Particle Filters

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Many slides adapted from Thrun, Burgard and Fox, Probabilistic Robotics

Motivation

- For continuous spaces: often no analytical formulas for Bayes filter updates
- Solution 1: Histogram Filters: (not studied in this lecture)
  - Partition the state space
  - Keep track of probability for each partition
  - Challenges:
    - What is the dynamics for the partitioned model?
    - What is the measurement model?
    - Often very fine resolution required to get reasonable results
- Solution 2: Particle Filters:
  - Represent belief by random samples
  - Can use actual dynamics and measurement models
  - Naturally allocates computational resources where required (~ adaptive resolution)
  - Aka Monte Carlo filter, Survival of the fittest, Condensation, Bootstrap filter

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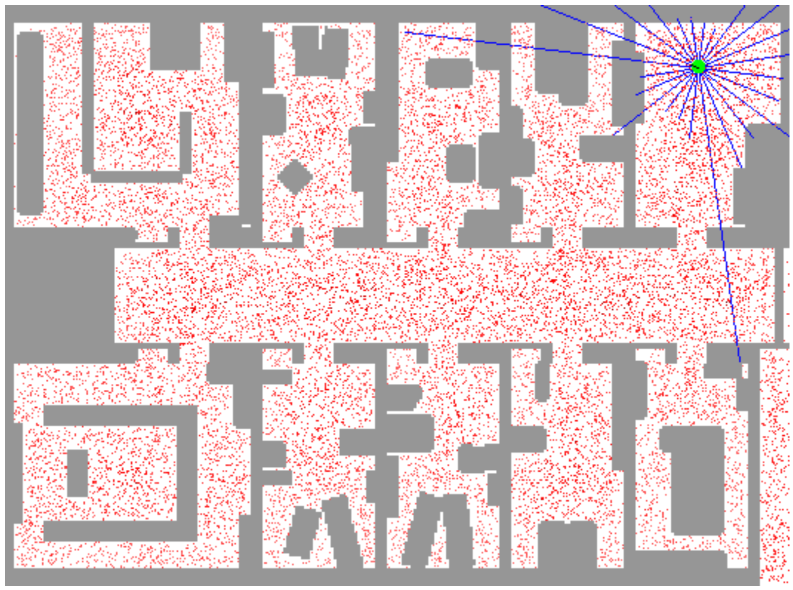
Sample-based Localization (sonar)



Problem to be Solved

- Given a sample-based representation $S_t = \{x_t^1, x_t^2, \ldots, x_t^N\}$

  of $Bel(x_t) = P(x_t \mid z_1, \ldots, z_t, u_1, \ldots, u_t)$

  Find a sample-based representation $S_{t+1} = \{x_{t+1}^1, x_{t+1}^2, \ldots, x_{t+1}^N\}$

  of $Bel(x_{t+1}) = P(x_{t+1} \mid z_1, \ldots, z_t, \mathbf{z_{t+1}}, u_1, \ldots, \mathbf{u_{t+1}})$

Dynamics Update

- Given a sample-based representation $S_t = \{x_t^1, x_t^2, ..., x_t^N\}$

  of Bel(x_t) = P(x_t | z_1, …, z_t, u_1, …, u_t)

  Find a sample-based representation

  of P(x_{t+1} | z_1, …, z_t, u_1, …, **u_{t+1}**)

- Solution:
  - For i=1, 2, …, N
    - Sample x^i_{t+1} from P(X_{t+1} | X_t = x^i_t)

Sampling Intermezzo

Observation update

- Given a sample-based representation of $\{x_{t+1}^1, x_{t+1}^2, \ldots, x_{t+1}^N\}$

  $P(x_{t+1} \mid z_1, \ldots, z_t)$

  Find a sample-based representation of

  $P(x_{t+1} \mid z_1, \ldots, z_t, z_{t+1}) = C * P(x_{t+1} \mid z_1, \ldots, z_t) * P(z_{t+1} \mid x_{t+1})$

- Solution:
  - For i=1, 2, …, N
    - $w^{(i)}_{t+1} = w^{(i)}_t * P(z_{t+1} \mid X_{t+1} = x^{(i)}_{t+1})$
  - the distribution is represented by the weighted set of samples

$$\{< x_{t+1}^1, w_{t+1}^1 >, < x_{t+1}^2, w_{t+1}^2 >, \ldots, < x_{t+1}^N, w_{t+1}^N >\}$$

Sequential Importance Sampling (SIS) Particle Filter

- Sample $x^1_1, x^2_1, \ldots, x^N_1$ from $P(X_1)$
- Set $w^i_1 = 1$ for all i=1,…,N
- For t=1, 2, …
  - Dynamics update:
    - For i=1, 2, …, N
      - Sample x^i_{t+1} from $P(X_{t+1} \mid X_t = x^i_t)$
  - Observation update:
    - For i=1, 2, …, N
      - $w^i_{t+1} = w^i_t * P(z_{t+1} \mid X_{t+1} = x^i_{t+1})$
- At any time t, the distribution is represented by the weighted set of samples

  $\{ <x^i_t, w^i_t> ; i=1,\ldots,N\}$

SIS particle filter major issue

- The resulting samples are only weighted by the evidence
- The samples themselves are never affected by the evidence

→ Fails to concentrate particles/computation in the high probability areas of the distribution $P(x_t \mid z_1, \ldots, z_t)$

Sequential Importance Resampling (SIR)

- At any time t, the distribution is represented by the weighted set of samples

  $\{ <x^i_t, w^i_t> ; i=1,\ldots,N\}$

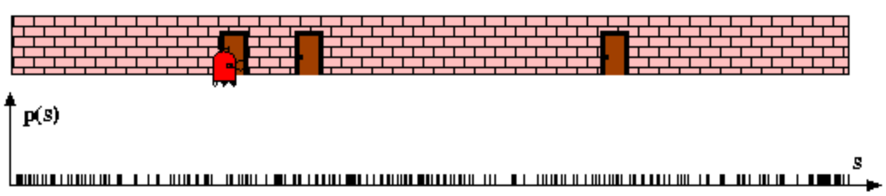
→ Sample N times from the set of particles

