

Hints for homework # 1.

1. If f has an integer root m , f can be factored as

$$f(x) = (x - m)g(x),$$

where g is again a polynomial with integer coefficients.

2. What linear combination of the three integrals is equal to zero?
3. Rewrite the set $x^4 + 36 \leq 13x^2$ as $A \leq |x| \leq B$ for some numbers A and B .
4. Consider the binomial expansion of $(1 + x)^{2n}$.
5. Let $x := \frac{1}{n}$. Rewrite $\frac{n^2}{n^2+1}$ in terms of x .
6. $\det(A)$ is the product of all eigenvalues of A . What eigenvalues does a rank-one matrix have?
7. For each n , how many n -digit numbers do not contain digit 9?
8. Can the maximum value of a continuous function on each interval $[n, n + 1]$ stay equal to, say, 1 while the assumption of the problem is satisfied?
9. Use integration by parts to mollify the rapidly oscillating factor.
10. For every four points, consider the 16 configurations obtained by replacing some of the points by the opposites.
11. First count solutions to $x^2 = y^2 + 1$ by rewriting as $(x + y)(x - y) = 1$, and solving the system $x + y = r$, $x - y = r^{-1}$ for each nonzero r .
12. Consider $(\sum_{j=1}^r M_j)^2$.