Bayes Filters

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Many slides adapted from Thrun, Burgard and Fox, Probabilistic Robotics

Actions

- Often the world is **dynamic** since
  - **actions carried out by the robot**,
  - **actions carried out by other agents**,
  - or just the **time** passing by

  change the world.

- How can we **incorporate** such **actions**?

2

Typical Actions

- The robot **turns its wheels** to move
- The robot **uses its manipulator** to grasp an object
- Plants grow over **time**…

- Actions are **never carried out with absolute certainty**.
- In contrast to measurements, **actions generally increase the uncertainty**.

3

Modeling Actions

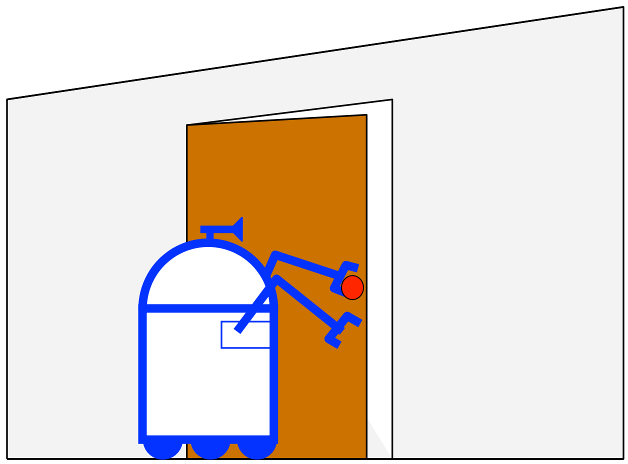
- To incorporate the outcome of an action *u* into the current "belief", we use the conditional pdf

$$P(x|u,x')$$

- This term specifies the pdf that **executing *u* changes the state from *x'* to *x***.

4

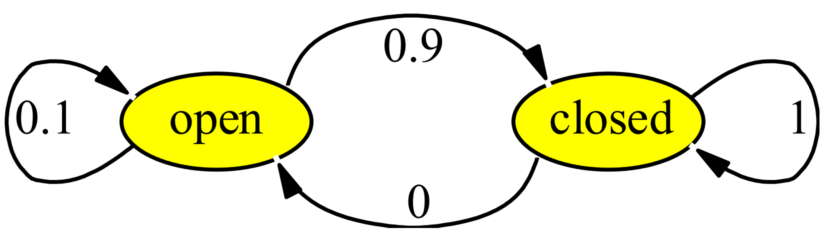
Example: Closing the door



5

State Transitions

$P(x|u,x')$ for u = "close door":



If the door is open, the action "close door" succeeds in 90% of all cases.

6

Integrating the Outcome of Actions

Continuous case:

$$P(x \mid u) = \int P(x \mid u, x')P(x')dx'$$

Discrete case:

$$P(x \mid u) = \sum P(x \mid u, x')P(x')$$

7

Example: The Resulting Belief

$$\begin{aligned} P(closed \mid u) &= \sum P(closed \mid u, x')P(x') \\ &= P(closed \mid u, open)P(open) \\ &\quad + P(closed \mid u, closed)P(closed) \\ &= \frac{9}{10} * \frac{5}{8} + \frac{1}{1} * \frac{3}{8} = \frac{15}{16} \\ P(open \mid u) &= \sum P(open \mid u, x')P(x') \\ &= P(open \mid u, open)P(open) \\ &\quad + P(open \mid u, closed)P(closed) \\ &= \frac{1}{10} * \frac{5}{8} + \frac{0}{1} * \frac{3}{8} = \frac{1}{16} \\ &= 1 - P(closed \mid u) \end{aligned}$$

8

Measurements

- Bayes rule

$$P(x \mid z) = \frac{P(z \mid x)\, P(x)}{P(z)} = \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}$$