→ The probability of drawing each particle is given by its importance weight

→ More particles/computation focused on the parts of the state space with high probability mass

1. Algorithm **particle_filter**(S_{t-1}, u_t , z_t):
2. $S_t = \emptyset, \quad \eta = 0$
3. **For** $i = 1 \ldots n$ ***Generate new samples***
4. Sample index *j(i)* from the discrete distribution given by w_{t-1}
5. Sample x_t^i from $p(x_t \mid x_{t-1}, u_t)$ using $x_{t-1}^{j(i)}$ and u_t
6. $w_t^i = p(z_t \mid x_t^i)$ ***Compute importance weight***
7. $\eta = \eta + w_t^i$ ***Update normalization factor***
8. $S_t = S_t \cup \{< x_t^i, w_t^i >\}$ ***Insert***
9. **For** $i = 1 \ldots n$
10. $w_t^i = w_t^i / \eta$ ***Normalize weights***

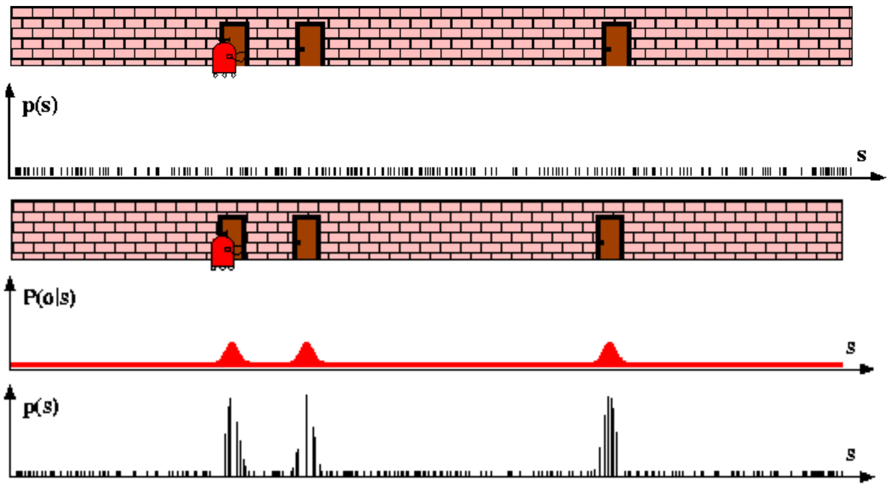
Particle Filters



Sensor Information: Importance Sampling

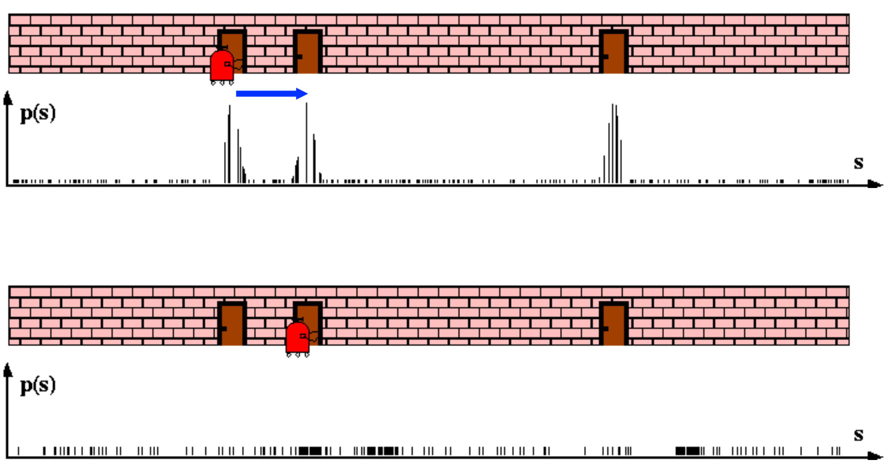
$$Bel(x) \leftarrow \alpha\, p(z\,|\,x)\, Bel^{-}(x)$$

$$w \leftarrow \frac{\alpha\, p(z\,|\,x)\, Bel^{-}(x)}{Bel^{-}(x)} = \alpha\, p(z\,|\,x)$$



Robot Motion

$$Bel^{-}(x) \leftarrow \int p(x\,|\,u, x')\, Bel(x')\, \mathrm{d}x'$$



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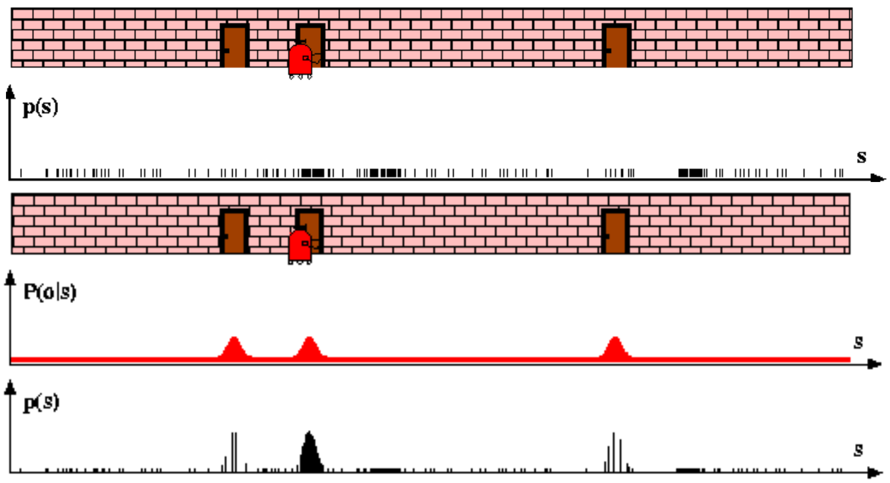

