

Nonlinear Optimization for Estimation

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Problem Setting

- General: find most likely state sequence given observations z :

$$\begin{aligned} & \max_{x,v,w} \log P(v, w) \\ \text{s.t. } & \forall t \quad x_{t+1} = f(x_t, u_t) + w_t \\ & z_t = g(x_t) + v_t \end{aligned}$$

- For Gaussian v, w :

$$\begin{aligned} & \min_{x,v,w} \sum_t \|w_t\|_2^2 + \sum_t \|v_t\|_2^2 \\ \text{s.t. } & \forall t \quad x_{t+1} = f(x_t, u_t) + w_t \\ & z_t = g(x_t) + v_t \end{aligned}$$

Nonlinear Optimization for Estimation

- Problem:
$$\min_{x,v,w} \sum_t \|w_t\|_2^2 + \sum_t \|v_t\|_2^2$$
$$\text{s.t. } \forall t \quad x_{t+1} = f(x_t, u_t) + w_t$$
$$z_t = g(x_t) + v_t$$

- Initialize by setting $x^{(0)}, v^{(0)}, w^{(0)}$

- Iterate for $i=1, 2, \dots$

- Linearize around current $x^{(i-1)}, u^{(i-1)} \rightarrow A_t, B_t, c_t, G_t, h_t$

- Solve

$$\min_{x,v,w} \sum_t \|w_t\|_2^2 + \sum_t \|v_t\|_2^2$$
$$\text{s.t. } \forall t \quad x_{t+1} = A_t x_t + B_t u_t + c_t + w_t$$
$$z_t = G_t x_t + h_t + v_t$$
$$\|x_t - x_t^{(i-1)}\|_2 \leq \varepsilon$$
$$\|u_t - u_t^{(i-1)}\|_2 \leq \varepsilon$$

“Model Predictive Estimation”

- Given: \bar{x}_0
- For $k=0, 1, 2, \dots, T$

- Solve

$$\min_{x,v,w} \sum_{t=0}^k \|w_t\|_2^2 + \sum_{t=0}^k \|v_t\|_2^2$$

$$\text{s.t. } \forall t : 0 \leq t \leq k \quad x_{t+1} = f(x_t, u_t) + w_t$$
$$z_t = g(x_t) + v_t$$

- Observe z_{t+1}