Bayes Filters: Framework

- **Given:**
  - Stream of observations z and action data *u:*

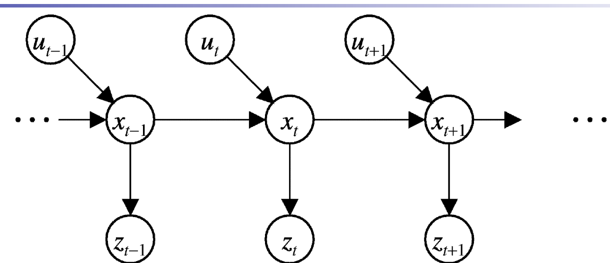
    $$d_t = \{u_1, z_1 \ldots, u_t, z_t\}$$

  - Sensor model *P(z|x)*.
  - Action model *P(x|u,x')*.
  - Prior probability of the system state *P(x)*.
- **Wanted:**
  - Estimate of the state *X* of a dynamical system.
  - The posterior of the state is also called **Belief**:

$$Bel(x_t) = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

10

Markov Assumption



$$p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t \mid x_t)$$

$$p(x_t \mid x_{1:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$$

Underlying Assumptions

- Static world
- Independent noise
- Perfect model, no approximation errors

11

Bayes Filters

z = observation
u = action
x = state

$$Bel(x_t) = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

Bayes $= \eta \; P(z_t \mid x_t, u_1, z_1, \ldots, u_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$

Markov $= \eta \; P(z_t \mid x_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$

Total prob. $= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_1, z_1, \ldots, u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$

Markov $= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$

Markov $= \eta P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \, P(x_{t-1} \mid u_1, z_1, \ldots, z_{t-1}) \; dx_{t-1}$

$$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

12

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

Algorithm **Bayes_filter**(*Bel(x),d*):

$\eta = 0$

If *d* is a perceptual data item *z* then

For all *x* do

$$Bel'(x) = P(z \mid x) Bel(x)$$

$$\eta = \eta + Bel'(x)$$

For all *x* do

$$Bel'(x) = \eta^{-1} Bel'(x)$$

Else if *d* is an action data item *u* then

For all *x* do

$$Bel'(x) = \int P(x \mid u, x') \; Bel(x') \; dx'$$

Return *Bel'(x)*

Bayes Filters are Familiar!

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- Partially Observable Markov Decision Processes (POMDPs)

Example Applications

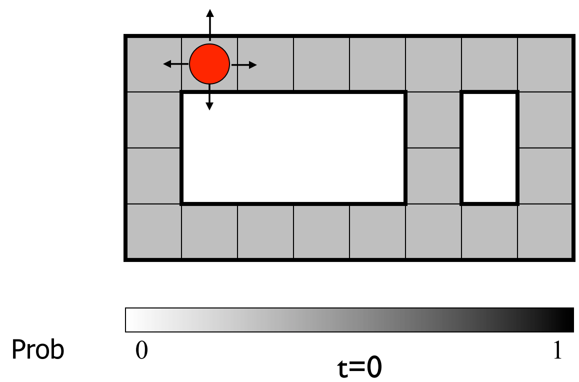
- Robot localization:
  - Observations are range readings (continuous)
  - States are positions on a map (continuous)
- Speech recognition HMMs:
  - Observations are acoustic signals (continuous valued)
  - States are specific positions in specific words (so, tens of thousands)
- Machine translation HMMs:
  - Observations are words (tens of thousands)
  - States are translation options

Summary

- Bayes rule allows us to compute probabilities that are hard to assess otherwise.
- Under the Markov assumption, recursive Bayesian updating can be used to efficiently combine evidence.
- Bayes filters are a probabilistic tool for estimating the state of dynamic systems.

Example: Robot Localization

Example from Michael Pfeiffer



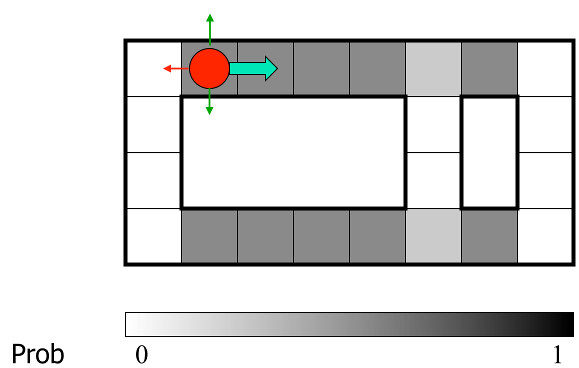
t=0

Sensor model: never more than 1 mistake

Know the heading (North, East, South or West)

Motion model: may not execute action with small prob.