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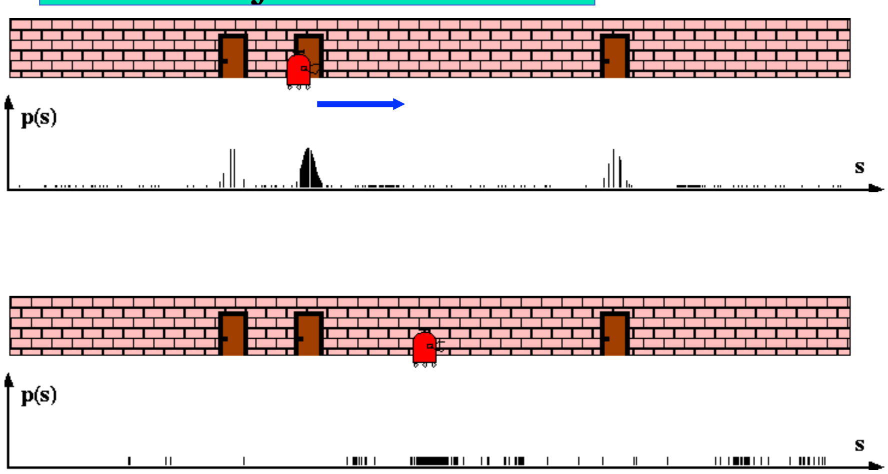

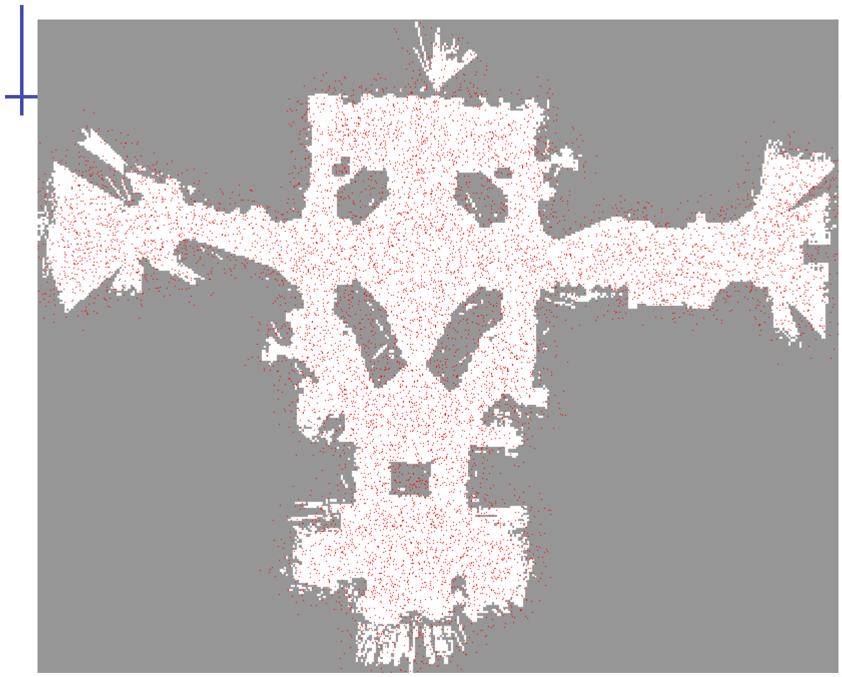

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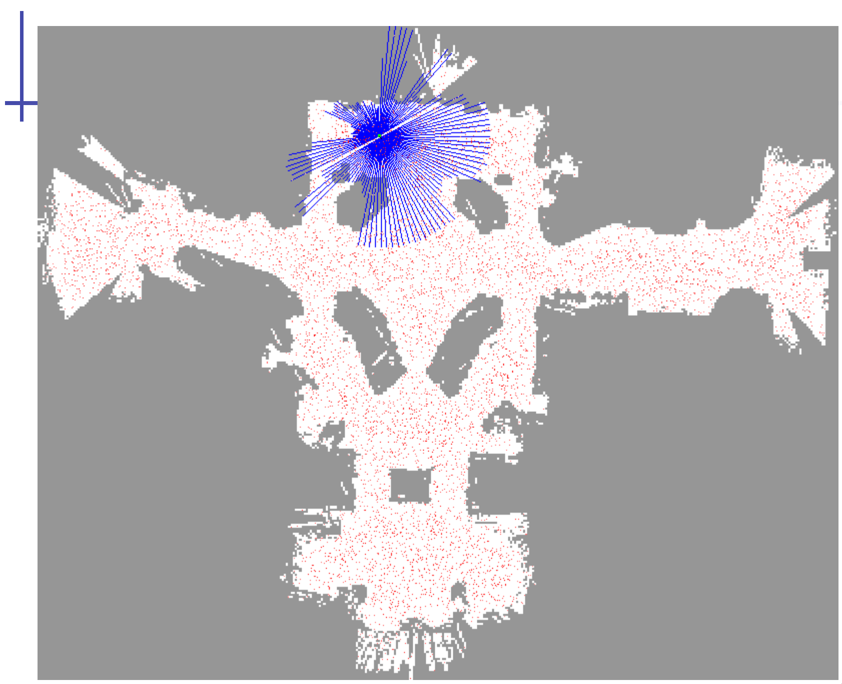

