Brief solutions to MATH 54 mock midterm test

1. Subtracting the first equation from the second leads to $-4x_2 = 0$, while subtracting the first equation from the third leaves $x_2 = 1$, so the system is obviously inconsistent.

2. The determinant of any 3×3 matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

is $aei+bfg+cdh-ceg-bdi-afh$. We compute $\det A = 1\cdot 1\cdot 1+3\cdot(-1)\cdot 1-2\cdot(-1)\cdot 1-1\cdot(-2)\cdot 1 = 2$. Since $\det A$ is non-zero, A in invertible. Since $\det(A^TA) = \det(A)^2 = 4 \neq 0$, A^TA is also invertible.

3. Each of the matrices A and B has determinant 2, so, in particular, they are both invertible, hence row-equivalent to the identity matrix, and to each other.

4. (a) False. The nonzero rows do span the row space, however they may be linearly dependent, in which case they cannot form a basis.

(b) False. Elementary row operations on a matrix A or, equivalently, on the matrix A augmented by a zero column, i.e., $[A|0]$, do not change the solution set of the linear system $A\vec{x} = \vec{0}$, which is the same as the null space of A.

(c) False. The nonpivot columns of a matrix do depend linearly on the pivot colcolumns, however they can still form an independent set, taken by themselves.

(d) True. If A is onto, then its column space equals its codomain R^m, Therefore, the rank of A, which is the same as the dimension of its column space, equals m.