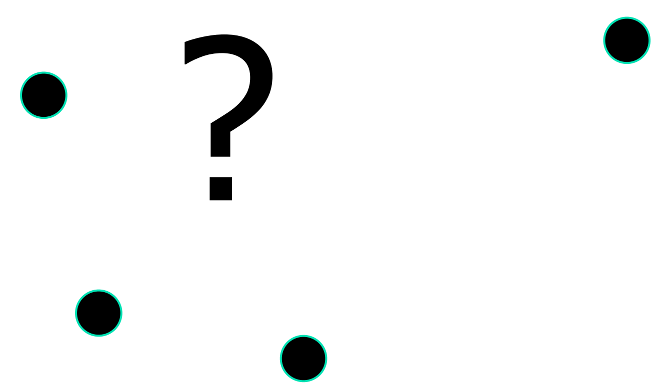

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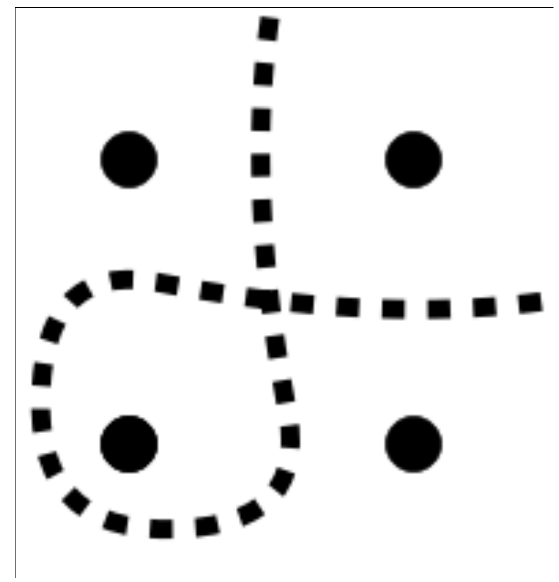

