1. Consider the vector space \( P \) of all polynomials in \( t \) and the subsets \( V \) consisting of those vectors (polynomials) \( f \) for which:

(a) \( f \) has exact degree 3,
(b) \( 2f(0) = f(1) \),
(c) \( f(t) \geq 0 \) whenever \( t \geq 0 \),
(d) \( f(t) = f(1-t) \) for all \( t \).

In which of these cases is \( V \) a subspace of \( P \)?

2. Suppose that \( m < n \) and that \( y_1, \ldots, y_m \) are linear functionals on an \( n \)-dimensional vector space \( V \). Under what conditions on the scalars \( \alpha_1, \ldots, \alpha_m \) is it true that there exists a vector \( x \) in \( V \) such that \([x, y_j] = \alpha_j \) for all \( j = 1, \ldots, m \)? What does this result say about solutions of linear equations?

3. Let \( T \) be a linear map on \( \mathbb{R}^2 \) with the matrix representation

\[
[T]_\mathcal{A} = \begin{bmatrix} 0 & 2 \\ 2 & -2 \end{bmatrix}
\]

in the standard basis \( \mathcal{A} \) and let

\( \mathcal{B} = \{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \} \).

Find the representation \([T]_\mathcal{B}\), the dual basis \( \mathcal{B}' \), and the matrix \([T']_{\mathcal{B}'}\).

4. What is the Jordan normal form of the differentiation operator on the space \( P_3 \) of polynomials of degree at most 3?