Hints for homework # 6.

1. Straightforward.
2. Determine the distance between the least and the biggest square. This restricts your choices quite a bit.
3. Pigeonhole principle.
4. Use vectors.
5. Use congruences mod 3 and mod 5.
6. Use Binomial theorem.
7. The limit exists and is directly related to the sum of the doubly infinite series $\sum_{-\infty}^{\infty} a_n$.
8. Examine the behavior of the sequence x, x^2+c, $(x^2+c)^2+c$, $\ldots$, for various values of x and c.
9. Begin by considering two subsets, A and B and by putting into A all the terms that do not divide any other term of S.
10.
11. Hard. First solve the following auxiliary problem: Let a and b be integers and let f be a function that is positive in the interval $a \le x \le b$. Find the number of integer points in the region
$$a \le x \le b, \qquad 0 < y \le f(x).$$
12. May be hard. Consider a new inner product defined by
$$\langle x, y\rangle_{\text{new}} := \sum_{j=0}^{n-1} \langle T^j x, T^j y\rangle.$$
The new space is isomorphic to the original space.