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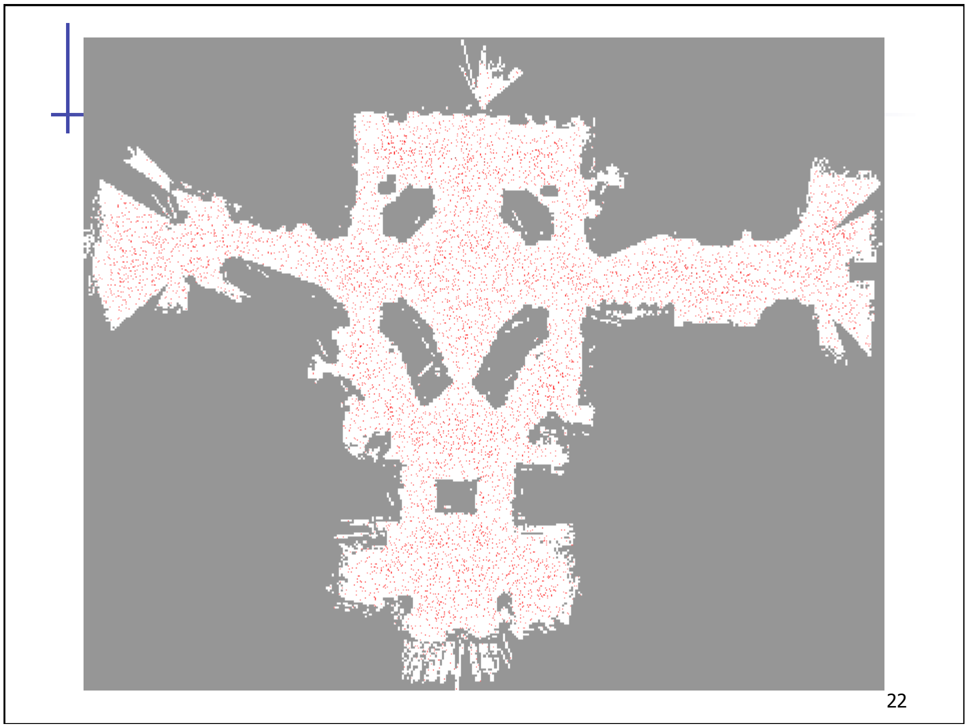

