An extended block Arnoldi algorithm for large-scale solutions of the continuous-time algebraic Riccati equation

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In this talk, we present a new iterative method for the numerical solution of the continuous-time algebraic Riccati equation (CARE in short) of the form

\[ A^T X + X A - X B B^T X + C^T C = 0, \] (1)

where \( A \in \mathbb{R}^{n \times n} \) is nonsingular, \( B \in \mathbb{R}^{n \times p} \) and \( C \in \mathbb{R}^{s \times n} \). The matrices \( B \) and \( C \) are assumed to be of full rank with \( p \ll n, s \ll n \).

Riccati equations play a fundamental role in many areas such as control, filter design theory, model reduction problems, differential equations and robust control problems. In applications, the so-called stabilizing solution of (1) is desired. Such a solution \( X \) is symmetric positive semidefinite and satisfies the fact that the eigenvalues of the resulting closed-loop matrix \( A - B B^T X \) are in the open complex left-half plane (i.e., each eigenvalue of the matrix \( A - B B^T X \) has a negative real part). The stabilizing solution exists and is unique under certain assumptions on the problem.

Let \( x(t) \) be the state vector of dimension \( n \), \( u(t) \) the control vector of \( \mathbb{R}^p \) and \( y(t) \) the output vector of length \( s \). We consider the following problem:

Minimize

\[ J(x_0, u) = \frac{1}{2} \int_0^{+\infty} (y(t)^T y(t) + u(t)^T u(t)) \, dt, \] (2)

under the dynamic constrains

\[
\begin{aligned}
  \dot{x}(t) &= A x(t) + B u(t) \text{ with } x(0) = x_0. \\
  y(t) &= C x(t).
\end{aligned}
\] (3)

Under some hypotheses a unique optimal solution \( \tilde{u} \) that minimize the functional \( J(x_0, u) \) exists and can be determined through a feedback operator \( K \) such that \( \tilde{u}(t) = K x(t) \), where \( K = -B^T X \) and \( X \in \mathbb{R}^{n \times n} \) is the unique symmetric positive semidefinite and stabilizing solution of the matrix equation (1).

We present a new projection method that allows us to compute low rank approximations to the stabilizing solution of (1). We project the initial problem onto an extended block Krylov subspace, generated by the matrices \( A \) and \( A^{-1} \) and we obtain a low dimensional CARE that is solved by a standard algorithm such as the Schur method.

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