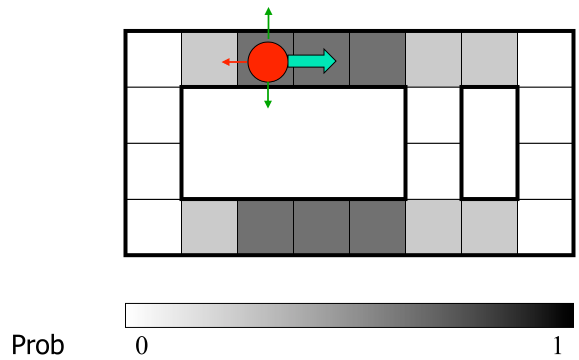
Example: Robot Localization



t=1

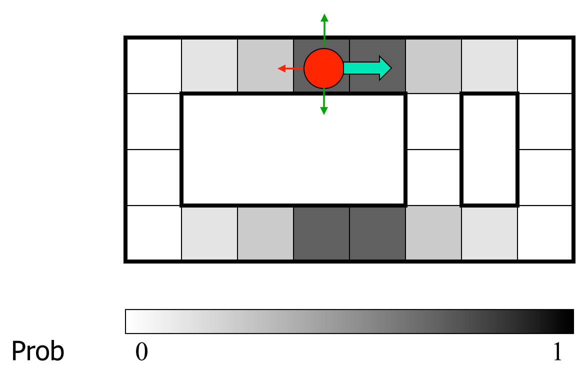
Lighter grey: was possible to get the reading, but less likely b/c required 1 mistake

Example: Robot Localization



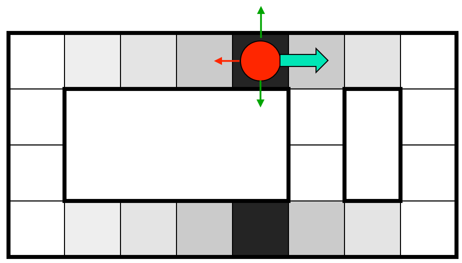
t=2

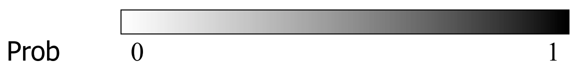
Example: Robot Localization



t=3

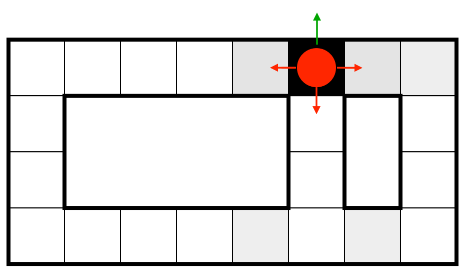
Example: Robot Localization

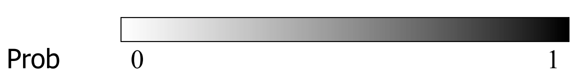




t=4

Example: Robot Localization





t=5

The likelihood of the observations

$$P(z_{1:t}) = \sum_{x_1, x_2, \ldots, x_t} P(x_{1:t}, z_{1:t}) = \sum_{x_1, x_2, \ldots, x_t} \prod_{k=1}^{t-1} P(x_{k+1}|x_k) P(z_k|x_k) P(z_t|x_t)$$

- The forward algorithm first sums over x_1, then over x_2 and so forth, which allows it to efficiently compute the likelihood at all times t, indeed:

$$P(z_{1:t}) = \sum_{x_t} P(x_t, z_{1:t})$$

- Relevance:
  - Compare the fit of several HMM models to the data
  - Could optimize the dynamics model and observation model to maximize the likelihood
  - Run multiple simultaneous trackers --- retain the best and split again whenever applicable (e.g., loop closures in SLAM, or different flight maneuvers)