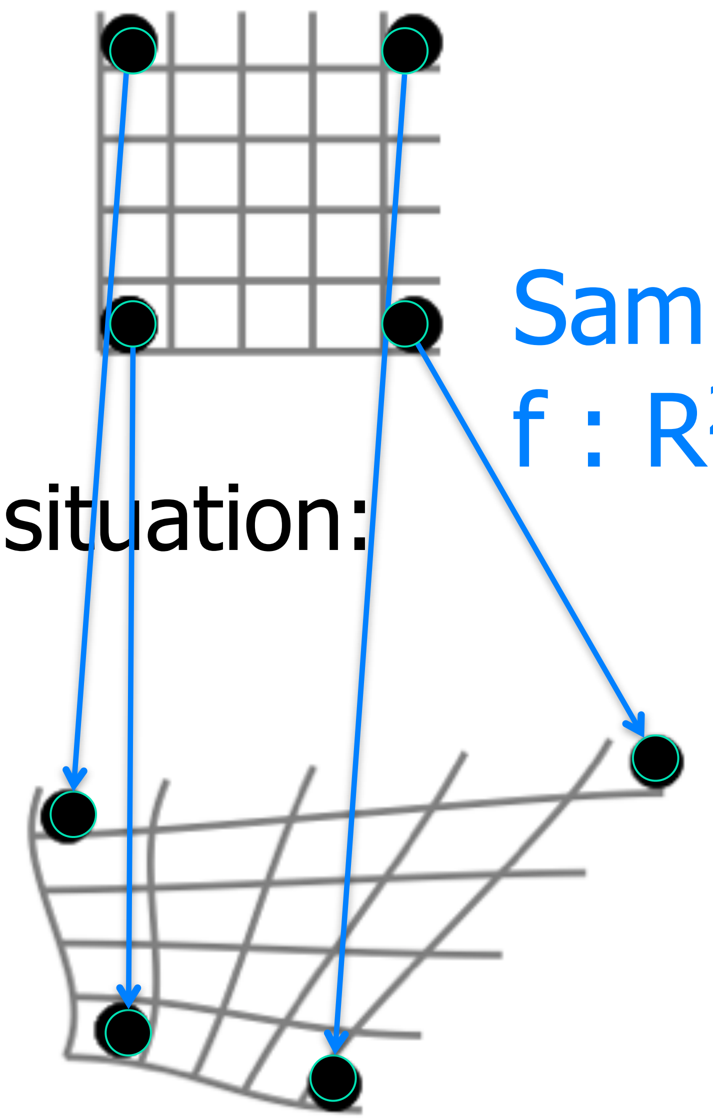

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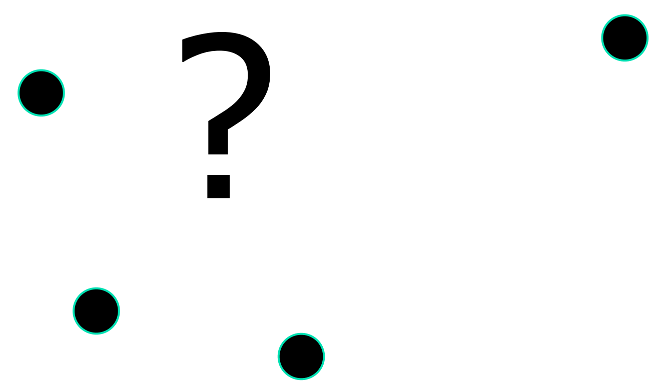

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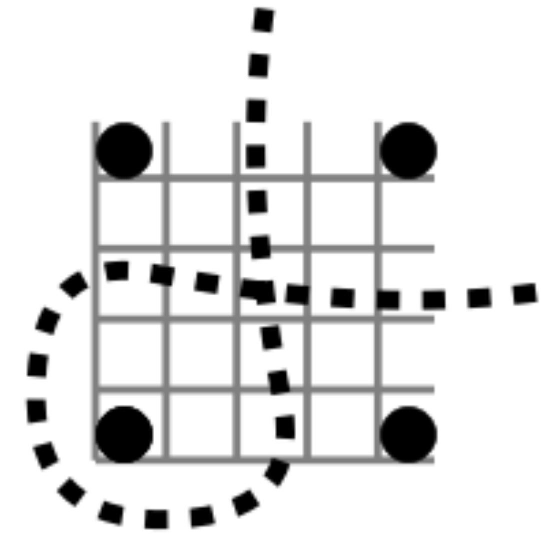

