Hints for homework # 6.

1. Easy. Draw a picture and compare the x-coordinate of the center of the square with its y-coordinate.

2. Easy. Mean value theorem or direct integration.

3. Notice that the b_n's are second differences of the a_n's. Express the given series as a double series.

4. Look at consecutive terms where cos stays bounded away from zero.

5. Find nice expressions for a_n as a function of all preceding a_j's and for the sum $\sum_{n=1}^{N} 1/a_n$.

6. Very easy.

7. Rewrite each fraction $x_i/(x_{i+1}+x_{i+2})$ as

$$\frac{x_i + \frac{1}{2}x_{i+1}}{x_{i+1}+x_{i+2}} + \frac{\frac{1}{2}x_{i+1} + x_{i+2}}{x_{i+1}+x_{i+2}} - 1.$$

8. Write the assumption in terms of a condition on some polynomial.

9. Easy. Consider the preceding totals.

10. This is not hard. Use induction combined with the formula

$$\cos x + \cos 2x + \cdots + \cos nx = \frac{\sin((n+1/2)x) - \sin x/2}{2\sin x/2}.$$

11. This is somewhat tricky. Consider the number ϕ_x of sets containing the point x. Show first that $\sum_{x\in A_j} \phi_x \le n(k+1)$. Then estimate $\sum_{x\in A} \phi_x^2$ from above and from below.

12. Not too hard. First show, using parity, that f is π-periodic. Then consider $g := f(\cdot + \pi/2)$, check that it is even and that $f'g - fg'$ is constant.