1. Expanding the given expression, we get

\[ 2(2a^4+2b^4+4a^3b+6a^2b^2+4ab^3) = 4(a^4+b^4+(ab)^2+2a^2(ab)+2b^2(ab)+2a^2b^2) = 4(a^2+b^2+ab)^2. \]

2. The condition is equivalent to

\[ (a - b)^2 + (b - c)^2 + (c - a)^2 = 0. \]  

(1)

Since (1) is invariant under any translation \( z \mapsto z - z_0 \) and rotation \( z \mapsto z_0z, \ z_0 \in \mathbb{C} \setminus \{0\} \), we may assume without loss of generality that \( a = 0 \) and that \( b \) is a positive real number. Note that \( c \) cannot be real, since then the left-hand side of (1) would be strictly positive. Writing \( c \) in its polar form \( re^{i\theta} \), we obtain

\[ b^2 + \rho^2(\cos 2\theta + i \sin 2\theta) = b\rho(\cos \theta + i \sin \theta). \]  

(2)

Comparing imaginary parts of both sides and using the double angle formula, we get

\[ (2\rho \cos \theta - b) \sin \theta = 0. \]

We know already that \( c \) is not real, so \( \sin \theta \neq 0 \). Hence \( \cos \theta = \frac{b}{2\rho} \) and \( \cos 2\theta = \frac{b^2}{4\rho^2} - 1 \). Now comparing the real parts in (2), we get \( b^2 + \frac{b^2}{2} - \rho^2 = \frac{b^2}{2} \). Hence \( b = r \) and \( \cos \theta = \frac{1}{r} \), so the triangle formed by points \( a, b, c \) is equilateral. The argument also shows that situation to be the unique one where (1) holds.

3. If a polynomial identity holds in \( \mathbb{R} \), it also olds over \( \mathbb{C} \). Hence \( p(z^2) = p(z)p(z-1) \) for all \( z \in \mathbb{C} \). Since \( p \) is nonconstant, it has roots in \( \mathbb{C} \). First note that 0 cannot be a root of \( p \), since then we would have

\[
\begin{align*}
p(1^2) &= p(1)p(1-1) = 0, \\
p(2^2) &= p(2)p(2-1) = 0, \\
p(5^2) &= p(5)p(5-1) = 0, \quad \text{etc.,}
\end{align*}
\]

so \( p \) would have infinitely many (nonnegative real) roots. So, \( p(0) \neq 0 \).

Now, if \( a \) is a root of \( p \), so are \( a^2, a^4, \ldots, a^{2^n} \). Therefore this sequence must have a finite range and, since \( a \neq 0, a \) must be a root of unity. Note also that \( (a+1)^2 \) is a root of \( p \) as well, hence \( a + 1 \) is also a root of unity. In particular, \( |a| = |a + 1| = 1 \), which is possible only for \( a = e^{\pm 2i\pi/3} \). Since \( p \) has real coefficients, both values, which are conjugates of each other, are (the only) roots of \( p \) and occur with the same multiplicity. Thus

\[ p(x) = c((x - e^{2i\pi/3})(x - e^{-2i\pi/3}))^m = c(x^2 + x + 1)^m. \]

By the functional equation, \( c = 1 \), so \( p(x) = (x^2 + x + 1)^m \) for some \( m \in \mathbb{N} \).

**Answer:** \( p(x) = (x^2 + x + 1)^m, m \in \mathbb{N} \).
4. If a function \( f \) is analytic, then \( \frac{\partial f(z)}{\partial z} = 0 \) (this is in fact just a reformulation of the Cauchy-Riemann conditions). By the same token, \( \frac{\partial f(z)}{\partial \overline{z}} = 0 \). On the other hand, \( \frac{\partial f(z)}{\partial z} = f'(z) \). So,

\[
\frac{\partial}{\partial z} \frac{\partial}{\partial \overline{z}} |f_j(z)|^2 = \frac{\partial}{\partial z} f_j(z)f_j(z) = \frac{\partial}{\partial \overline{z}} f_j(z)f_j(z) = f'_j(z)f'_j(z) = |f'_j(z)|^2.
\]

Differentiating the right-hand side, we get \( \frac{\partial}{\partial z} \frac{\partial}{\partial \overline{z}} (az + b\overline{z} + c) = 0 \), so

\[
\sum_{j=1}^{n} |f'_j(z)|^2 = 0,
\]

hence each \( f'_j \) is zero, and since the domain \( D \) is connected, each \( f_j \) is constant in \( D \).

5. The set of all 0–1 matrices has cardinality \( 2^{n^2} \). From the definition of determinant,

\[
(det A)^2 = \left( \sum_{\sigma} \varepsilon(\sigma) \prod_{i=1}^{n} a_{i,\sigma(i)} \right) \left( \sum_{\tau} \varepsilon(\tau) \prod_{i=1}^{n} a_{i,\tau(i)} \right) = \sum_{\sigma,\tau} \varepsilon(\sigma)\varepsilon(\tau) \prod_{i=1}^{n} a_{i,\sigma(i)} a_{i,\tau(i)},
\]

where \( \varepsilon(\sigma) \) is \( \pm 1 \) depending on whether \( \sigma \) is even or odd. This sum can be rewritten more simply if we denote the permutation \( \sigma^{-1}\tau \) by \( \pi \). Then \( \varepsilon(\sigma)\varepsilon(\tau) = \varepsilon(\sigma^{-1})\varepsilon(\tau) = \varepsilon(\pi) \), and the formula becomes

\[
(det A)^2 = \sum_{\sigma,\pi} \varepsilon(\pi) \prod_{i=1}^{n} a_{i,\sigma(i)} a_{i,\sigma\pi(i)}.
\]

Let \( f(\pi) \) denote the number of indices fixed by \( \pi \). Then, for each \( \sigma \), the corresponding product has \( 2n - f(\pi) \) distinct factors, and there are \( 2^{n^2-2n+f(\pi)} \) matrices in which all these elements are equal to 1. So the expectation of each term is

\[
E\left( \prod_{i=1}^{n} a_{i,\sigma(i)} a_{i,\sigma\pi(i)} \right) = \frac{2f(\pi)}{2^{2n}}
\]

and, by the linearity of expectation,

\[
E((det A)^2) = \frac{n!}{2^{2n}} \sum_{\pi} \varepsilon(\pi)2f(\pi).
\]

It remains to find the sum in the right-hand side. Notice that that sum is the value of a determinant itself, namely, the determinant of the \( n \times n \) matrix \( C \) whose entries are

\[
c_{i,j} = \begin{cases} 
1 & i \neq j \\
2 & i = j.
\end{cases}
\]

The value of \( det C \) is \( n+1 \), which can be seen by replacing its first row by the sum of all rows, factoring out \( n+1 \) and then subtracting the first row from every other row. So, the resulting expectation value is \( E((det A)^2) = (n+1)!/2^{2n} \).

Answer: \( (n+1)!/2^{2n} \).