Hints for homework # 5.

1. Consider dyadic sums $\sum_{j=2^{m-1}}^{2^m-1} a_j$.

2. The numerator is a difference of squares.

3. Consider congruences mod p for all primes p.

4. It is easy to open the safe in 48 trials. The actual minimum is 32, which needs more work.

5. Consider monochromatic intervals.

6. Define an inner product on the space of matrices.

7. Switch to contour integration.

8. Modify the matrix by adding some rank-one matrix times a variable to obtain a linear polynomial in that variable whose values at two points can be found easily. Then the original determinant is the value of that polynomial at zero.

9. By Stone-Weierstrass, we need to approximate the powers x^n in the ∞-norm. The obvious method gives the remainder $x^{n^{2^m}}$ which has value 1 at 1. Change coefficients slightly to avoid that problem.

10. Rolle's theorem.

11. Use Fourier series (I don't see a direct method, but it may be my fault).

12. Hard. Requires at least two facts: 1) if K is an algebraically closed field, char$K = 0$, and K is a finite extension of some proper subfield L, then $|K : L| = 2$; 2) every ordering of any subfield of the field of all algebraic numbers is Archimedean. The answer has to do with transcendentality and nonreality.