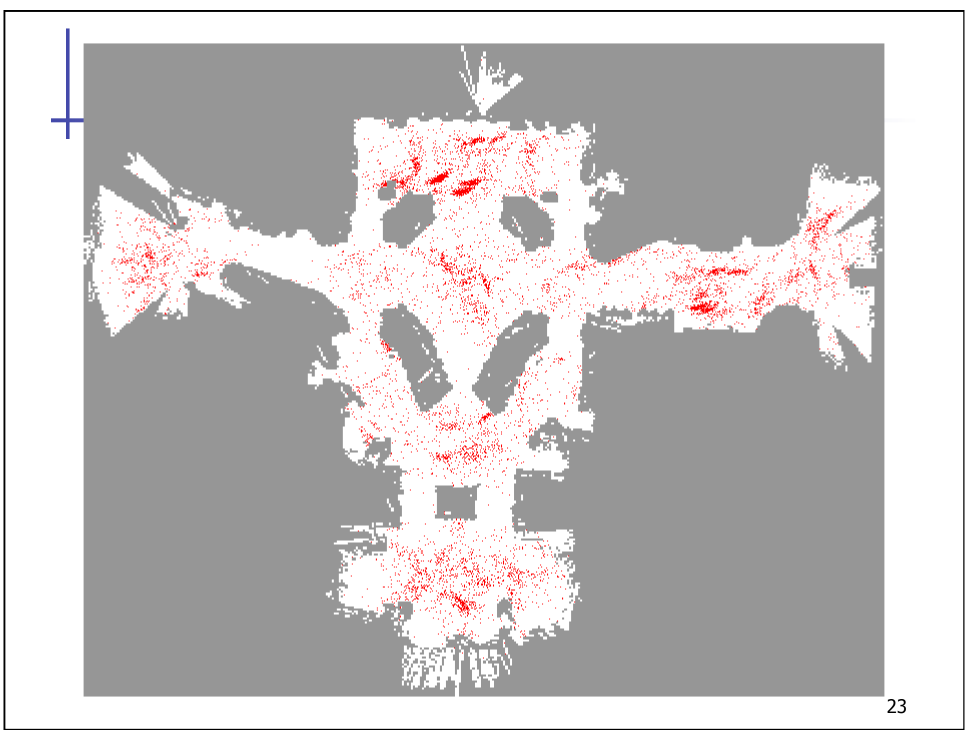

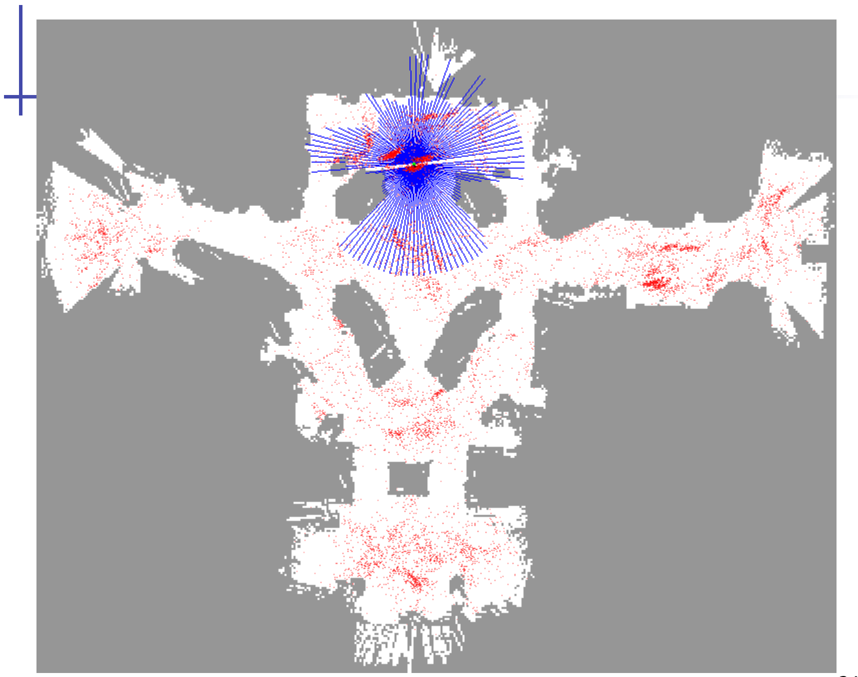

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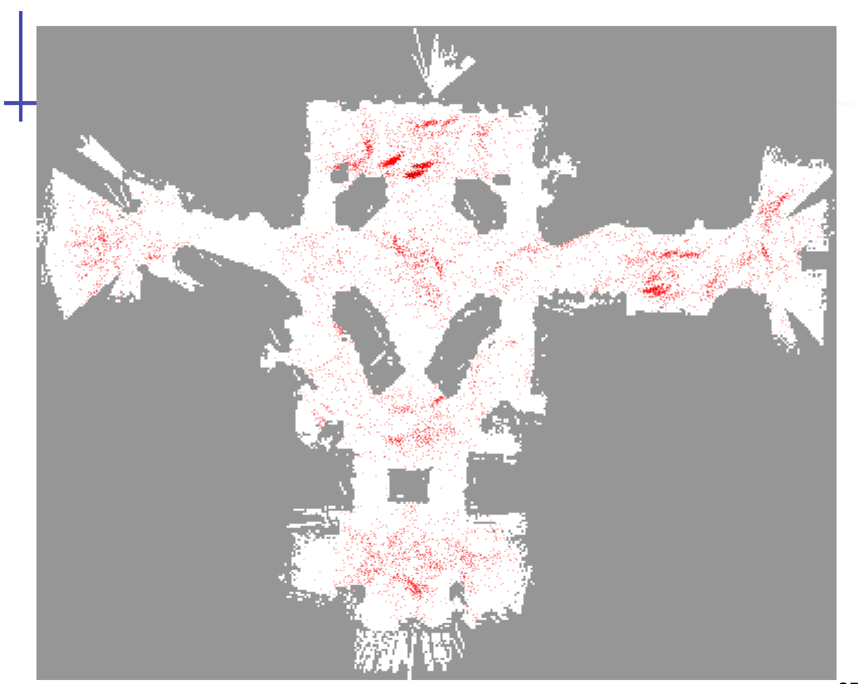

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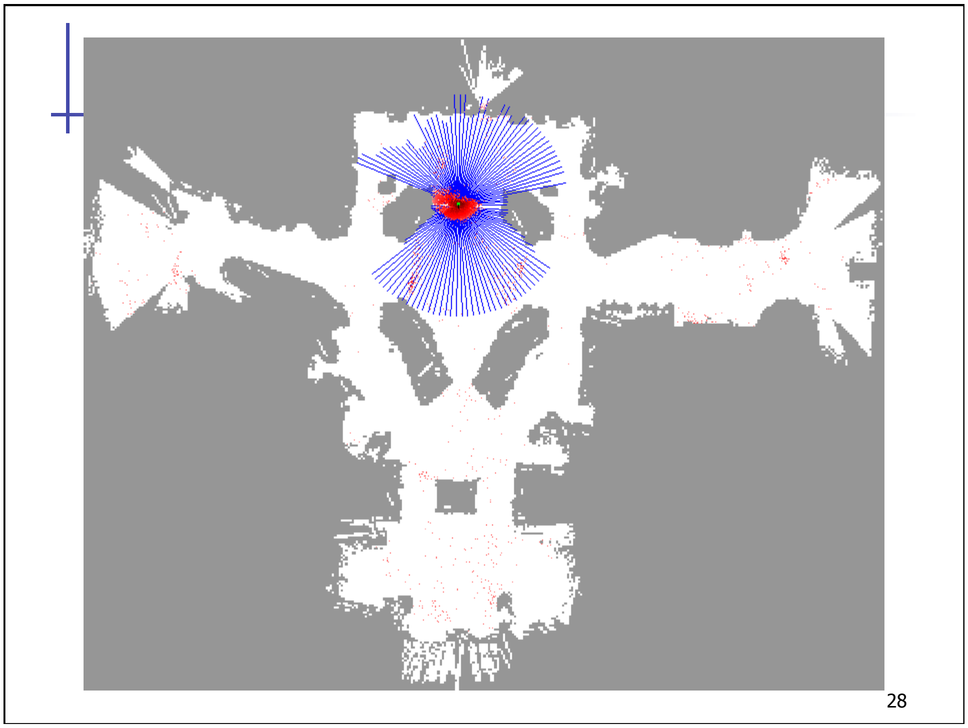

