CS 287: Advanced Robotics
Fall 2009

Lecture 14: Reinforcement Learning with Function Approximation and TD Gammon case study

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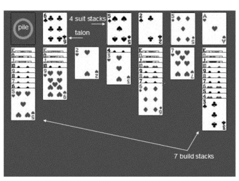
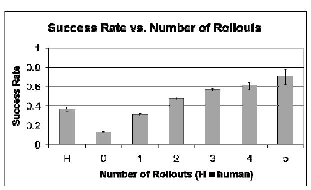
Generalization and function approximation

- Represent the value function using a parameterized function $V_\theta(s)$, e.g.:
  - Neural network: θ is a vector with the weights on the connections in the network
  - Linear function approximator: $V_\theta(s) = \theta^\top \phi(s)$
    - Radial basis functions $\phi_i(s) = \exp\left(\frac{1}{2}(s-s_i)^\top \Sigma^{-1}(s-s_i)\right)$
    - Tilings: (often multiple) partitions of the state space
    - Polynomials: $\phi_i(s) = x_1^{j_1^i} x_2^{j_2^i} \ldots x_n^{j_n^i}$
    - Fourier basis $\{1, \sin(2\pi\frac{x_1}{L_1}), \cos(2\pi\frac{x_1}{L_1}), \sin(2\pi\frac{x_2}{L_2}), \cos(2\pi\frac{x_2}{L_2}), \ldots\}$
    - [Note: most algorithms kernelizable]
    - Often also specifically designed for the problem at hand

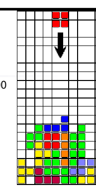
Assignment #1

- **Roll-out**: nice example paper: X. Yan, P. Diaconis, P. Rusmevichientong, and B. Van Roy, ``Solitaire: Man Versus Machine,'' *Advances in Neural Information Processing Systems 17*, MIT Press, 2005.





Example: tetris



- state: board configuration + shape of the falling piece ~2^{200} states!
- action: rotation and translation applied to the falling piece

$V(s) = \sum_{i=1}^{22} \theta_i \phi_i(s)$

- 22 features aka basis functions ϕ_i
  - Ten basis functions, 0, . . . , *9, mapping the state to the height h[k] of each of the ten* columns.
  - Nine basis functions, 10, . . . , *18, each mapping the state to the absolute difference* between heights of successive columns: *|h[k+1] – h[k]|, k = 1, . . . , 9.*
  - One basis function, 19, that maps state to the maximum column height: $\max_k$ *h[k]*
  - One basis function, 20, that maps state to the number of 'holes' in the board.
  - One basis function, 21, that is equal to 1 in every state.

[Bertsekas & Ioffe, 1996 (TD); Bertsekas & Tsitsiklis 1996 (TD);Kakade 2002 (policy gradient); Farias & Van Roy, 2006 (approximate LP)]

Recap RL so far

- When model is available:
  - VI, PI, GPI, LP
- When model is not available:
  - Model-based RL: collect data, estimate model, run one of the above methods for estimated model
  - Model-free RL: learn V, Q directly from experience:
    - TD(λ), sarsa(λ): on policy updates
    - Q: off policy updates
- What about large MDPs for which we cannot represent all states in memory or cannot collect experience from all states?

  → Function Approximation

Objective

- A standard way to find θ in supervised learning, optimize MSE:

$$\min_\theta \sum_s P(s)\,(V(s) - V_\theta(s))^2$$

- Is this the correct objective?

Evaluating the objective

- When performing policy evaluation, we can obtain samples by simply executing the policy and recording the empirical (discounted) sum of rewards

$$\min_\theta \sum_{\text{encountered states } s} \left(\hat{V}^\pi(s) - V^\pi_\theta(s)\right)^2$$

- TD methods: use $v_t = r_t + \gamma V^\pi_\theta(s_{t+1})$ as a substitute for $V^\pi(s)$

TD(λ) with function approximation

```
Initialize θ⃗ arbitrarily
Repeat (for each episode):
    e⃗ = 0
    s ← initial state of episode
    Repeat (for each step of episode):
        a ← action given by π for s
        Take action a, observe reward, r, and next state, s'
        δ ← r + γV(s') − V(s)
        e⃗ ← γλe⃗ + ∇_θ⃗ V(s)
        θ⃗ ← θ⃗ + αδe⃗
        s ← s'
    until s is terminal
```

Can similarly adapt sarsa(λ) and Q(λ) eligibility vectors for function approximation

Stochastic gradient descent

- Stochastic gradient descent to optimize MSE objective:
  - Iterate
    - Draw a state s according to P
    - Update:

$$\theta \leftarrow \theta - \tfrac{1}{2}\alpha\nabla_\theta\left(V^\pi(s) - V^\pi_\theta(s)\right)^2 = \theta + \alpha(V^\pi(s) - V^\pi_\theta(s))\nabla_\theta V^\pi_\theta(s)$$