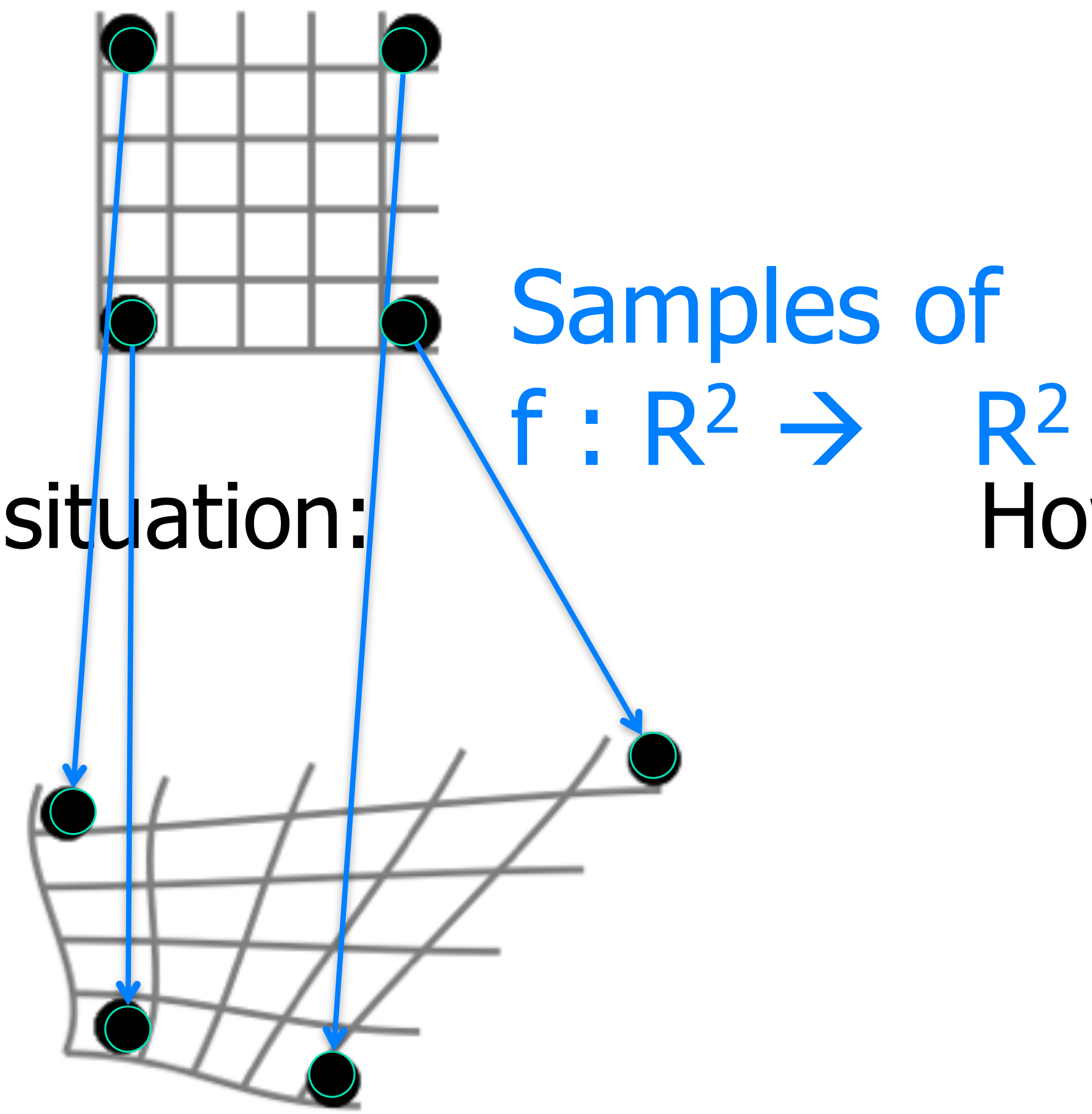

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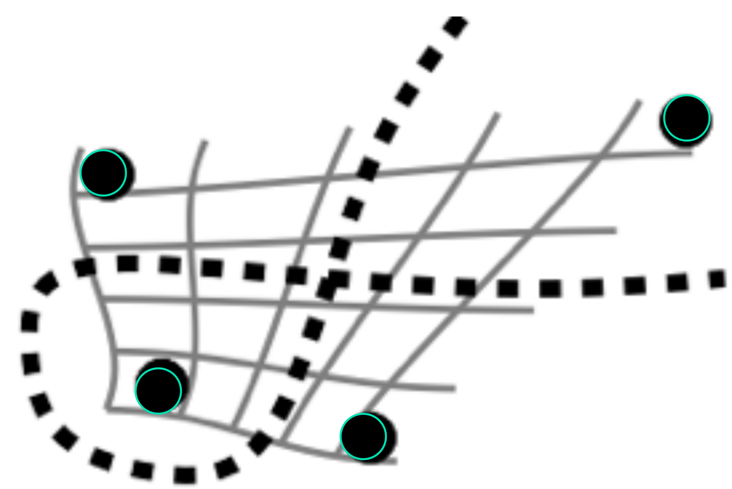


Thin plate splines

- Global smoothness is very important, since this function will determine the gripper trajectory and orientation
- Thin plate splines: regularize function by Frob norm of second derivatives matrix

$$J(f) = \sum_i (y_i - f(\mathbf{x}_i))^2 + \lambda \int d^3\mathbf{x} \|D_2 f(\mathbf{x})\|^2$$

- Kernel expansion (1D):

$$f(x) = \sum_{i=1}^{m} a_i K(x_i, x) + b^\top x + c,$$

$$K(x, y) = \begin{cases} c_0 r^{4-d} \ln r, & d = 2 \text{ or } d = 4 \\ c_1 r^{4-d}, & \text{otherwise} \end{cases} \qquad \text{with} \quad r = \|x - y\|_2.$$