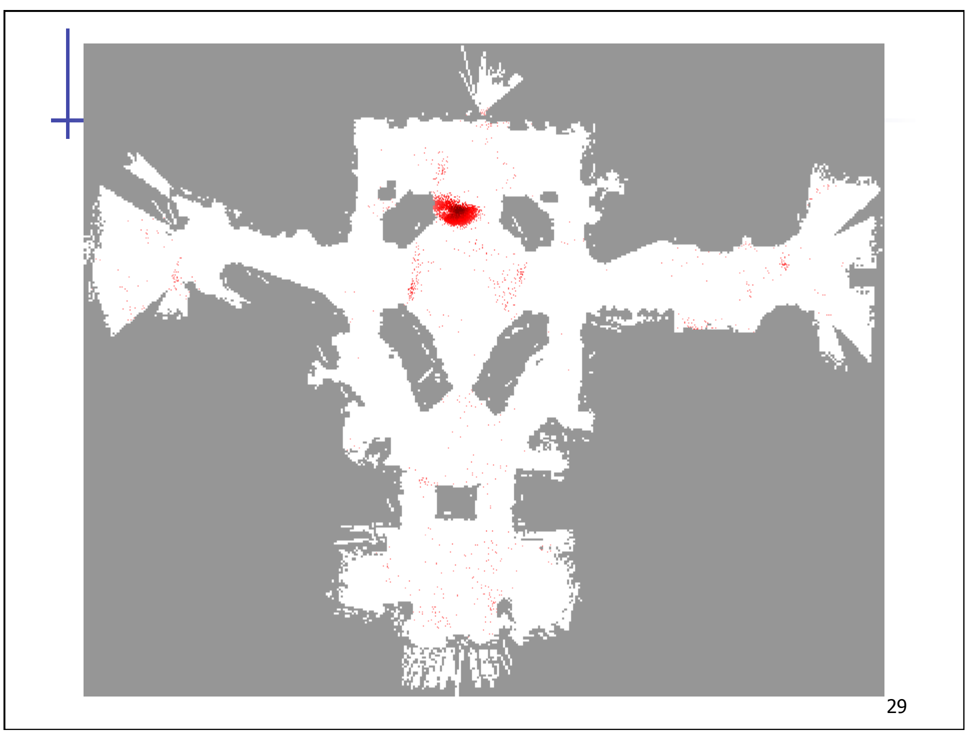

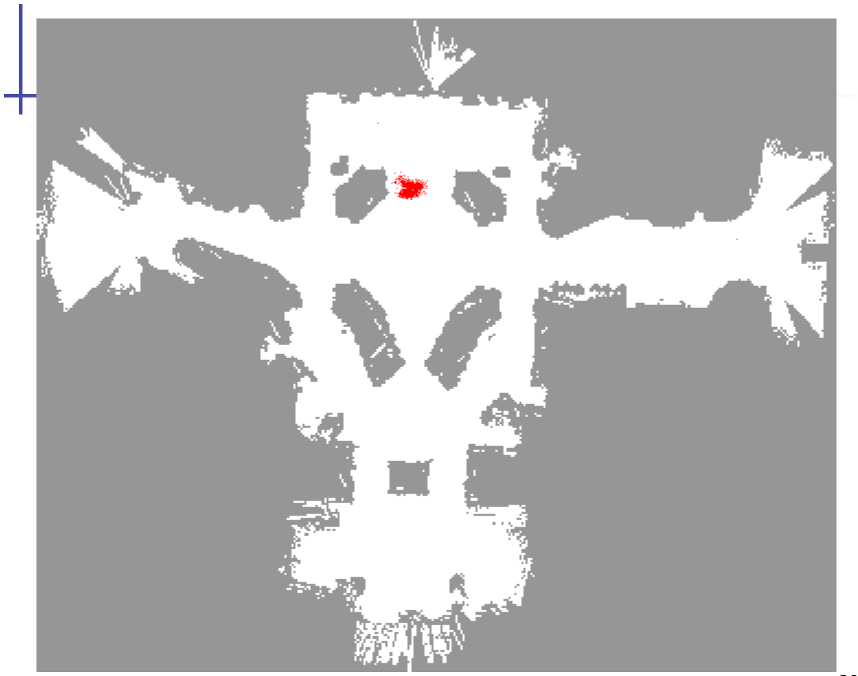

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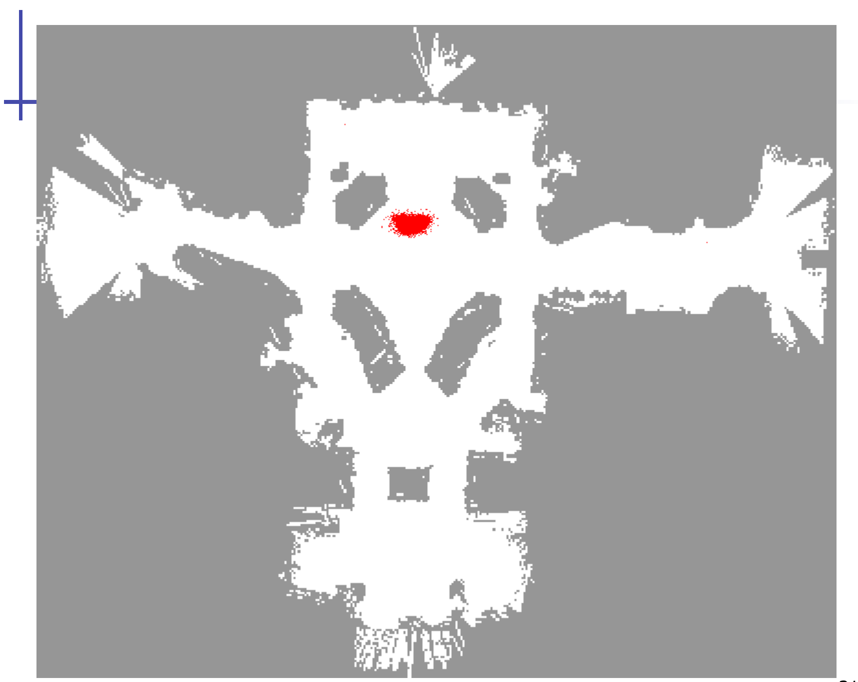