- TD(0): use $v_t = r_t + \gamma V^\pi_\theta(s_{t+1})$ as a substitute for $V^\pi(s)$

$$\theta_{t+1} \leftarrow \theta_t + \alpha\left(R(s_t, a_t, s_{t+1}) + \gamma V^\pi_{\theta_t}(s_{t+1}) - V^\pi_{\theta_t}(s_t)\right)\nabla_\theta V^\pi_{\theta_t}(s_t)$$

Guarantees

- Monte Carlo based evaluation:
  - Provides unbiased estimates under current policy
  - Will converge to true value of current policy
- Temporal difference based evaluations:
  - **TD(λ) w/linear function approximation: [Tsitsiklis and Van Roy, 1997]** ***If*** samples are generated from traces of execution of the policy π, and for appropriate choice of step-sizes α, TD(λ) converges and at convergence: [D = expected discounted state visitation frequencies under policy π]

$$\|V_\theta - V^*\|_D \leq \frac{1-\lambda\gamma}{1-\gamma}\|\Pi_D V^* - V^*\|_D$$

  - **Sarsa(λ) w/linear function approximation:** same as TD
  - **Q w/linear function approximation: [Melo and Ribeiro, 2007]** Convergence to "reasonable" Q value under certain assumptions, including: $\forall s, a \|\phi(s,a)\|_1 \leq 1$
  - [Could also use infinity norm contraction function approximators to attain convergence --- see earlier lectures. However, this class of function approximators tends to be more restrictive.]

TD(λ) with function approximation

- time t:

$$\theta_{t+1} \leftarrow \theta_t + \alpha\underbrace{\left(R(s_t, a_t, s_{t+1}) + \gamma V^\pi_{\theta_t}(s_{t+1}) - V^\pi_{\theta_t}(s_t)\right)}_{\delta_t}\underbrace{\nabla_\theta V^\pi_{\theta_t}(s_t)}_{e_t}$$

- time t+1:

$$\theta_{t+2} \leftarrow \theta_{t+1} + \alpha\underbrace{\left(R(s_{t+1}, a_{t+1}, s_{t+2}) + \gamma V^\pi_{\theta_{t+1}}(s_{t+2}) - V^\pi_{\theta_{t+1}}(s_{t+1})\right)}_{\delta_{t+1}}\nabla_\theta V^\pi_{\theta_{t+1}}(s_{t+1})$$

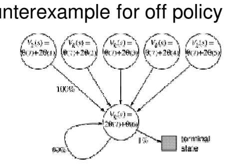
$\theta_{t+2} \leftarrow \theta_{t+2} + \alpha\delta_{t+1}\gamma\lambda e_t$ ["improving previous update"]

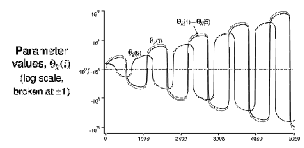
Combined:

$$\begin{aligned}\delta_{t+1} &= R(s_{t+1}, a_{t+1}, s_{t+2}) + \gamma V^\pi_\theta(s_{t+2}) - V^\pi_{\theta_{t+1}}(s_{t+1}) \\ e_{t+1} &= \gamma\lambda e_t + \nabla_{\theta_{t+1}} V^\pi_{\theta_{t+1}}(s_{t+1}) \\ \theta_{t+2} &= \theta_{t+1} + \alpha\delta_{t+1}e_{t+1}\end{aligned}$$

Off-policy counterexamples

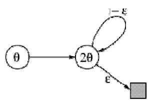
- Baird's counterexample for off policy updates:





Off-policy counterexamples

- Tsitsiklis and Van Roy counterexample: complete back-up with “off-policy” linear regression [i.e., uniform least squares, rather than waited by state visitation rates]



Should we use TD than well Monte Carlo?

- At convergence: $\|V_\theta - V^*\|_D \le \frac{1-\lambda\gamma}{1-\gamma}\|\Pi_D V^* - V^*\|_D$

Intuition behind TD(0) with linear function approximation guarantees

- Stochastic approximation of the following operations:
  - Back-up: $(T^\pi V)(s) = \sum_{s'} T(s,\pi(s),s')\,[R(s,\pi(s),s') + \gamma V(s')]$
  - Weighted linear regression: $\min_\theta \sum_s D(s)((T^\pi V)(s) - \theta^\top \phi(s))^2$

