Rao-Blackwellized Particle Filtering

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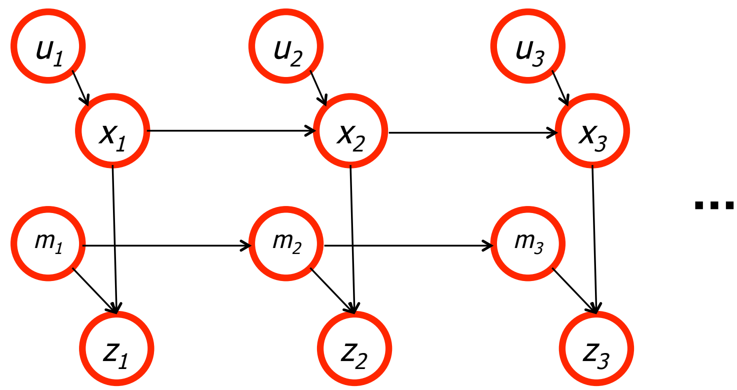
Many slides adapted from Thrun, Burgard and Fox, Probabilistic Robotics

Particle Filters Recap

1. Algorithm particle_filter(S_{t-1}, u_t , z_t):
2. $S_t = \emptyset, \quad \eta = 0$
3. **For** $i = 1 \ldots n$ ***Generate new samples***
4. Sample index $j(i)$ from the discrete distribution given by w_{t-1}
5. Sample x_t^i from $\pi(x_t \mid x_{t-1}^{j(i)}, u_t, z_t)$
6. $w_t^i = \frac{p(z_t \mid x_t^i) p(x_t^i \mid x_{t-1}^i, u_t)}{\pi(x_t^i \mid x_{t-1}^i, u_t, z_t)}$ ***Compute importance weight***
7. $\eta = \eta + w_t^i$ ***Update normalization factor***
8. $S_t = S_t \cup \{< x_t^i, w_t^i >\}$ ***Insert***
9. **For** $i = 1 \ldots n$
10. $w_t^i = w_t^i / \eta$ ***Normalize weights***
11. ***Return*** S_t

Motivating Example: Simultaneous Localization and Mapping (SLAM)



- m_t: map at time t
  - Often map is assumed static, then denoted by *m*

Naive Particle Filter for SLAM

- Each particle < (x^i, m^i), w^i > encodes a weighted hypothesis of robot pose and map
- E.g., 20m x 10m space, mapped at 5cm x 5cm resolution

  → 400 x 200 = 80,000 cells

  → 2^{80,000} possible maps
- Impractical to get sufficient coverage of such a large state space

Particle Filter Revisited

Let's consider just the robot pose:

- Sample from $\pi(x_t \mid x_{t-1}^i, u_t, z_t)$
- Reweight $$w_t^i = \frac{p(z_t \mid x_t^i)\, p(x_t^i \mid x_{t-1}^i, u_t)}{\pi(x_t^i \mid x_{t-1}^i, u_t, z_t)}$$

Recall a particle really corresponds to an entire history, this will matter going forward, so let's make this explicit, also account for the fact that by ignoring the other state variable, we lost Markov property:

- Reweight $$w_t^i = \frac{p(z_t \mid x_{1:t}^i, z_{1:t-1})\, p(x_t^i \mid x_{1:t-1}^i, u_t, z_{1:t-1})}{\pi(x_t^i \mid x_{t-1}^i, u_t, z_t)}$$

Still defines a valid particle filter just for x, BUT as z depends both on x and m, some quantities are not readily available (yet).

Weights Computation

$$w_t^i = \frac{p(z_t \mid x_{1:t}^i, z_{1:t-1})\, p(x_t^i \mid x_{1:t-1}^i, u_t, z_{1:t-1})}{\pi(x_t^i \mid x_{t-1}^i, u_t, z_t)}$$

- $$p(z_t \mid x_{1:t}^i, z_{1:t-1}) = \int p(z_t \mid x_{1:t}^i, m_t, z_{1:t-1})\, p(m_t \mid z_{1:t-1}, x_{1:t-1}^i)\, dm_t$$
  $$= \int p(z_t \mid x_t^i, m_t)\, p(m_t \mid z_{1:t-1}, x_{1:t-1}^i)\, dm_t$$

  sensor model — mapping with KNOWN poses

  This integral is over large space, but we'll see how to still compute it efficiently (sometimes approximately).