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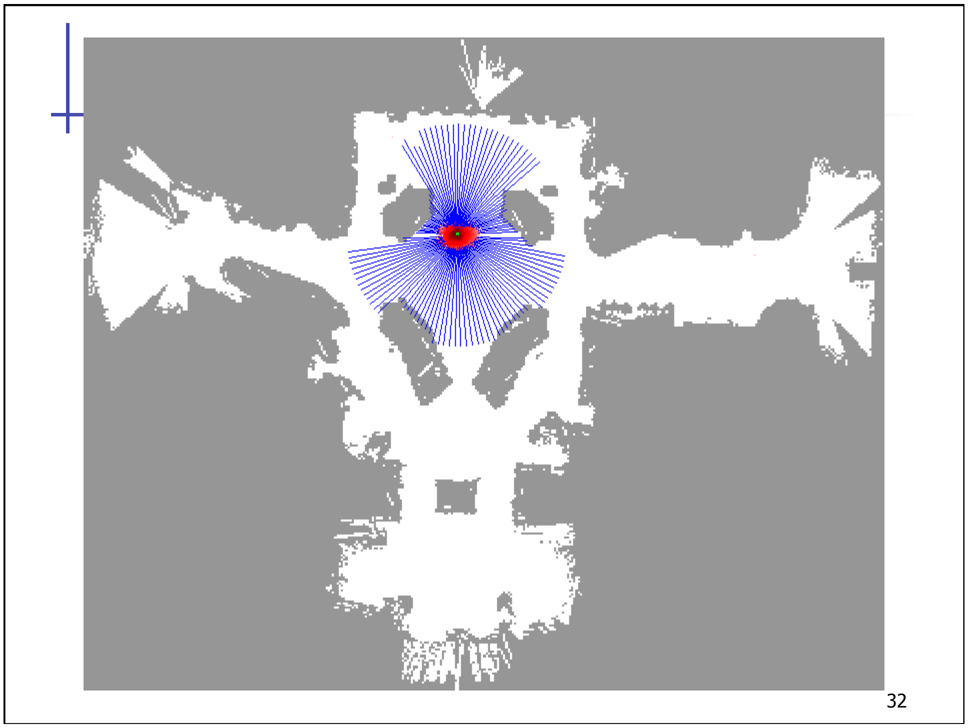