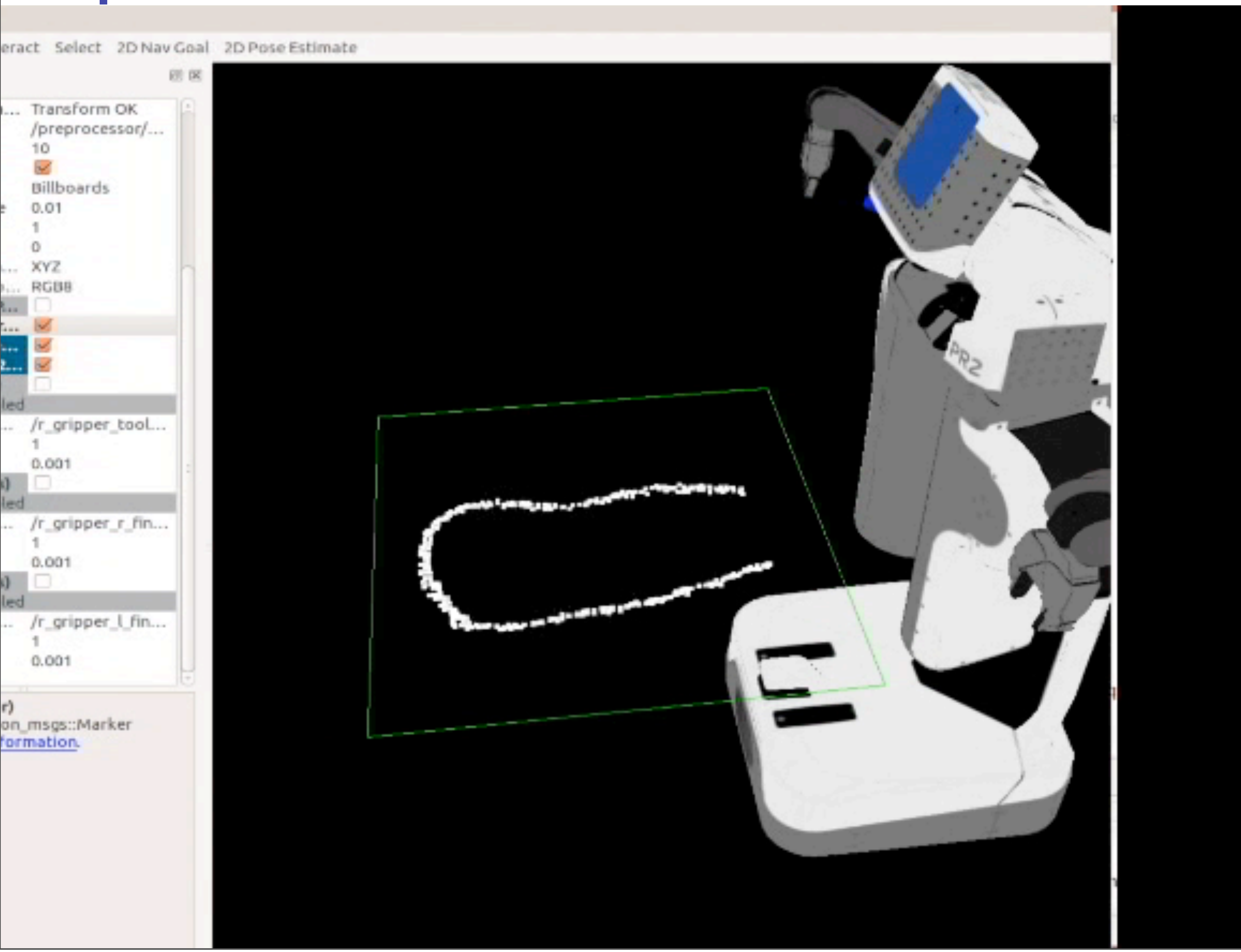
Knot tying procedure

- Look up nearest demonstration

$$ClosestDemoRope = \arg\min_i \mathrm{dist}(DemoRope_i, NewRope)$$

- Fit a non-rigid transformation f that maps from ClosestDemoRope to NewRope
- Apply f to the end-effector trajectory (positions and orientations) to get a “warped” trajectory
- Execute warped trajectory

Visualization during knot tie



Point cloud registration

- Find a non-rigid transformation between two point clouds
- Given two point clouds X, Y, find a non-rigid transformation f that minimizes dist(f(X), Y)
  - for some meaningful distance measure dist(.) on un-organized point clouds
- TPS-RPM Algorithm (Chui & Ragnaran, 2003)
  - Correspondence: find matrix of correspondences between X and Y points
    - C_{ij} = correspondence between x_i and y_j
  - Fit thin plate spline transformation that maps each x_i to weighted sum of points y_j it corresponds to

Application to other tasks

- Want to apply this method to a wide assortment of everyday tasks. e.g. in the kitchen:
  - pour, open container, pour, sprinkle, dip, stir, scoop, skewer, unskewer, stack, toss, cover, uncover, press, shake, grind, dump out, slice
- Still need to use non-rigid registration, even if the objects themselves are rigid