- $$p(x_t^i \mid x_{1:t-1}^i, u_t, z_{1:t-1}) = p(x_t^i \mid x_{t-1}^i, u_t)$$

  motion model

Examples

- We'll consider $\pi(x_t \mid x_{t-1}^i, u_t, z_t) = p(x_t \mid x_{t-1}^i, u_t)$

  hence $w_t^i = \int p(z_t \mid x_t^i, m_t) p(m_t \mid z_{1:t-1}, x_{1:t-1}^i) dm_t$
- Examples for which w_t^i can be computed efficiently
  - "Color-tile" SLAM
  - FastSLAM:
    - Not in this lecture. Need to cover multi-variate Gaussians first.
  - SLAM with gridmaps

"Color-tile" SLAM

- Robot lives in MxN discrete grid:
  - → Robot pose space = {1,…,M} x {1,…, N}
- Every grid-cell can be red or green
  - → Map space = $\{R, G\}^{MN}$
- Motion model: robot can try to move to any neighboring cell, and succeeds with probability a, stays in place with probability 1-a.
- Sensor model: robot can measure the color of the cell it is currently on. Measurement is correct with probability b, incorrect with probability 1-b.

"Color-tile" SLAM

- Challenge in running the Rao-Blackwellized Particle Filter:

efficiently evaluate $w_t^i = \int p(z_t \mid x_t^i, m_t) p(m_t \mid z_{1:t-1}, x_{1:t-1}^i) dm_t$

$$\begin{aligned} w_t^i &= \sum_m p(z_t | x_t^i, m) p(m | z_{1:t-1}, x_{1:t-1}^i) \\ &= \sum_m p(z_t | x_t^i, m^{x_t^i}) p(m | z_{1:t-1}, x_{1:t-1}^i) \\ &= \sum_{m^{x_t^i}} p(z_t | x_t^i, m^{x_t^i}) p(m^{x_t^i} | z_{1:t-1}, x_{1:t-1}^i) \end{aligned}$$

sensor model

posterior for the coloring of the cell the robot is currently at, which we can efficiently keep track of over time (mapping w/known poses)

Note: FastSLAM follows same derivation, difference being that (gridcell → landmark), (gridcell color → landmark location), (multinomial over color → Gaussian over location)

"Color-tile" SLAM

Challenge in running the Rao-Blackwellized Particle Filter:

efficiently evaluate $w_t^i = \int p(z_t \mid x_t^i, m_t) p(m_t \mid z_{1:t-1}^i, x_{1:t-1}^i) dm_t$

$$\begin{aligned} w_t^i &= \sum_m p(z_t | x_t^i, m) p(m | z_{1:t-1}, x_{1:t-1}^i) \\ &= \sum_m p(z_t | x_t^i, m^{x_t^i}) p(m | z_{1:t-1}, x_{1:t-1}^i) \\ &= \sum_{m^{y-x_t^i}} \sum_{m^{x_t^i}} p(z_t | x_t^i, m^{x_t^i}) p(m^{x_t^i}, m^{y-x_t^i} | z_{1:t-1}, x_{1:t-1}^i) \\ &= \sum_{m^{x_t^i}} p(z_t | x_t^i, m^{x_t^i}) \sum_{m^{y-x_t^i}} p(m^{x_t^i}, m^{y-x_t^i} | z_{1:t-1}, x_{1:t-1}^i) \\ &= \sum_{m^{x_t^i}} p(z_t | x_t^i, m^{x_t^i}) p(m^{x_t^i} | z_{1:t-1}, x_{1:t-1}^i) \end{aligned}$$

Sensor reading only depends on current cell

y: all gridcells
y-x^i_t: all gridcells except for x^i_t

Bring out shared factor

Sum out over other cell values

sensor model

posterior for the coloring of the cell the robot is currently at, which we can efficiently keep track of over time (mapping w/known poses)

Note: FastSLAM follows same derivation, difference being that (gridcell → landmark), (gridcell color → landmark location), (multinomial over color → Gaussian over location)

SLAM with Gridmaps

- Robot state (x, y, θ)
- Map space $\{0,1\}^{MN}$ where M and N is number of grid cells considered in X and Y direction
- Challenge in running the Rao-Blackwellized Particle Filter:

  efficiently evaluate $w_t^i = \int p(z_t \mid x_t^i, m_t) p(m_t \mid z_{1:t-1}, x_{1:t-1}^i) dm_t$

- Let $m^{*i} = \arg\max_m p(m|z_{1:t-1}, x^i_{1:t-1})$

  then assuming a peaked posterior for the map, we have

  $w_t^i \approx p(z_t|x_t^i, m^{*i})$ which is a sensor model evaluation