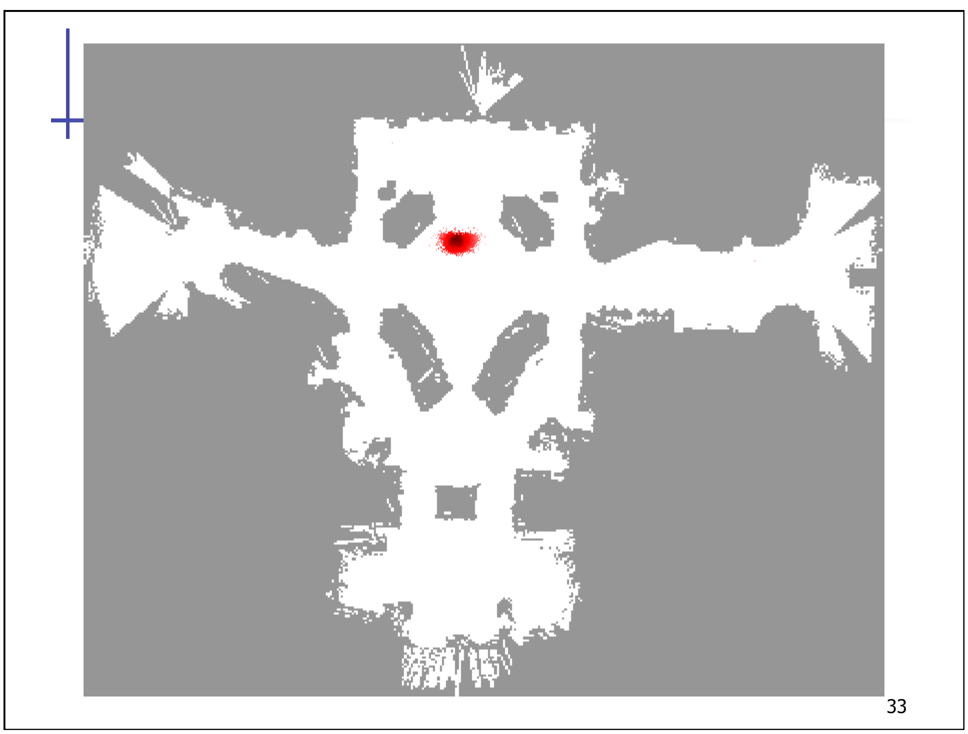

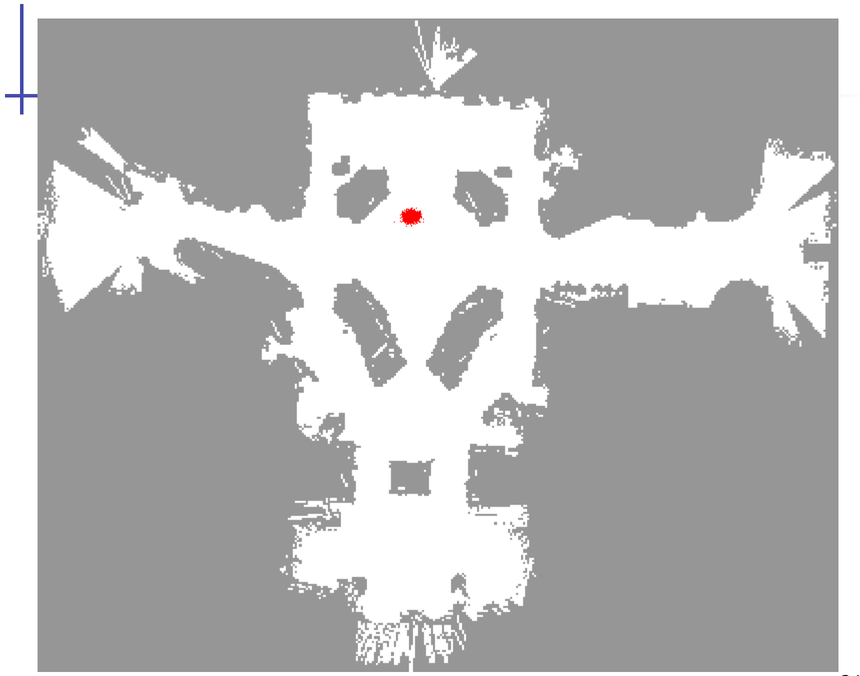

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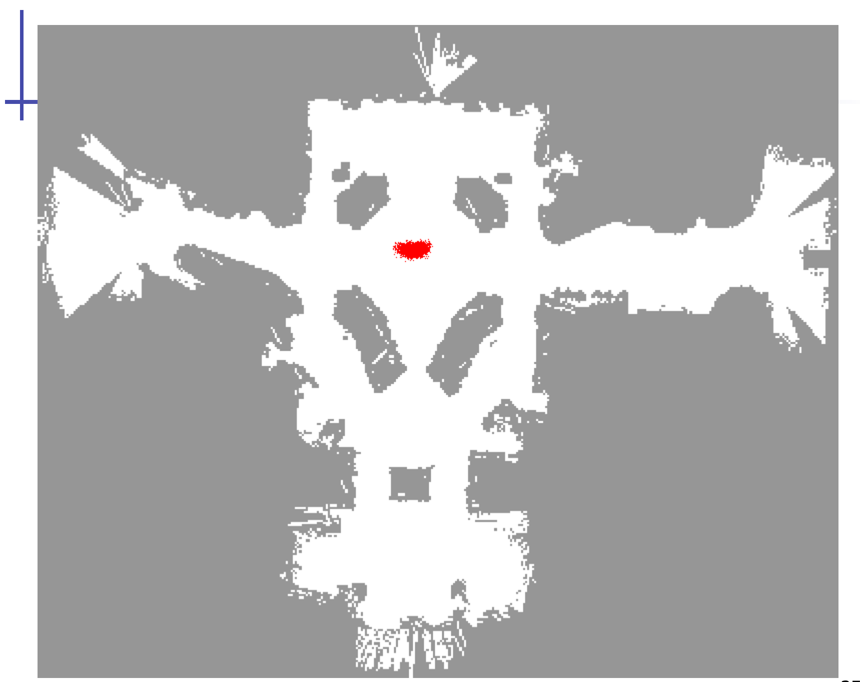

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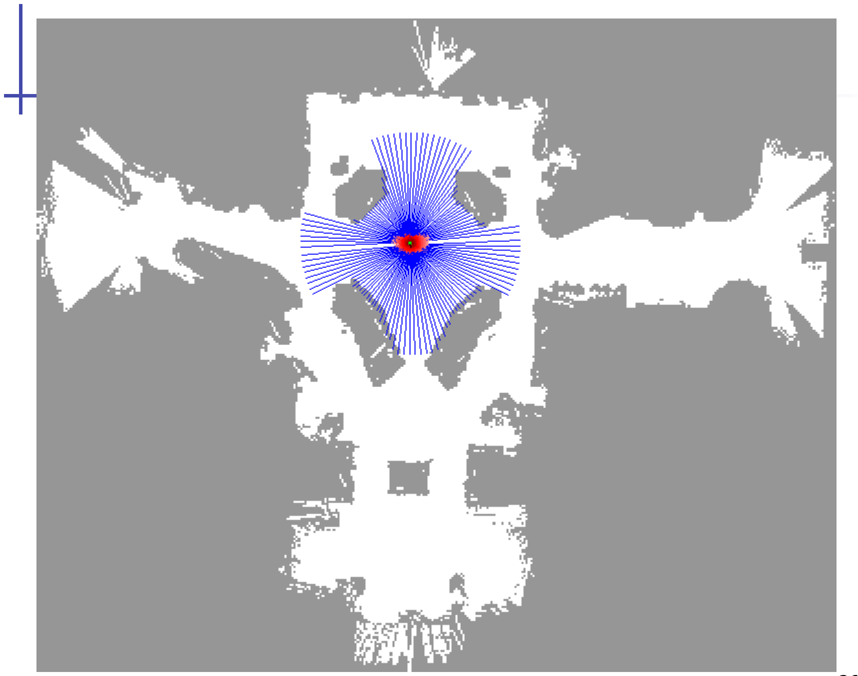

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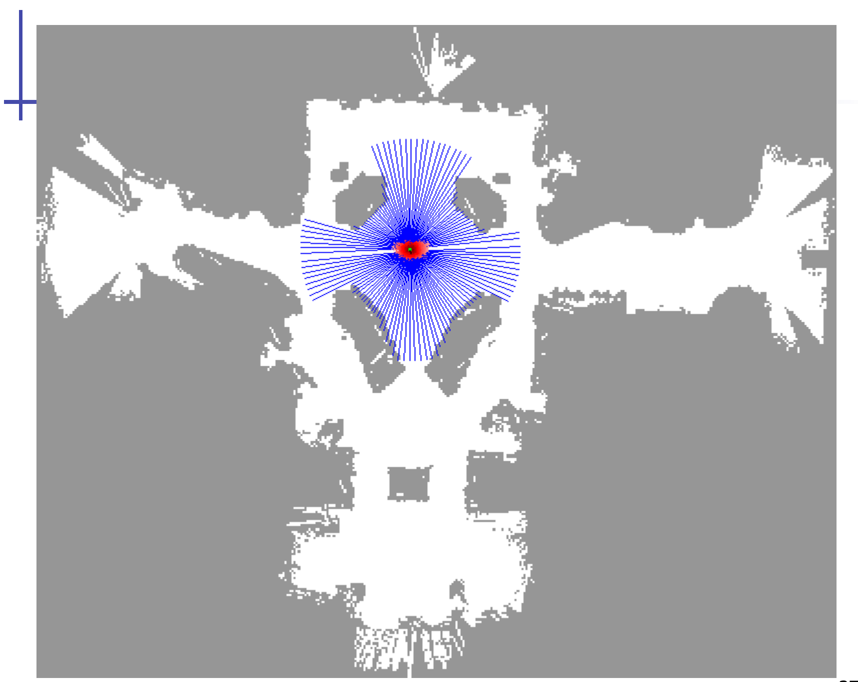

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Summary – Particle Filters

- Particle filters are an implementation of recursive Bayesian filtering
- They represent the posterior by a set of weighted samples
- They can model non-Gaussian distributions
- Proposal to draw new samples
- Weight to account for the differences between the proposal and the target

Summary – PF Localization

- In the context of localization, the particles are propagated according to the motion model.
- They are then weighted according to the likelihood of the observations.
- In a re-sampling step, new particles are drawn with a probability proportional to the likelihood of the observation.