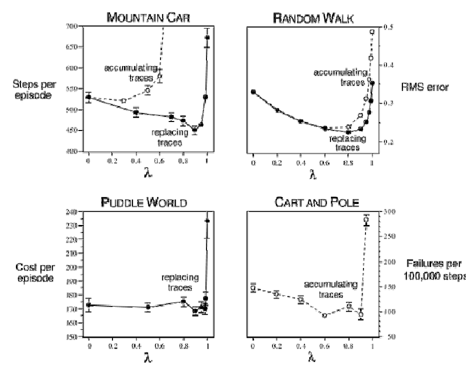
    with solution: $\Phi\theta = \underbrace{\Phi(\Phi^\top D\Phi)^{-1}\Phi^\top D}_{\Pi_D}(T^\pi V)$
- Key observations:

$$\forall V_1, V_2\ : \|T^\pi V_1 - T^\pi V_2\|_D \le \gamma \|V_1 - V_2\|_D, \quad \text{here}: \|x\|_D = \sqrt{\sum_i D(i)x(i)^2}$$

$$\forall V_1, V_2\ : \|\Pi_D V_1 - \Pi_D V_2\|_D \le \|V_1 - V_2\|_D$$

Empirical comparison

(See, Sutton&Barto p.221 for details)



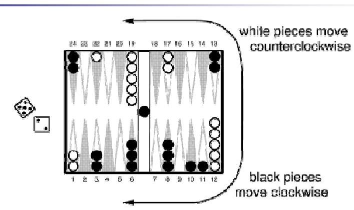
Intuition behind TD(λ) guarantees

- Bellman operator:

  $(T^\pi J)(s) = \sum_{s'} P(s'|s,\pi(s))\,[g(s) + \gamma J(s')] = \mathrm{E}\,[g(s) + \gamma J(s')]$
- T$^\lambda$ operator:

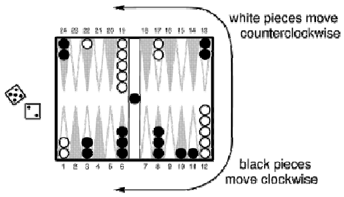
  $T^\lambda J(s) = (1-\lambda)\sum_{m=0}^{\infty} \lambda^m \mathrm{E}\left[\sum_{k=0}^{m} \gamma^t g(s_k) + \gamma^{m+1} J(s_{m+1})\right]$
- T$^\lambda$ operator is contraction w.r.t. \| \|_D   for all $\lambda \in$ [0,1]

Backgammon



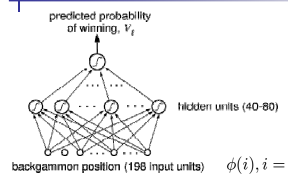
- 15 pieces, try go reach “other side”
- Move according to roll of dice
- If hitting an opponent piece: it gets reset to the middle row
- Cannot hit points with two or more opponent pieces

Backgammon

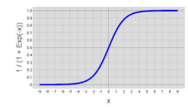


- 30 pieces, 24+2 possible locations
- For typical state and dice roll: often 20 moves

Neural net



$\phi(i), i = 1, \ldots, 198$



- Each hidden unit computes:

  $h(j) = \sigma(\sum_i w_{ij}\phi(i)) = \frac{1}{1+\exp\left(-\sum_i w_{ij}\phi(i)\right)}$

- Output unit computes:

  $o = \sigma(\sum_j w_j h(j)) = \frac{1}{1+\exp\left(-\sum_j w_j h(j)\right)}$

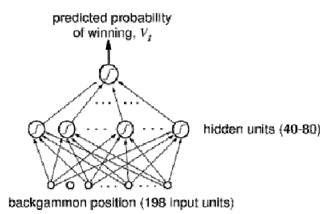
- Overall: $o = f(\phi(1), \ldots, \phi(198); w)$

TD Gammon [Tesauro 92,94,95]

- Reward = 1 for winning the game

  = 0 other states

- Function approximator: 2 layer neural network



Neural nets

- Popular at that time for function approximation / learning in general
- Derivatives/Gradients are easily derived analytically
  - Turns out they can be computed through backward error propagation --- name “error backpropagation”
- Susceptible to local optima!

Input features

- For each point on the backgammon board, 4 input units indicate the number of white pieces as follows:
  - 1 piece → unit1=1;
  - 2 pieces → unit1=1, unit2=1;
  - 3 pieces → unit1=1, unit2=1, unit3=1;
  - n>3 pieces → unit1=1, unit2=1, unit3=1, unit4 = (n-3)/2
- Similarly for black

[This already makes for 2*4*24 = 192 input units.]

- Last six: number of pieces on the bar (w/b), number of pieces that completed the game (w/b), white’s move, black’s move

Learning

- Initialize weights randomly
- TD(λ) [λ = 0.7, α = 0.1]
- Source of games: self-play, greedy w.r.t. current value function [results suggest game has enough stochasticity built in for exploration purposes]

Results

- After 300,000 games as good as best previous computer programs
  - Neurogammon: highly tuned neural network trained on large corpus of exemplary moves
- TD Gammon 1.0: add Neurogammon features
  - Substantially better than all previous computer players; human expert level
- TD Gammon 2.0, 2.1: selective 2-ply search
- TD Gammon 3.0: selective 3-ply search, 160 hidden units